Computational Models
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Assignment 2

Handed: 24/11/03, Due: 9/12/03

1. For each of the following languages decide whether it is regular, and prove your claims.

   (a) $L_1 = \{a^n b^m \mid m > n\}$, $\Sigma = \{a, b\}$.

   (b) $L_2 = \{a^n b^m \mid m < n\}$, $\Sigma = \{a, b\}$.

   (c) $L_3 = \{x\#y\#z \mid x, y, z \in \{0,1\}^* \wedge x + y = z\}$, $\Sigma = \{0, 1, \#\}$.

   (d) $L_4 = \{x_1 y_1 z_1 \ldots x_n y_n z_n \,|\forall i : x_i, y_i, z_i \in \Sigma,\ x = x_1 \ldots x_n, y = y_1 \ldots y_n, z = z_1 \ldots z_n \wedge x + y = z\}$, $\Sigma = \{0, 1\}$.

2. Let $P = \{w : w \in \{0,1\}^*, w = w^R\}$. For each of the following languages prove regularity or non-regularity.

   (a) PP.

   (b) P^*.

3. In each of the following items, L is a regular language. Decide whether L' is also regular, and prove your claim.

   (a) $L' = \{xy \,|x, y \in \Sigma^*,\ x \in L \wedge y \notin L\}$

   (b) $L' = \{xy \,|x, y \in \Sigma^*,\ \exists a \in \Sigma : xay \in L\}$

   (c) $L' = \{x_1 x_3 x_5 \ldots x_{2n-1} \,|\forall i : x_i, y_i, z_i \in \Sigma,\ \exists x_2 x_4 x_6 \ldots x_{2n} : x_1 x_2 \ldots x_{2n-1} x_{2n} \in L\}$

4. In each of the following items, r is a regular expression. Describe $L(r)$ in words, and write a regular expression for $\overline{L(r)}$.

   (a) $r = (0 \cup 1)^* 1 (0 \cup 1)^* 0 (0 \cup 1)^*$

   (b) $r = (1 \cup 01 \cup 001)^* (\epsilon + 0 \cup 00)$

5. Let $L = \{a^i b^j c^k \mid i, j, k \in \mathrm{N}, i + k = j\}$:

   (a) Write a Context Free Grammar (CFG) G, such that $L = L(G)$.

   (b) Describe a Push Down Automata (PDA) M, such that $L = L(M)$.

6. For each of the following languages, decide whether it is Context Free or not. Prove your claims.

   (a) $L_1 = \{x \in \{0,1\}^* \mid \#_0(x) = \#_1(x)\}$

   (b) $L_2 = \{a^n b^{2n} c^{3n} \mid n \in \mathrm{N}\}$

   (c) $L_3 = \{x_1 \# x_2 \# \ldots \# x_n \mid n \geq 2 \wedge \forall i : x_i \in \{0,1\}^* \wedge \exists i \neq j : x_i = x_j\}$

   (d) $L_4 = \{x_1 \# x_2 \# \ldots \# x_n \mid n \geq 2 \wedge \forall i : x_i \in \{0,1\}^* \wedge \exists i \neq j : x_i = x_j^R\}$

7. For each of the following operations:

   (a) Decide whether the regular languages are closed under it.

   (b) Decide whether the context free languages are closed under it.

   Prove your claims:

   (a) Reverse$(L) = \{w \mid w^R \in L\}$

   (b) DropMiddle$(L) = \{xy \mid |x| = |y| \wedge \exists a \in \Sigma : xay \in L\}$

   (c) PP$(L) = \{x \mid x \in L \wedge$ no proper prefix of x is in L$\}$

8. Show that the language $F = \{a^i b^j c^k | i, j, k \geq 0$ and if $i = 1$ then $j = k\}$ satisfies the three conditions of the pumping lemma even though it is not regular. Explain why this fact does not contradict the pumping lemma.

9. Bonus: Give a family of languages E_n, where each E_n can be recognized by an n-state NFA but requires at least c^n states on a DFA for some constant $c > 1$. Prove that your languages has this property.