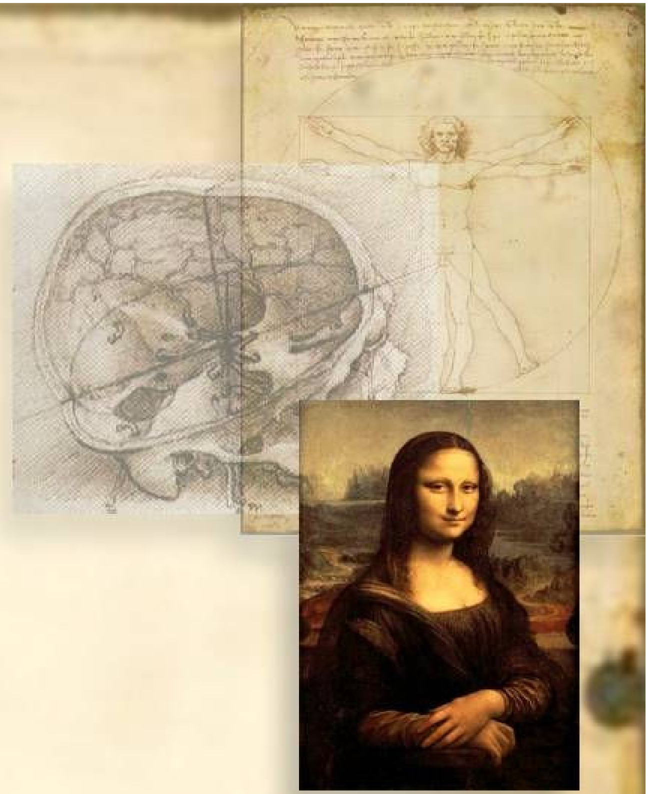
Applications of Epistemic Logic

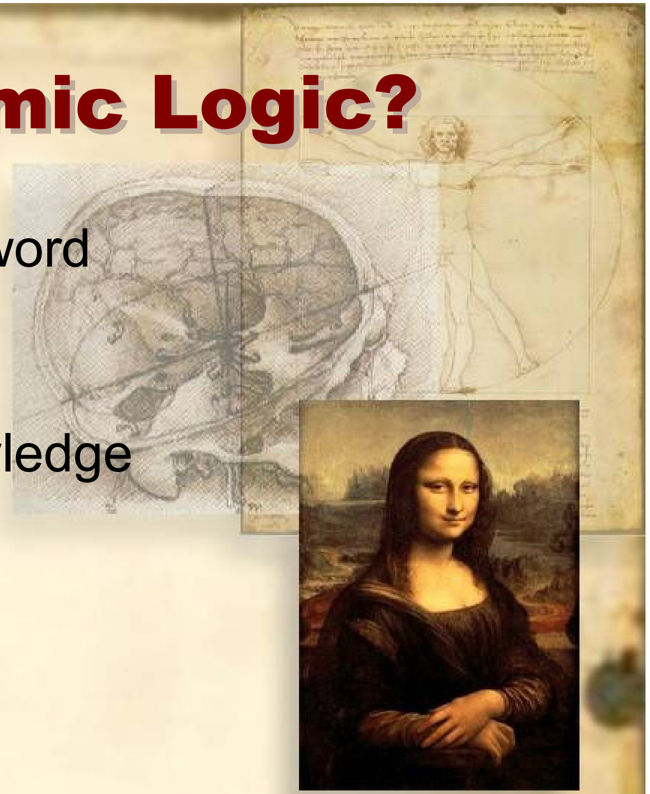
By Hadar Kaufman

Today

- Introduction – Knowledge and Belief
- Motivation – Why Epistemic Logic?
- Semantics – introducing of two conceptualizations:
  - **Sentential** conceptualization
  - **Possible Worlds** conceptualization
- Applications of Epistemic Logic



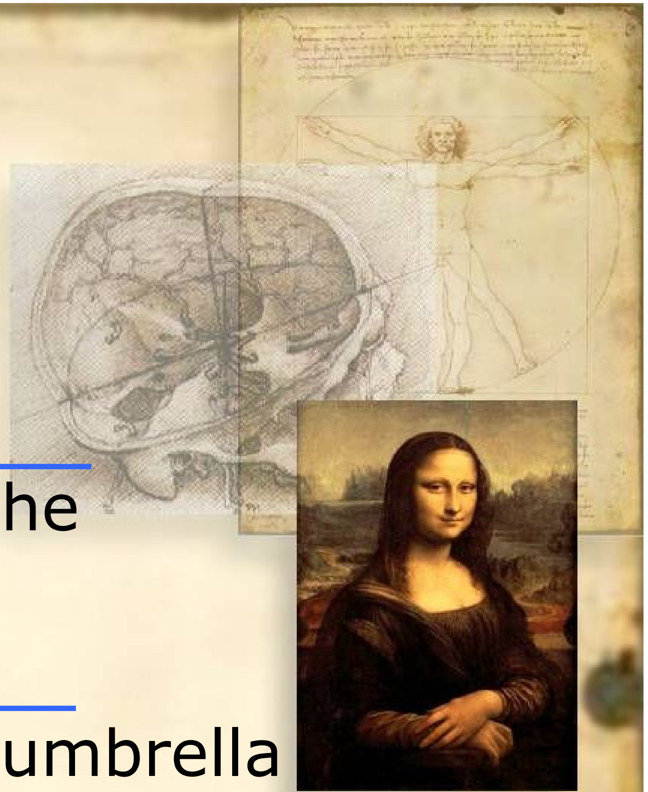
Introduction – What is Epistemic Logic?



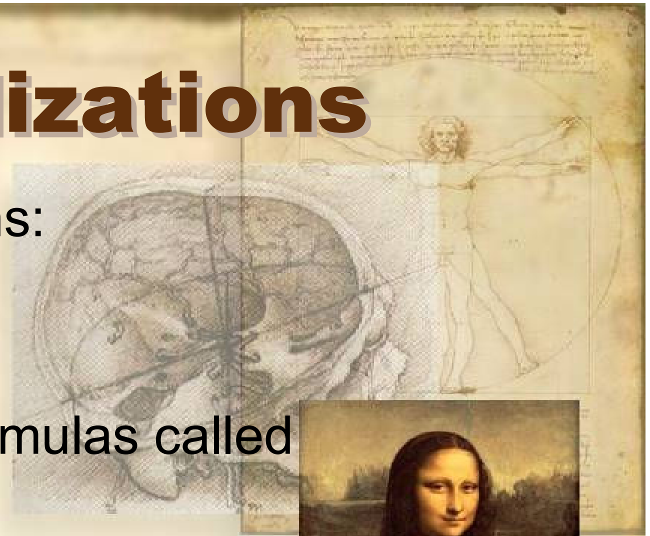
- The term “**Epistemic**” is based on the Greek word “*Epistem*” – knowledge.
- Classical Logic cannot handle belief and knowledge statements
  - Not enough delicate
  - Lack of conceptualization
- Epistemic Logic defines special conceptualizations to deal with knowledge and belief.
- We will distinguish between *knowledge* concepts to *belief* concepts – Can you think why?

