Write short but full and accurate answers. Each solution should appear on a separate page and each of its parts should not exceed a page.

1. Consider the on-line load balancing problem on (even) $m$ related machines. The speed of each of the first $m/2$ machines is 1 and the speed of each of the other $m/2$ machines is 100. The goal is to minimize the maximum load.

   (a) Show a deterministic 2.01 competitive algorithm.
   (b) Show a lower bound of $5/3$ for any deterministic algorithm for any (even) $m \geq 6$.

2. Consider the on-line load balancing problem of tasks on $m$ machines in the restricted assignment model. Consider the case where for each $i$ the set of machines which is associated with job $i$ is $[1 \ldots k_i]$ for some $1 \leq k_i \leq m$. The goal is to minimize the maximum load.

   (a) Design a constant competitive algorithm.
   (b) Show a lower bound of 2 for deterministic algorithms, for any $m \geq 6$, using only unit jobs. Hint: start with $m = 6$. (you will a partial credit for a lower bound of 11/6.)
   (c) Show a lower bound of $2 - O(1/m)$ for randomized algorithms, for any $m$, using only unit jobs.

3. Consider the on-line load balancing problem in the restricted assignment model where all jobs have unit size. The goal is to minimize the maximum load.

   (a) Design an algorithm which is at most $\log m + 2$ competitive.
   (b) Assume that all jobs are of sizes between 1 and $1 + \epsilon$ for some fixed $\epsilon > 0$. Design $(1 + \epsilon) \log m + O(1)$ competitive algorithm.

4. We are given a connected graph $G = (V, E)$. All edges have unit capacity. At round $i$ we receive a request to connect $s_i$ to $t_i$ with bandwidth $q_i$ on a path $Q_i$ or with bandwidth $r_i$ on a path $R_i$ ($Q_i$ and $R_i$ are given disjoint paths from $s_i$ to $t_i$).

   (a) Assume we must assign the request to one of the two paths and the goal is to minimize the maximum load over the edges. Design an $O(\log |V|)$ competitive algorithm.
   (b) We may either assign the request to one of the two paths or reject it and pay $\beta_i > 0$ for the rejection. The goal is to minimize the sum of the costs of all rejected requests plus the maximum load. Design an $O(\log |V|)$ competitive algorithm.

   **Remark:** Do not re-prove theorems proved in class for (a)(b).

5. Consider the problem of on-line load balancing of jobs on $kr$ machines that are partitioned into $k$ groups, each consists of $r$ identical machines. Job $i$ has a weight vector of length $k$ were $w_{ij}$ (for $1 \leq j \leq k$) is the load that the job adds to a machine if it is assigned to a machine in the group $j$.

   (a) Show an $O(\log k)$ competitive algorithm where the goal is to minimize the maximum group volume (group volume is the total load of machines in a group).
   (b) Show an $O(\log k)$ competitive algorithm for minimizing the maximum load over all machines (Note that a solution that only balances the volume between groups cannot be transformed in general into a solution that balances the loads).
   (c) Show a lower bound for minimizing the maximum load over all machines of $\Omega(\log k)$ for any $k$ and $r$.

Exercise # 2 is due May 24, 2020 at 7PM.