Algorithmic Methods 10/01/2011

Lecture Notes 13: PTAS for identical machines sched. (cont.)

Professor: Yossi Azar *Scribe:Yoni Moses*

Introduction

In this (short) lecture, we'll continue showing a PTAS (polynomial-time approximation scheme) for scheduling on identical machines.

Discrete Assignment

We need to solve the (accurate) decision problem for discrete values. Given an input of jobs of different sizes, we want to find a job assignment for which the max load is at most T.
The input is a vector $\vec{n} = (n_1, n_2, ..., n_Q)$, where n_i is the number of jobs of size t_i, for $t_k = \epsilon \cdot T + k \cdot \epsilon^2 \cdot T$ and $0 \leq k \leq \frac{1-\epsilon}{\epsilon^2}$.

First, we'll define $R = \{\vec{r} | \vec{r} = (r_1, r_2, ..., r_Q), \sum r_k \cdot t_k \leq T\}$, the collection of vectors of jobs that can be assigned on a single machine. In other words, each vector in R is a feasible job assignment for a single machine.
Since we can only assign $\frac{T}{\epsilon \cdot T} = \frac{1}{\epsilon}$ jobs on a single machine, and the number of job types is Q, R's size is $|R| \leq |Q|^{\frac{1}{\epsilon}} \leq (\frac{1}{\epsilon^2})^{\frac{1}{\epsilon}}$.

Next, we'll define the boolean function $V(\vec{n}, k)$ whose value is true iff the jobs in $\vec{n}$ can be performed by k machines: $V(\vec{n}, k) = \vee_{r \in R} V(\vec{n} - \vec{r}, k-1)$.
This can be computed using dynamic programming. Note that the layer size is bounded, i.e the number of vectors in each layer is bounded: The original input vector is $(n_1, n_2, ..., n_Q)$ and the other vectors are $(n'_1, n'_2, ..., n'_Q)$ where $n'_i \leq n_i$. Also, $n_i \leq n$.
Therefore, the number of vectors in each layer is bounded by $(n+1)^Q$, and the total run time is $O(n^{\frac{1}{\epsilon^2}} \cdot (\frac{1}{\epsilon^2})^{\frac{1}{\epsilon}} \cdot m)$.
Explanation: the first member of the product is the layer size. The second member is the number of possibilities in the big-or ($\vee$). The third member is the number of layers (which is the number of machines).