Motivation – Example

- Recall this example:
  - John knows it's raining. (p)
  - John knows that if it's raining, he should take an umbrella. (q)
  - John knows he should take an umbrella (r)
  - Does $p, q \vdash_{CPL} r$? **NO!** $p, q \nvdash_{CPL} r$



Our Two Main Conceptualizations



- We will present two different conceptualizations:
  - <u>**Sentential Conceptualization**</u>
    - Associates with each agent a set of formulas called the agent's *base beliefs*.
    - An agent <u>**believes**</u> a proposition just in case the agent can <u>**prove**</u> the proposition from his base beliefs.
  - <u>***Possible Worlds Conceptualization***</u>
    - Associates with each agent set of possible worlds.
    - An agent <u>**knows**</u> in a proposition only if the proposition is true in each world that is <u>**accessible**</u> from his own world.

B & K Modal Operators

- We want to handle propositions of this form:

  A   **B**

  – Yossi Believes that it's raining outside

- We shall define new modal operator to enrich the classical logic – the **B** operator:

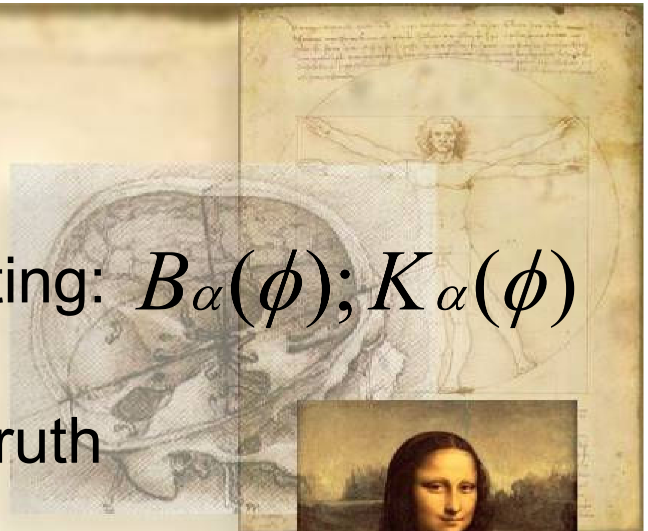
B (Yossi, it's raining outside)

Our agent   His belief

- In a same way, we define the **K** modal operator:

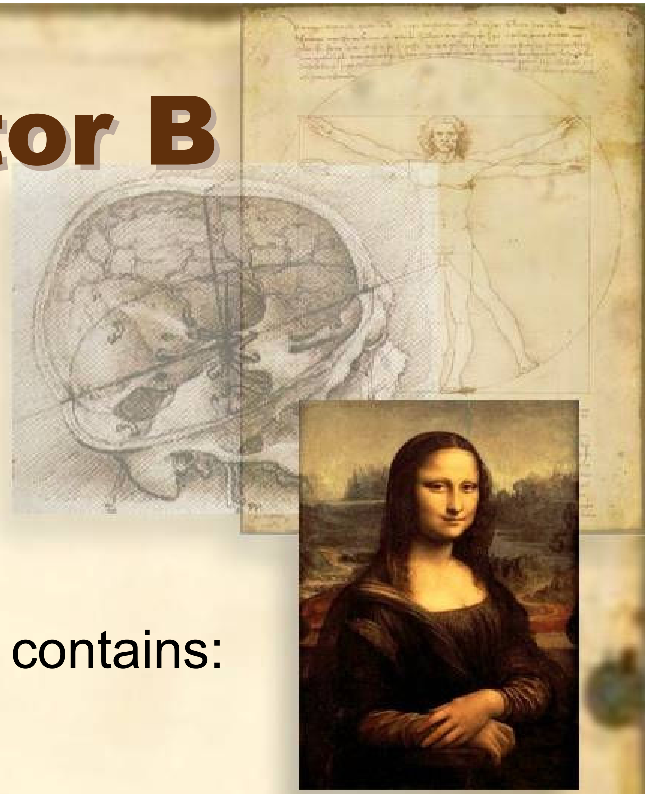
K (Yossi, It's raining outside)

B & K Modal Operators



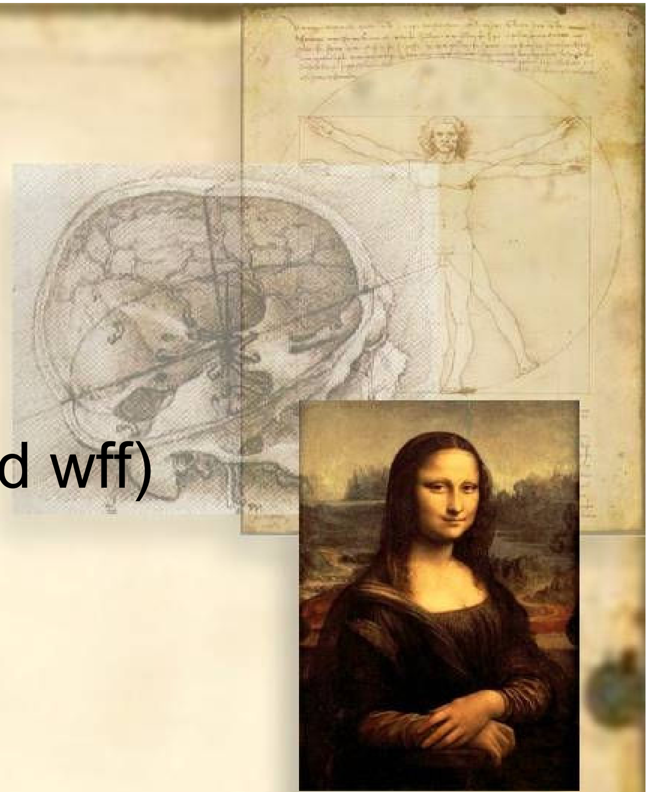
- We will sometimes use this form of writing: $B_\alpha(\phi); K_\alpha(\phi)$
- A reminder: Modal Operators are not Truth Functional. Therefore we will present them a special semantics which is not based on truth tables.
- Note that the difference between both conceptualization is how to define the semantics of **K** & **B**

Using the Modal Operator B



- “X believes Y”
- We need to present **syntax** and **semantics**
- **Syntax:**
  - New formal language is based on FOL and contains:
    - All ordinary wff are wff
    - If ϕ is a an ordinary, closed wff (one with no free variables) and if α is a ground term, then $B(\alpha,\phi)$ is a wff and is called ***belief atom***.
    - If ϕ and ψ are wffs, then so are any expressions constructed from them by the usual propositional connectives.

