Algorithmic Methods	**Spring Semester, 2022/23**

Exercise 3: Jun 12, 2023

Lecturer: Prof. Yossi Azar

Write short but full and accurate answers. Each question should start on a new **separate** page and each of its parts should not exceed a page.

1. We are given n jobs and m unrelated machines. The load of job i on machine j is w_{ij}. The load of a machine is the sum of the weights of the jobs assigned to it. In contrast to the standard problem here each job i has two copies and they should be assigned exactly to TWO **different** machines say $j_1 \neq j_2$ (then the load of j_1 would increase by w_{ij_1} and the load j_2 would increase by w_{ij_2}). The goal is to minimize the maximum load.
   (a) Write the appropriate LP formulation.
   (b) Round the LP and provide a 2 approximation algorithm. (recall that the two machines each job is assigned to must be **different**)

2. Suppose we are given a regular graph $G = (V, E)$ of degree Δ. Each vertex has a different i.d (which initially is unknown to the others) between 0 to $2^n - 1$ where $|V| = n$. Recall that a local algorithm with k rounds is an algorithm where each vertex decides on its output after k synchronized communication rounds with its neighbors. Find a **local** algorithm that colors the graph in $\Delta + 1$ colors in $\log^* n + 2^{O(\Delta)}$ rounds.
   Remark: a solution in $\log^* n + 2^{O(\Delta \log \Delta)}$ rounds will receive almost all points.

3. You are given a set of tasks where task i has a width b_i and a benefit v_i for $i \in \{1, 2, \ldots, n\}$. For some fixed k task i is associated with intervals set, $\{(x_i^1, y_i^1), (x_i^2, y_i^2), \ldots, (x_i^k, y_i^k)\}$ where $x_i^j < y_i^j$ for all $1 \leq j \leq k$. A feasible solution is a set $S \subseteq \{1, 2, \ldots, n\}$ and $j_i \in \{1, \ldots, k\}$ for each $i \in S$ such that for any t we have $\sum_{i \in S, x_i^{j_i} < t < y_i^{j_i}} b_i \leq 1$. The benefit of the solution is $\sum_{i \in S} v_i$. The goal is to find a feasible subset with maximum benefit. Design a 5 approximation algorithm.

4. We are given a tree and requests (s_i, t_i) with a bandwidth w_i and a value v_i for $1 \leq i \leq n$. The goal is to maximize the total value of requests in a feasible subset.
   (a) Design a 3 approximation algorithm where a feasible set is a subset of the requests with total bandwidth at most 1 on each vertex.
   (b) Modify the algorithm and the proof to design a 6 approximation algorithm where a feasible set is a subset of the requests with bandwidth at most 1 on each edge.

Exercise # 3 is due Jun 25, 2023 at 11pm.