Algorithmic Methods	**Fall Semester, 2010/11**

Final Exam: January 31, 2011

Lecturer: Prof. Yossi Azar

Solve **4 out of the 5** questions. Write short but full and accurate answers. Each question should start on a new page and each of its parts should not exceed a page. No extra material is allowed.

1. We are given n jobs and m unrelated machines. The load of job i on machine j is w_{ij}. The load of a machine is the sum of the weights of the jobs assigned to it. In contrast to the standard problem here each job i has two copies and they should be assigned exactly to TWO **different** machines say $j_1 \neq j_2$ (then the load of j_1 would increase by w_{ij_1} and the load j_2 would increase by w_{ij_2}). The goal is to minimize the maximum load.
   (a) Write the appropriate LP formulation.
   (b) Round the LP and provide a 2 approximation algorithm. (recall that the two machines each job is assigned to must be **different**)
2. We are given a DAG-Directed Acyclic Graph $G = (V, E)$ (directed graph with no directed cycles) with non-negative weight w_e on each edge $e \in E$. The cost of increasing or decreasing the weight of an edge e by each unit is c_e for each $e \in E$. One need to modify the weights (increase or decrease) such that for all vertices $u, v \in V$ the lengths of any two paths from u to v (if exist) differ by a factor of at most 2. The weight of each edge must stay non-negative after the modification. The goal is to minimize total cost of the modification.
   (a) Form an LP for the problem and show how to solve it by a polynomial time algorithm.
   (b) How (a) and (b) would change if the initial weights as well as the final weights may also be negative.
3. Suppose we are given a graph $G = (V, E)$ with arbitrary degrees. Each vertex has at most 4 neighbors whose degree is more than 4. Each vertex has a different i.d (which initially is unknown to the others) between 0 to $2^n - 1$ where $|V| = n$. Recall that a local algorithm with k rounds is an algorithm where each vertex decides on its output after k synchronized communication rounds with its neighbors. Find a local algorithm that colors the graph in 10 colors in $\log^* n + O(1)$ rounds. Do not forget to prove that the coloring is valid.
4. (a) Consider an approximation algorithm for MAX-SAT which is based on LP and randomized rounding. Let x be the LP optimal fractional solution. Show that if we randomly round each variable independently according to the function $p_i = 1 - 4^{-x_i}$ then we get an 3/4-approximation for MAX-SAT.
   (b) Consider an approximation algorithm for MIN-SAT (CNF formula) problem where your goal is to find an assignment for the variables that **minimizes** the total weight of satisfied clauses. Write a fractional LP formulation and show a **simple** deterministic rounding for the LP that yields an 2-approximation for the MIN-SAT.
5. We are given an undirected graph $G = (V, E)$ with non-negative weights on the edges and non-negative penalties on the nodes. The goal is to find a non-empty tree T (i.e. a tree with at least one node) of minimum cost where a cost of a tree T is the sum of the weights of its edges plus the sum of the penalties of the nodes that are NOT in the tree.
   (a) Consider a degenerate instance where all penalties are either 0 or 1. In addition all edges have weights equal to the sum of penalties of its two vertices (i.e. the weight is either 0 or 1 or 2). Define a "green" solution and show that any "green" solution is 2-approximation.
   (b) Design a 2 approximation algorithm for the general problem using the local ratio based on (a).

The duration of the exam is 3 hours. GOOD LUCK