Examples



- The following are **NOT** valid wffs:
  - $\exists x\ \boldsymbol{B}(R, P(x))$ ($P(x)$ is not a closed wff)
  - $\boldsymbol{B}(R1, \boldsymbol{B}(R2, P(A)))$

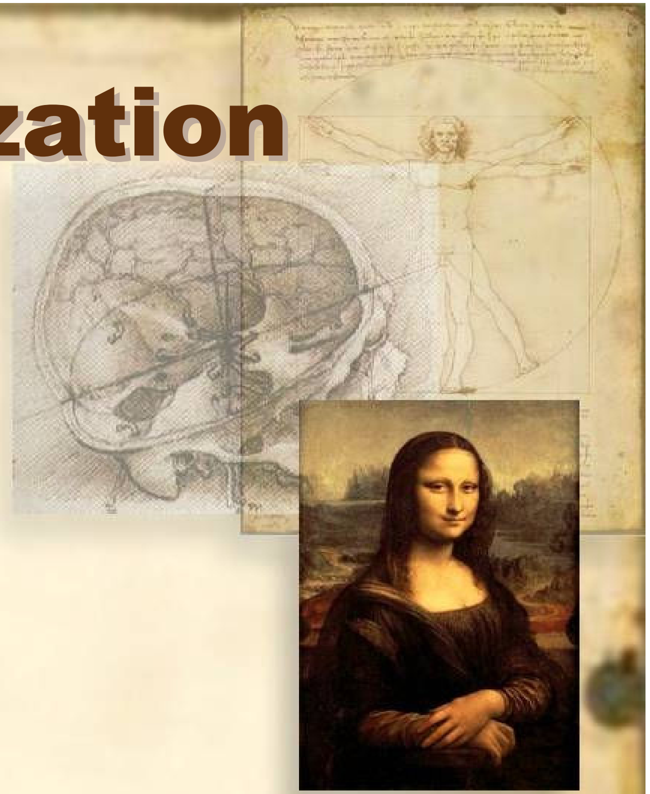
    $\boldsymbol{B}(R2, P(A))$ is not an ordinary wff)

  –($\boldsymbol{B}((\exists x\ G(x)), P(A))$ ($\exists x\ G(x)$ is not a ground term)
- Valid wffs:

  $$\boldsymbol{B}(R, \exists x\ P(x))$$

  $$P(A) \Rightarrow \boldsymbol{B}(R, P(A))$$

Sentential Conceptualization

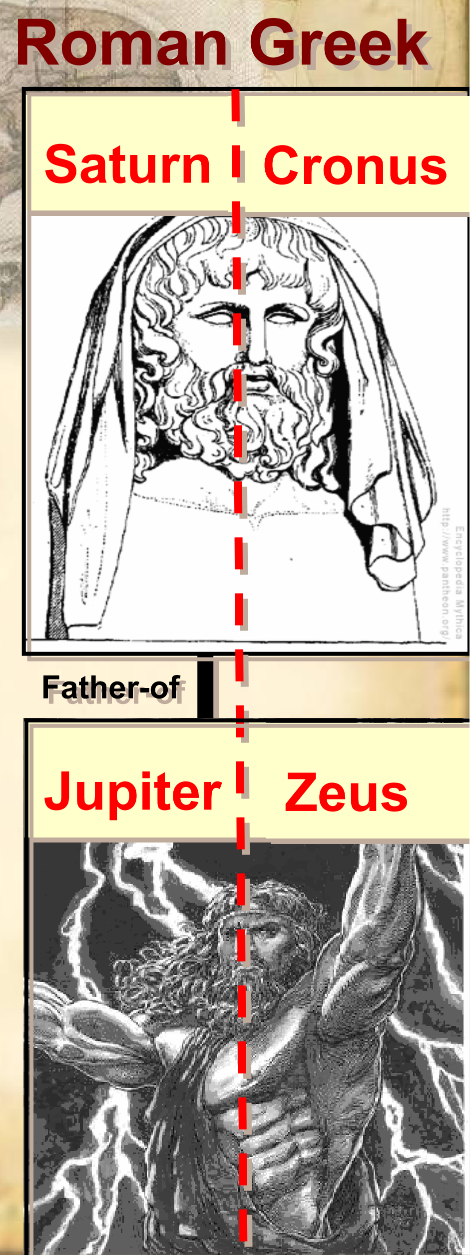


- Semantics
- Proof Methods
  - ***Attachment*** rule
  - Example of using Attachment
- Nested Beliefs
  - Change of semantic
  - Example: “Mud Children”

- Coffee Break!

Modal Operator B - Semantics

- Semantics of the propositional connectives remains.
- **What is the difference between belief atoms to ordinary wff atoms?**
- Let's have an example:
  - Roman: Saturn is the father of Jupiter
  - Greek: Cronus is the father of Zeus.
- Saturn and Cronus are equivalent, as Jupiter and Zeus are.
- Yossi is familiar only with the Greek mythology.



B operator Semantics – Cont’d

- Are the following also equivalent?

