Computational Models
Winter Semester, 2003/4
Instructor: Benny Chor
T.A.: Gadi Kimmel

Assignment 3

Handed: 15/12/03, Due: 30/12/03

1. For each of the following languages, write down a complete description (including transition function) of a TM that decides it:

   (a) $L_1 = \{w \in \{0,1\}^* \mid w$ does not include two consecutive zeros$\}$

   (b) $L_2 = \{a^n b^n c^n \mid n \in \mathrm{N}\}$

2. Let A, B be languages, and suppose B is regular, and $A \leq_m B$. Is A also regular? Why or why not?

3. Prove each of the following statements, or disprove it by giving a counter example.

   (a) R is closed under infinite union, that is, if $L_1, L_2, \ldots \in R$ then $\bigcup_{i=1}^{\infty} L_i \in R$.

   (b) R is closed under infinite intersection.

   (c) RE is closed under infinite union.

   (d) RE is closed under infinite intersection.

4. Let $L_1, L_2 \in RE \setminus R$. Can it be that:

   (a) $L_1 \cap L_2 \in R$.

   (b) $L_1 \cup L_2 \in R$.

   (c) $L_1 \cap L_2 \in R$ and $L_1 \cup L_2 \in R$.

5. Prove that $L \in RE$ if and only if $L \leq_m halt_0$.

6. Let $L_1, L_2 \subseteq \{0,1\}^*$ be RE languages such that $L_1 \cup L_2 = \{0,1\}^*$ and $L_1 \cap L_2 \neq \emptyset$. Show that $L_1 \propto_m (L_1 \cap L_2)$.

7. On existential proofs vs. explicit constructions (**bonus**):

   The game of Ghung! is a two players game, played on an chocolate bar of size M-by-N, with a *poisonous* leftmost/topmost square. Players take turns picking chocolate squares. In her (or his) turn, a player must pick one of the remaining squares, and eat it along with all the squares that are "below it and to its right". Using matrix-notation, the poisonous square is denoted by entry $(1,1)$, and the initial "state" of the brand new bar consists of the whole bar $\{(i,j) | 1 \leq i \leq M, 1 \leq j \leq N\}$. Picking the square (i_0, j_0) means that one has to eat all the remaining squares (i,j) for which *both* $i \geq i_0$ and $j \geq j_0$ hold. The player that eats the poisonous (topmost/leftmost) square dies in excruciating pains, and consequently loses the game. Picking $(1, 1)$ to be your move kills you (in pains...), so a player who is non-suicidal will not play that move (unless she/he is forced to).

For example, if $M = 5$ and $N = 3$, then the initial game-position is

$$\begin{matrix} X & X & X \\ X & X & X \\ X & X & X \\ X & X & X \\ X & X & X \end{matrix} .$$

The first player may choose to play $(5, 3)$, in which case the chocolate bar (game) state becomes

$$\begin{matrix} X & X & X \\ X & X & X \\ X & X & X \\ X & X & X \\ X & X & \end{matrix} ,$$

or she may choose to play $(2, 2)$, which shrinks the chocolate bar to

$$\begin{matrix} X & X & X \\ X & X & X \\ X & & \\ X & & \\ X & & \end{matrix} ,$$

and so on.

A *strategy* for a player can be thought of as a table (possibly huge) which tells the player which move to make in each given position (state of the game). A well known result from game theory states that in every two person game like the one we deal with (*i.e.* finite and deterministic), either the first player or the second one has a *winning strategy.* This theorem applies, for example, to the game of chess (but we are still a long way from knowing which player has a winning strategy, let alone *describing* such winning strategy).

(a) Suppose $N > 1$ and our chocolate bar is a square of size N-by-N. Describe (in words) a simple winning strategy for the first player in a game of N-by-N Ghung!

(b*) Suppose $M, N > 1$ and our bar is a rectangle of size M-by-N. Prove the existence of winning strategy for the first player in an a game of M-by-N Ghung!

The existence proof we know of is very simple, short, and extremely elegant. Unfortunately, the proof gives *no clue* what that winning strategy might be. Finding such winning strategy for the general case ($M \neq N$) will make a very impressive M.Sc., or maybe even Ph.D., thesis.