Write short but full and accurate answers. Each question should start on a new page and each of its parts should not exceed a page. Solving 5 questions results in score 90. One page A4 is allowed.

1. Suppose you are in the middle of an infinite building. To go up or down one floor it takes one minute. To scan a floor it takes 2 minutes. You need to find some treasure in some unknown floor. You would find it once you scan completely the correct floor. When you go up or down you do not necessarily need to scan all floors. The optimum can go (up or down) directly to the correct floor and scan completely only that floor. The goal is to minimize the time until you find the treasure. Design a 15 competitive algorithm for the problem (no additive constant is allowed).

2. Consider the following algorithm MoveMid for the list update problem. Given a request to an element \(x\) at position \(i\), MoveMid moves \(x\) to position \(\lfloor i/2 \rfloor\).
   
   (a) Show that MoveMid is 4 competitive.
   
   (b) Show that MoveMid is at least \(4 - O(1/n)\) competitive where \(n\) is the size of the list (this can be done by an example of a request sequence for any given \(n\)).

   Hint: consider what the algorithm is doing when you request the last element of the list.

3. Consider the on-line load balancing problem on \(m\) (divisible by 3) machines. Job \(i\) has weight \(w_i\) and a subset \(S_i\) of size \(m/3\) of the \(m\) machines. The job has to be assigned to one machine in \(S_i\).
   
   (a) Design a 4 competitive algorithm.
   
   (b) Show a deterministic lower bound of 2 for any \(m \geq 6\) (divisible by 3). Hint: start with \(m = 6\).

4. Consider the on-line load balancing problem in the restricted assignment model where all jobs have unit size. The goal is to minimize the maximum load.
   
   (a) Design an algorithm which is at most \(\log m + 2\) competitive.
   
   (b) Show a lower bound of 3 for \(m = 4\) for any deterministic algorithm.

5. Consider the following variant of on-line load balancing of permanent jobs on \(m\) machines. We are given some \(1 \leq k \leq m\). To serve a request you need to assign the job to precisely \(k\) different machines out of the \(m\). If job \(i\) is assigned to machine \(j\) then the job adds a load of \(w_{i,j}\) on that machine. The goal is to minimize the maximum load.
   
   (a) Assume that \(k = 3\) and the indexes of the three machines to which each job is assigned must be consecutive. Design an \(O(\log m)\) competitive algorithm.
   
   (b) Design an \(O(\log m)\) competitive algorithm for the case that \(k\) is arbitrary and the \(k\) machines to which each job is assigned are arbitrary.

Remark: Do not re-prove theorems proved in class for (a)(b).

6. Consider an admission control problem for requests on a line which consists of \(n\) unit capacity edges. A request is a segment with a bandwidth. The goal is to maximize the throughput, i.e. the total bandwidth of the accepted requests.
   
   (a) For the case \(n = 1\) (single edge) design a 2 competitive deterministic non-preemptive algorithm for requests of bandwidth of at most \(1/2\).
   
   (b) Design an \(O(\log n)\) competitive randomized non-preemptive algorithm for requests of bandwidth of at most \(1/2\).
   
   (c) Design an \(O(\log n)\) competitive randomized non-preemptive algorithm for requests of any bandwidth.

   Hint: first solve the case for which all requests are of bandwidth larger than \(1/2\).

The duration of the exam is 3 hours. GOOD LUCK