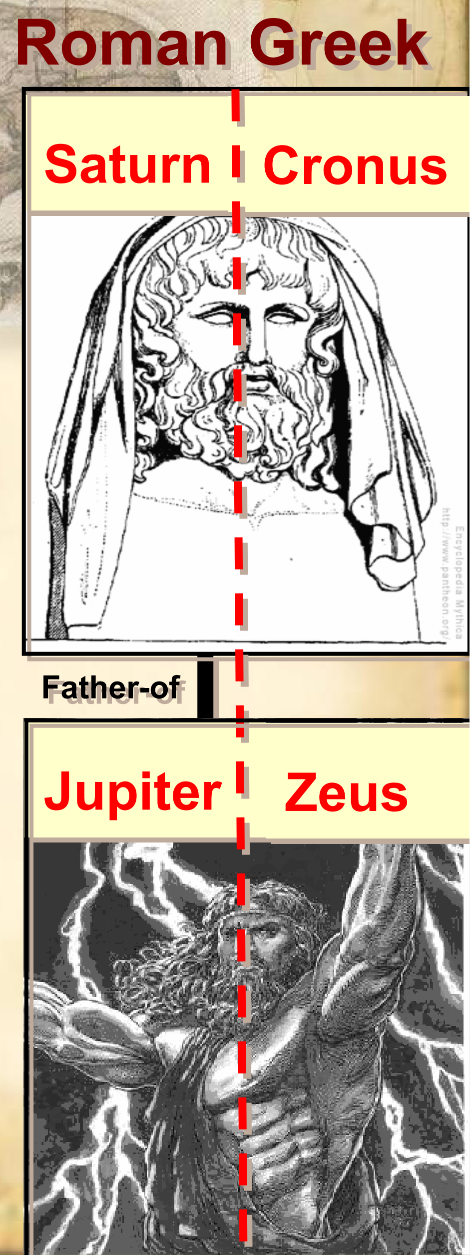
  $B(Yossi, Father-of(Zeus, Cronus))$

  $B(Yossi, Father-of(Jupiter, Saturn))$

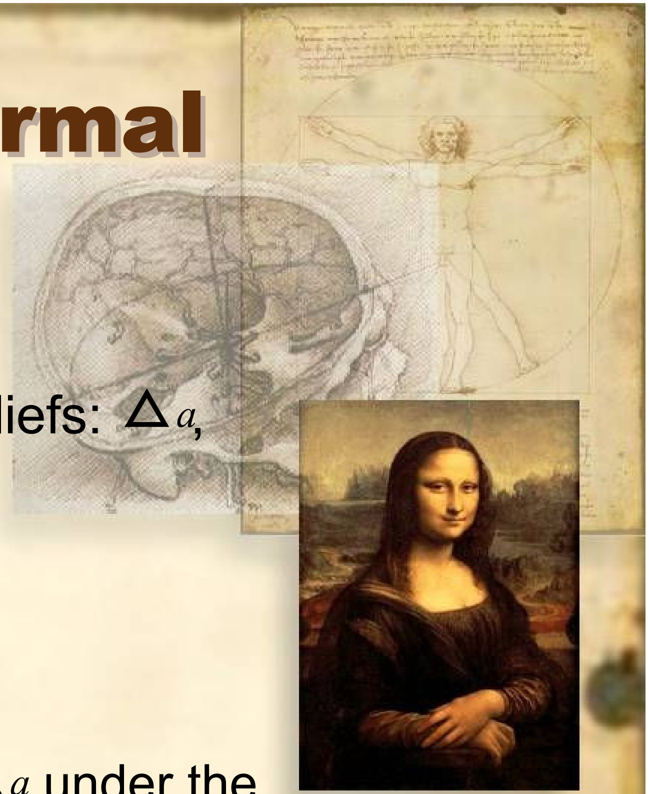
- ***Opaque Context***.
- In FOL, $\vdash_{\mathcal{FOL}}(t=s) \Rightarrow \vdash_{\mathcal{FOL}}(\phi(t)=\phi(s))$

  Transparent Context

- Belief Atom to be TRUE depends on:
  - the proposition itself
  - Agent’s “intentions”.

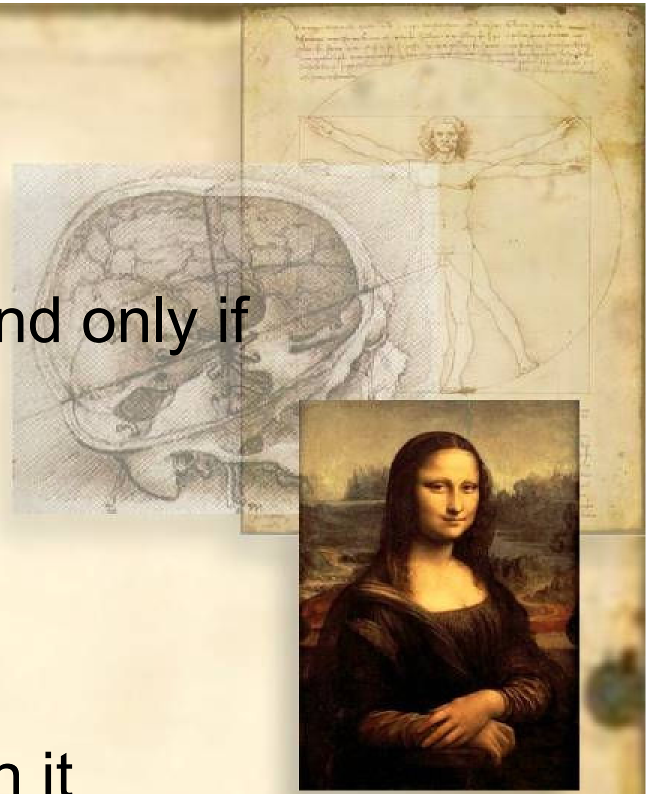


B operator Semantics – Formal



- Extending our notion of a domain:
  - Denumerable set of *agents*
  - Agent ***a*** is associated with a base set of beliefs: Δ_a, composed of:
    - ordinary closed wffs
    - set of inference rules ρ_a
  - τ_a is the theory formed by the closure of Δ_a under the inference rules ρ_a .
  - Provability in agent *a's* theory, using *a*'s inference rules: $\vdash_a$
  - Thus, $p \in \tau_a$ iff $\Delta_a \vdash_a p$
- Different agents might have different theories&inference rules!
- An agent's theory is closed only under his <u>**own**</u> inference rules

B Semantics - Final

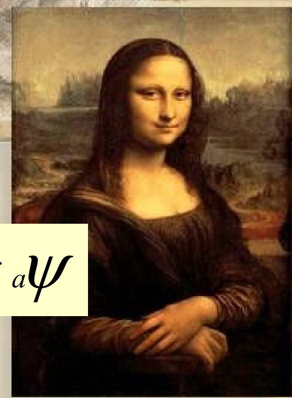


$B(\alpha, \phi)$ Will receive a TRUE value if and only if ϕ is in agent's α theory.

- Called "**Sentential Semantics**"
- Comes from the word "sentence" - α believes in sentence ϕ only when it belongs to it's theory.

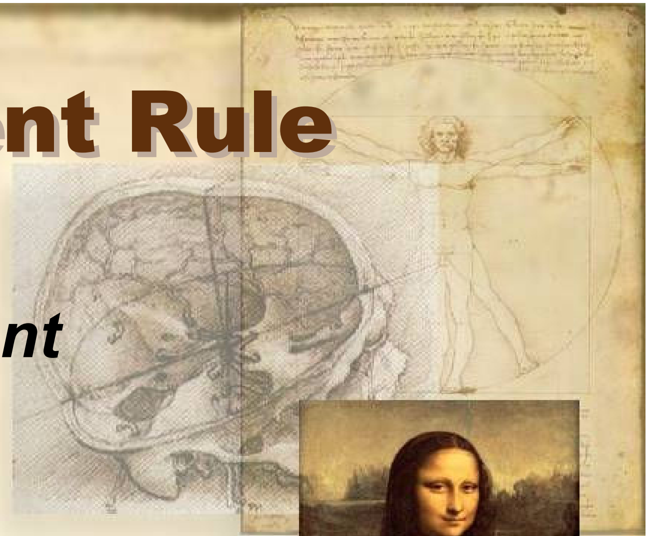
- Note that ***B*** semantics is referentially opaque as we required – substituting ϕ with equivalent ψ does not remain the truth value, which is depended if $\psi \in \tau_a$!

