Algorithmic Methods	**Spring Semester, 2012/13**

Final Exam: July 4, 2013

Lecturer: Prof. Yossi Azar

Solve **4 out of the 5** questions. Write short but full and accurate answers. Each question should start on a new page and each of its parts should not exceed a page. No extra material is allowed.

1. We are given a cycle of n nodes and edges and requests from s_i to t_i of demand d_i for $1 \leq i \leq m$. Each request should be routed either clockwise or counterclockwise. The load on an edge is the some of the demands routed through the edge.
   (a) (20 points) Design a 2-approximation algorithm if the goal function is minimizing the maximum load.
   (b) (5 points) What can be achieved if the goal function is the sum of the loads of all edges in the cycle.

2. We are given an undirected connected Graph $G = (V, E)$ with a non-negative capacity w_e on each edge $e \in E$ and an integer Z. The cost of increasing the capacity of an edge e by each unit is c_e for each $e \in E$. A cut in the graph is a partition of all the nodes to two non empty disjoint sets S and T. The capacity of a cut is $\sum_{\{e=(u,v)\in E | u \in S \& v \in T\}} w_e$. One need to modify (increase) the capacities of the edges with minimum total cost such that the capacity of each cut is at least Z .
   (a) (20 points) Form an LP for the problem and show how to solve it by a polynomial time algorithm.
   (b) (5 points) Assume that the goal function is to minimize the maximum cost of changing any edge. Show a poly-time algorithm without LP assuming $c_e = 1$ for all $e \in E$ and all w_e are integers.

3. Consider an approximation algorithm for MAX-SAT which is based on LP and randomized rounding. Let x be the LP optimal fractional solution.
   (a) (2 points) Show that $1 - 4^{a-1} \leq 4^{-a}$ for any a.
   (b) (18 points) Show that if we randomly round each variable independently according to the function $p_i = 4^{x_i - 1}$ then we get an 3/4-approximation for MAX-SAT.
   (c) (5 points) Show that this algorithm does not achieve a better than 3/4 approximation by providing for every n an example of a formula on n variables where each variable appears at least once and the algorithm achieves exactly 3/4.

4. Suppose we should assign n jobs to m machines with rational speeds in the range 1 to 2 (not necessarily integers). Assigning a job of weight w_i on machine j increases its load by w_i/v_j where v_j is the speed of machine j. Recall that PTAS stand for poly-time approximation scheme
   (a) (20 points) Describe a PTAS for minimizing the maximum load.
   (b) (5 points) Explain why your algorithm does not work for machines with arbitrary speeds (say in the range 1 to n).

5. We are given time intervals (s_i, t_i) with a value v_i and a parameter $w_i \leq 1/2$ for $1 \leq i \leq n$. The bandwidth of the an interval i is increasing linearly from 0 to w_i for $s_i \leq t \leq t_i$. More precisely, at time t its bandwidth is $\frac{t - s_i}{t_i - s_i} w_i$ where $s_i \leq t \leq t_i$ (for $t < s_i$ or $t > t_i$ its bandwidth is 0). A feasible solution consists of a subset of the intervals such that the total bandwidth at each time t is at most 1.
   (a) (20 points) Design a 2 approximation algorithm for a feasible solution with maximum total value.
   (b) (5 points) How to modify the solution if the bandwidth is $\frac{t_i - t}{t_i - s_i} w_i$ for $s_i \leq t \leq t_i$ (i.e decreasing linearly).

The duration of the exam is 3 hours. GOOD LUCK