

Exercise 2: May 15, 2023

Lecturer: Prof. Yossi Azar

Write short but full and accurate answers. Each section of a question should start on a new **separate** page and should not exceed a page.

1. We are given a cycle of n nodes and edges and requests from s_i to t_i of demand d_i for $1 \leq i \leq m$. Each request should be routed either clockwise or counterclockwise. The load on an edge is the sum of the demands routed through the edge.
 - (a) Design a 2-approximation algorithm if the goal function is minimizing the maximum load.
 - (b) What can be achieved if the goal function is the sum of the loads of all edges in the cycle.
2. We are given an undirected connected graph $G = (V, E)$ with a non-negative capacities c_e for each edge $e \in E$. You need to pack (fractionally) spanning tree, i.e. assign each tree a non-negative weight such that the total load on any edge does not exceed its capacity. The goal is to maximize the sum of the weights of the packed spanning trees.
 - (a) Show that there exists an optimal solution in which at most $|E|$ spanning trees get non-zero weights (Hint: you may write LP with non-polynomial number of variables).
 - (b) Show how to find the value of the optimal solution in polynomial time. (Hint: dual - you do not need to find the trees but only the optimal value).
 - (c) Assume we can add (fractional) capacities to the edges such that the total addition to all edges is at most 2. Find the new value of the optimal solution of the packing in polynomial time.
3. Consider the $1 - 1/e$ approximation algorithm for MAX-SAT which is based on LP and randomly rounding using the function $p_i = x_i$ where x is the LP optimal fractional solution. The scheme was combined with random assignment to get a $3/4$ -approximation algorithm.
 - (a) Show how to get a $3/4$ -approximation by a deterministic algorithm.
 - (b) Combine the scheme and the random assignment in a slightly different way to get a randomized 0.77 -approximation algorithm for MAX-SAT without clauses of size exactly 2.
4. Consider an approximation algorithm for MAX-SAT which is based on LP and randomized rounding. Let x be the LP optimal fractional solution.
 - (a) Show that if we randomly round each variable independently according to the function $p_i = 4^{x_i-1}$ then we get an $3/4$ -approximation for MAX-SAT (first show that $1 - 4^{a-1} \leq 4^{-a}$ for any a).
 - (b) Show that this algorithm does not achieve a better than $3/4$ approximation by providing for every n an example of a formula on n variables where each variable appears at least once and the algorithm achieves exactly $3/4$.
5. We are given a DAG-Directed Acyclic Graph $G = (V, E)$ (directed graph with no directed cycles) with non-negative weight w_e on each edge $e \in E$. The cost of increasing or decreasing the weight of an edge e by each unit is c_e for each $e \in E$. One need to modify the weights (increase or decrease) such that for all vertices $u, v \in V$ the lengths of any two paths from u to v (if exist) differ by a factor of at most 2. The weight of each edge must stay non-negative after the modification. The goal is to minimize total cost of the modification.
 - (a) Form an LP for the problem and show how to solve it by a polynomial time algorithm.
 - (b) Assume that the initial weights may be negative but the modified weights should be non-negative. Show how to use the solution to case (a) as a black box to solve fast the new problem.

Exercise # 2 is due May 28, 2023 at 11pm.