Proof Methods - Preliminaries



- We want to prove that an agent ***a*** that believes ϕ also believes in ψ
- Start a deductive process (calculation) using inference rules that ***a*** knows, in order to prove $\phi \vdash_a \psi$
- Will lead to a conclusion of the form $\boldsymbol{B}(a,\psi)$ from $\boldsymbol{B}(a,\phi)$
- We assumes that we have models of deduction process of each agent.
- We do not convert formulas inside of **B** operators. (atoms)

Proof Methods – Attachment Rule



- New inference-rule schema: ***attachment***

From

$$B(\alpha,\phi_1) \vee \psi_1$$

$$B(\alpha,\phi_2) \vee \psi_2$$

$$\vdots$$

$$B(\alpha,\phi_n) \vee \psi_n$$

$$\neg B(\alpha,\phi_{n+1}) \vee \psi_{n+1}$$

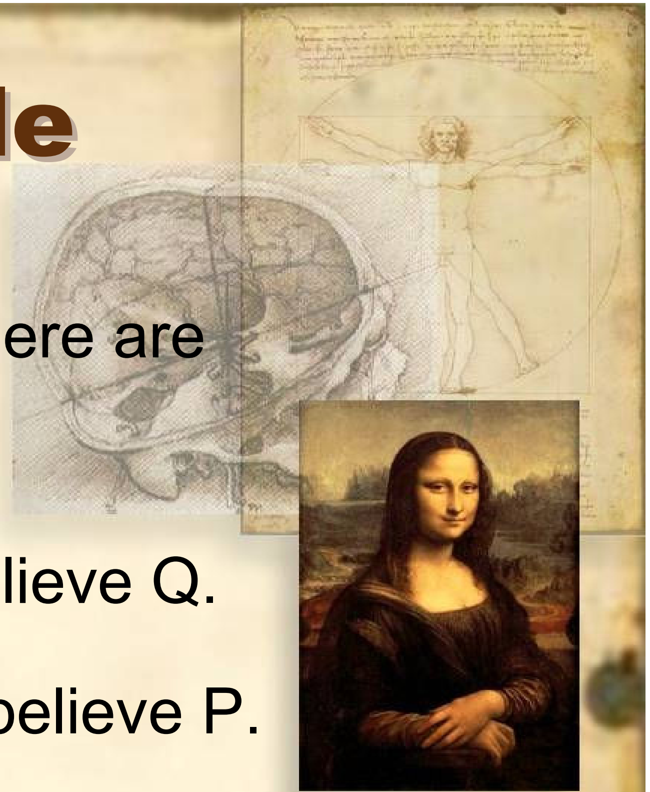
and

$$\phi_1 \wedge \phi_2 \wedge \ldots \wedge \phi_n \vdash_a \phi_{n+1}$$

conclude:

$$\psi_1 \vee \ldots \vee \psi_{n+1}$$

Proof Methods - Example



- We'll start with a special case where there are no ψ s.
- Nora believes $P \Rightarrow Q$ but does not believe Q.
- We want to prove that Nora does not believe P.
- Formal Clauses:

  1. $\boldsymbol{B}(Nora, P \Rightarrow Q)$
  2. $\neg\boldsymbol{B}(Nora, Q)$
  3. $\boldsymbol{B}(Nora, P)$ (Negation of what we want to prove)

Proof Methods – Example (Cont'd)

$1.\ \boldsymbol{B}(Nora, P \Rightarrow Q) \quad 2.\ \neg\boldsymbol{B}(Nora, Q) \quad 3.\ \boldsymbol{B}(Nora, P)$

$(P \Rightarrow Q) \in \tau_{Nora}$ $\qquad$ $(P) \in \tau_{Nora}$

So we got: $(P \Rightarrow Q) \wedge P \vdash_{Nora} Q$

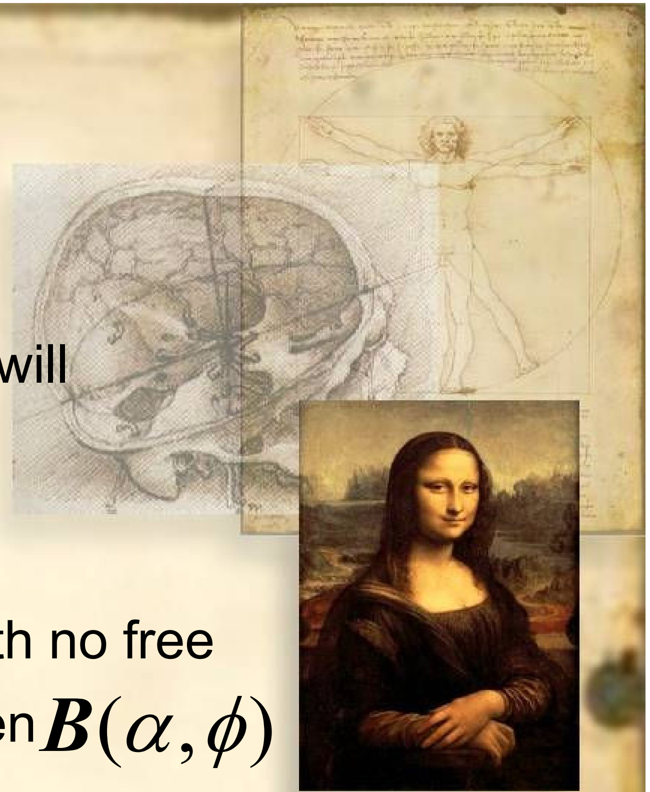
$$\boldsymbol{B}(\alpha, \phi_1) \vee \psi_1$$
$$\boldsymbol{B}(\alpha, \phi_2) \vee \psi_2$$
$$\vdots$$
$$\boldsymbol{B}(\alpha, \phi_n) \vee \psi_n$$
$$\neg\boldsymbol{B}(\alpha, \phi_{n+1}) \vee \psi_{n+1}$$

$$\phi_1 \wedge \phi_2 \wedge \ldots \wedge \phi_n \vdash_a \phi_{n+1}$$

$$\psi_1 \vee \ldots \vee \psi_n$$

- So we got an empty set of ψ's
  - proof completed.

