Write short but full and accurate answers. Each question should start on a new separate page and each of its parts should not exceed a page.

1. We are given \( n \) jobs and \( m \) unrelated machines. The load of job \( i \) on machine \( j \) is \( w_{ij} \). The load of a machine is the sum of the weights of the jobs assigned to it. In contrast to the standard problem here each job \( i \) has two copies and they should be assigned exactly to TWO different machines say \( j_1 \neq j_2 \) (then the load of \( j_1 \) would increase by \( w_{ij_1} \) and the load \( j_2 \) would increase by \( w_{ij_2} \)). The goal is to minimize the maximum load.

   (a) Write the appropriate LP formulation.
   (b) Round the LP and provide a 2 approximation algorithm. (recall that the two machines each job is assigned to must be different)

2. Suppose we are given a regular graph \( G = (V, E) \) of degree \( \Delta \). Each vertex has a different i.d (which initially is unknown to the others) between 0 to \( 2^n - 1 \) where \( |V| = n \). Recall that a local algorithm with \( k \) rounds is an algorithm where each vertex decides on its output after \( k \) synchronized communication rounds with its neighbors. Find a local algorithm that colors the graph in \( \Delta + 1 \) colors in \( \log^* n + 2^{O(\Delta)} \) rounds.
   Remark: a solution in \( \log^* n + 2^{O(\Delta \log \Delta)} \) rounds will receive almost all points.

3. You are given a set of tasks where task \( i \) has a width \( b_i \) and a benefit \( v_i \) for \( i \in \{1, 2, \ldots, n\} \). For some fixed \( k \) task \( i \) is associated with intervals set, \( \{(x^1_i, y^1_i), (x^2_i, y^2_i), \ldots, (x^k_i, y^k_i)\} \) where \( x^j_i < y^j_i \) for all \( 1 \leq j \leq k \). A feasible solution is a set \( S \subseteq \{1, 2, \ldots, n\} \) and \( j_i \in \{1, \ldots, k\} \) for each \( i \in S \) such that for any \( t \) we have \( \sum_{i \in S, x^j_i < t < y^j_i} b_i \leq 1 \). The benefit of the solution is \( \sum_{i \in S} v_i \). The goal is to find a feasible subset with maximum benefit. Design a 5 approximation algorithm.

4. We are given a tree and requests \((s_i, t_i)\) with a bandwidth \( w_i \) and a value \( v_i \) for \( 1 \leq i \leq n \). The goal is to maximize the total value of requests in a feasible subset.
   (a) Design a 3 approximation algorithm where a feasible set is a subset of the requests with total bandwidth at most 1 on each vertex.
   (b) Modify the algorithm and the proof to design a 6 approximation algorithm where a feasible set is a subset of the requests with bandwidth at most 1 on each edge.

Exercise # 3 is due Jun 25, 2023 at 11pm.