Nested Beliefs



- **Syntax:**
  - Formal language will be based on FOL and will contain:
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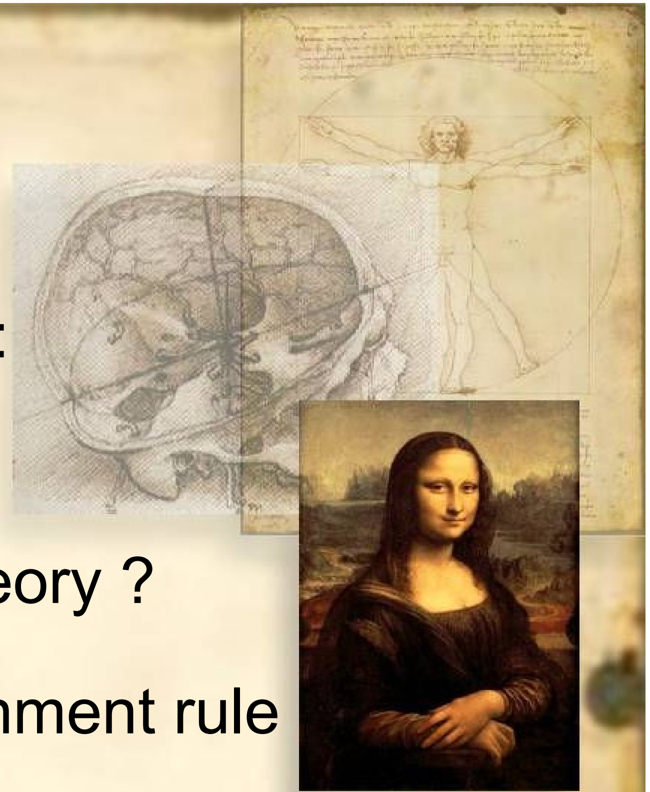
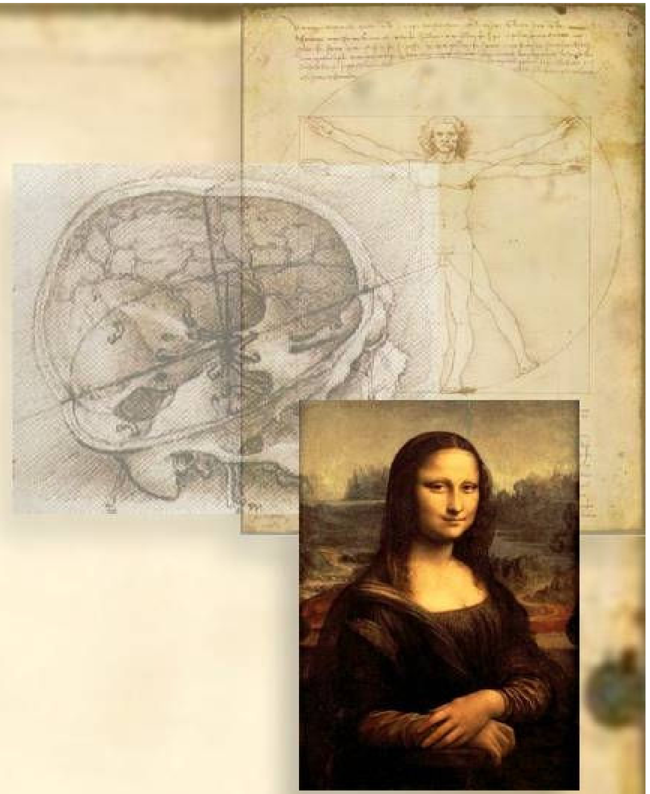
Nested Beliefs – Cont'd



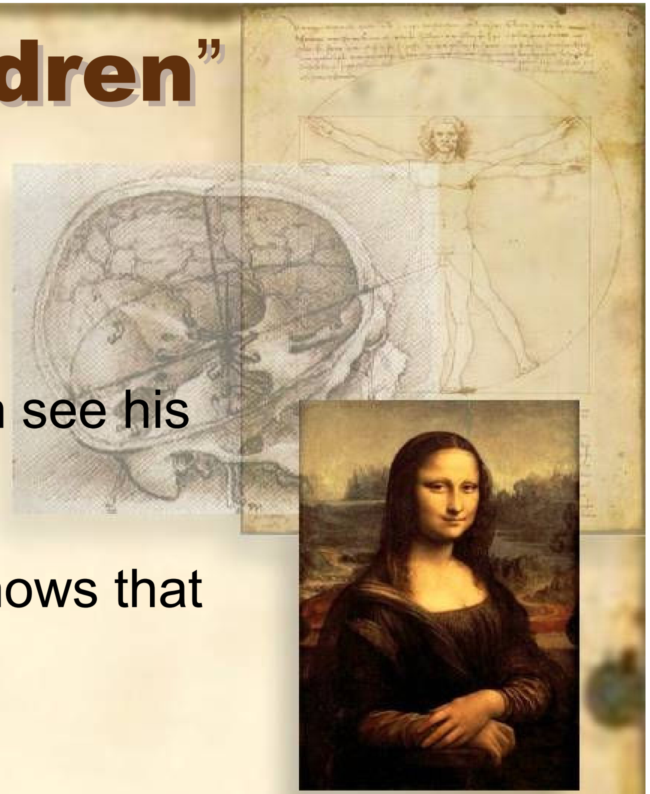
- We now allow expressions of this form:
  $\boldsymbol{B}(R1, \boldsymbol{B}(R2, P(A)))$
- What is different in the agents base theory ?
- Assume that each agent has the attachment rule
- Allows us to use attachment when we deduce,
  from agent a_i about agent a_j

Nested Beliefs – Cont'd



- Example:
  - Suppose $\boldsymbol{B}(a_i, \boldsymbol{B}(a_j, \phi))$
  - We get that: $\boldsymbol{B}(a_j, \phi) \in \tau_{ai}$
  - So according to a_i : $\phi \in \tau_{aj}$
  - From ϕ we can infer all kind of propositions, but just from a_i point of view. Therefore will use this symbol: $\vdash_{ai,\, aj}$
  - This can be nested to unlimited # of agents: $\vdash_{ai,\, aj,\, ak\ldots}$

Nested Beliefs – "Mud Children" Example



- We'll name the children Bat & Ben
- (1) Ben & Bat both know that each of them can see his sibling's forehead but not his own. Therefore:
  - (1a) if Bat doesn't have a mud spot, Ben knows that Bat doesn't have a mud spot.
  - (1b) Bat knows (1a)
- (2) Bat & Ben know that at least one of them has a mud spot, and they each know that the other knows that. In particular,
  - (2a) Bat knows that Ben knows that either Bat or Ben has a mud spot.
- (3) Ben says that he doesn't know whether he has a mud spot, and Bat thereby knows that Ben doesn't know.

Nested Beliefs – "Mud Children" Example – Cont'd

1b. $\boldsymbol{B}_{Bat}(\neg Mud(Bat) \Rightarrow B_{Ben}(\neg Mud(Bat)))$

2a. $\boldsymbol{B}_{Bat}(B_{Ben}(Mud(Bat) \vee Mud(Ben)))$

3. $\boldsymbol{B}_{Bat}(\neg B_{Ben}(Mud(Ben)))$

- Prove that: $\boldsymbol{B}_{Bat}(Mud(Bat))$
- For Bat, the following occurs:

$$\neg Mud(Bat) \Rightarrow B_{Ben}(\neg Mud(Bat))$$
$$\wedge\ B_{Ben}(Mud(Bat) \vee Mud(Ben))$$
$$\wedge\ \neg B_{Ben}(Mud(Ben)) \ \vdash_{Bat} Mud(Bat)$$

here we use a proposition that the agent does **not** believe. we assumed by negation that: $\neg \boldsymbol{B}_{Bat}(Mud(Bat))$

Nested Beliefs – "Mud Children" Example – Cont'd

- Assuming reasonable rules for $\vdash_{Bat}$, we next attempt this proof:

$$1.\ [B_{Ben}(\neg Mud(Bat))] \quad \vee \quad Mud(Bat)$$

$$2.\ B_{Ben}[(Mud(Bat) \vee Mud(Ben))]$$

$$3.\ \neg B_{Ben}(Mud(Ben))$$

$$\boldsymbol{B}(\alpha,\phi_1) \vee \psi_1$$
$$\boldsymbol{B}(\alpha,\phi_2) \vee \psi_2$$
$$\vdots$$
$$\boldsymbol{B}(\alpha,\phi_n) \vee \psi_n$$
$$\neg\boldsymbol{B}(\alpha,\phi_{n+1}) \vee \psi_{n+1}$$

$$\phi_1 \wedge \phi_2 \wedge \ldots \wedge \phi_n \vdash_a \phi_n$$

- So now we need to prove:

$$\psi_1 \vee \ldots \vee \psi_{n+1}$$

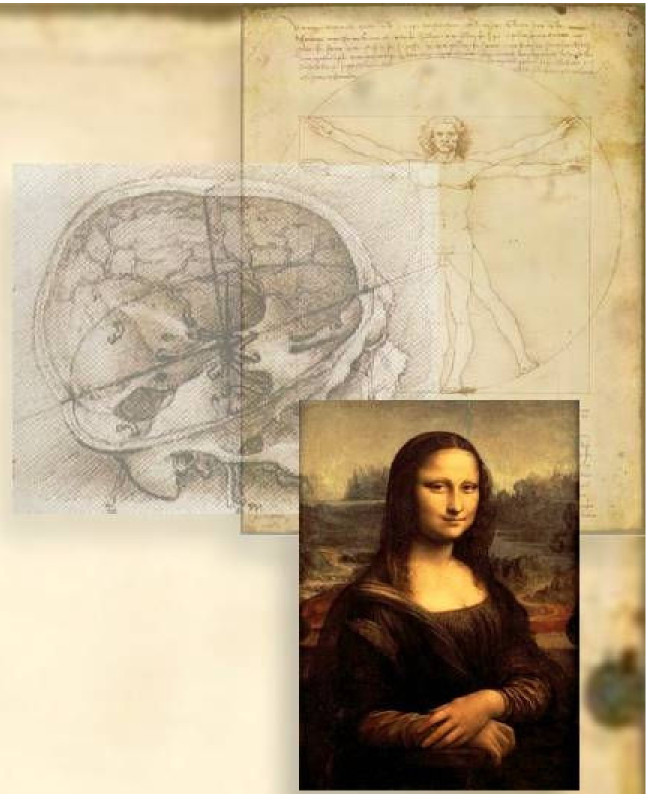
$$(\neg Mud(Bat) \quad \wedge \quad (Mud(Bat) \wedge Mud(Ben))) \vdash_{Bat,\,Ben} Mud(Ben)$$

- But by assuming reasonable rules for $\vdash_{Bat,\,Ben}$, proof is trivial.

$$\boldsymbol{B}_{Bat}(Mud(Bat))$$

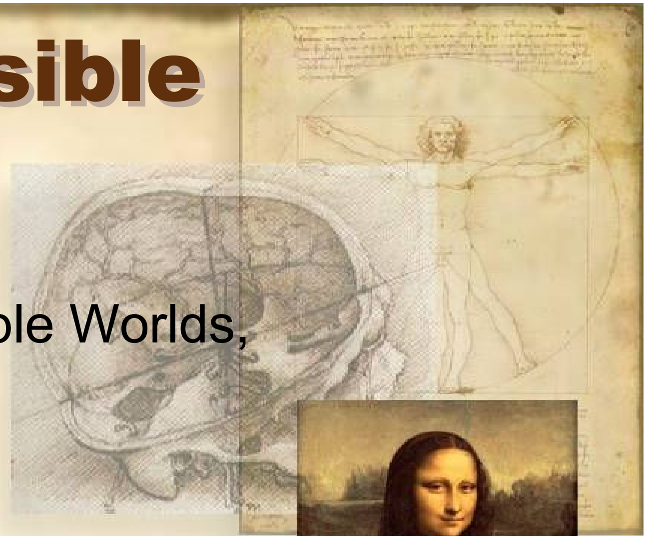
Time for break!

Possible Worlds Conceptualization



- Intro
- Semantics
  - Example
- Nested Knowledge
- Properties of Knowledge
  - Axioms
  - Rules
- Properties of Belief

2^{nd} Conceptualization: Possible Worlds Logic



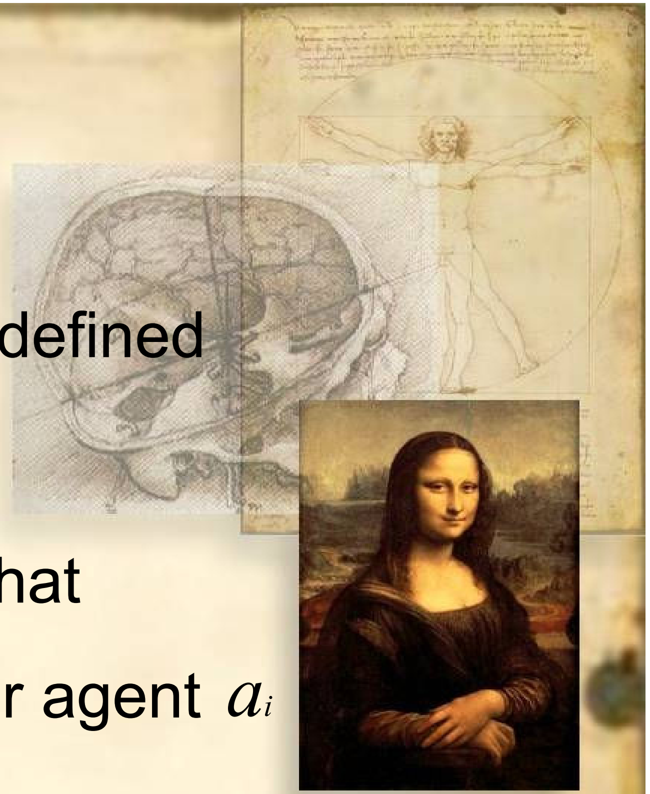
- We will present the conceptualization of Possible Worlds, to express **Knowledge**.
- Recall: In this conceptualization, we include objects $w_0, w_1, w_2, \ldots, w_i, \ldots$ called ***possible worlds***.
- We will use it to define, this time, the semantics of modal operator ***K***.
- We'll use the same language we defined before, that distinguish between ordinary wffs and belief & knowledge atoms.
- We will also define a new semantic for ordinary wff. Can you say why?

Possible Worlds: Changing ordinary wff semantics

- So far wffs had an absolute truth value, using truth tables and valuations
- We introduce a notion of a wff being true or false ***with respect to a possible world***.
- We'll now have a different interpretation for each possible world, each contain it's own set of functions, objects and relations.
- An ordinary wff ϕ has the value TRUE with respect to the world w_i only when it's evaluates to true using the interpretation associated with w

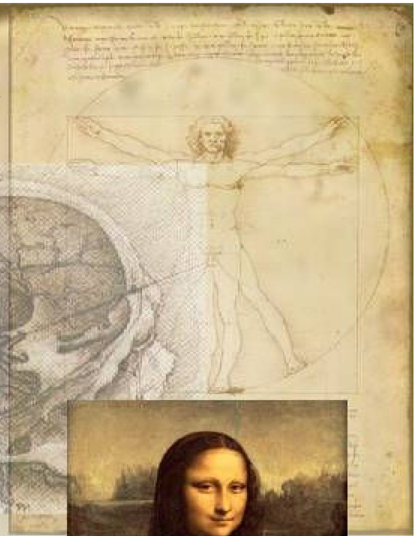
Example: White(Snow) will evaluate to TRUE only in worlds where the snow is white.

Possible Worlds – The Accessibility Relation



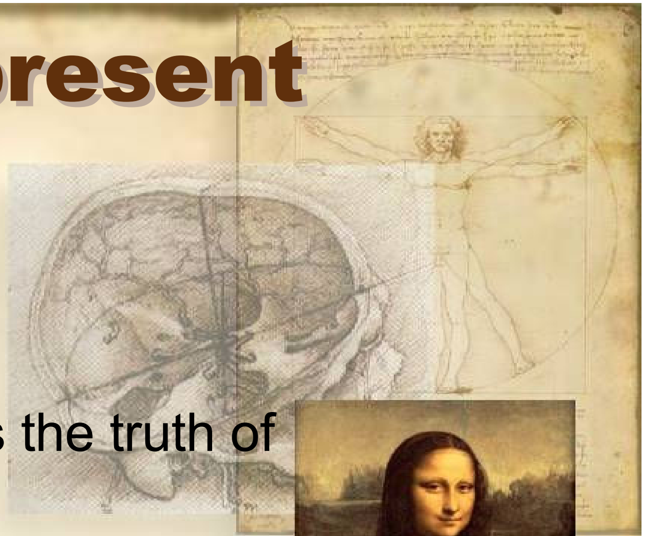
- Reminder: An Accessibility Relation is defined by the triple: $R(a_i, w_i, w_j)$
- When the relation is satisfied, we say that world w_i is accessible from world w_j for agent a_i
- Accessibility relation will define the semantics
- Opposed to the Sentential conceptualization, where sentences defined if.

Semantics of modal operator *K*



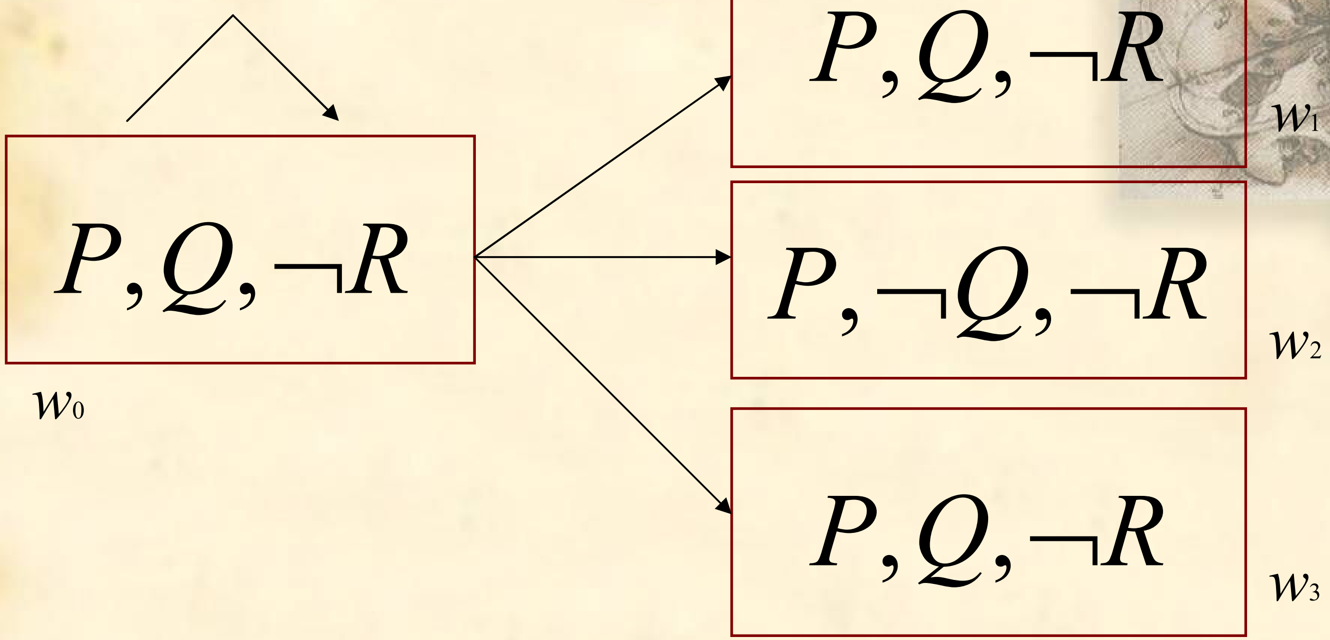
- The expression $K(\alpha, \phi)$ will receive TRUE with respect to world w_i iff ϕ receives TRUE in **all** worlds that are accessible from w_i to agent α
- Applied recursively
  – even for wffs that contain nested modal operators.
- Can we apply the accessibility relation also for belief?
  – Further we'll see it's not that trivial.

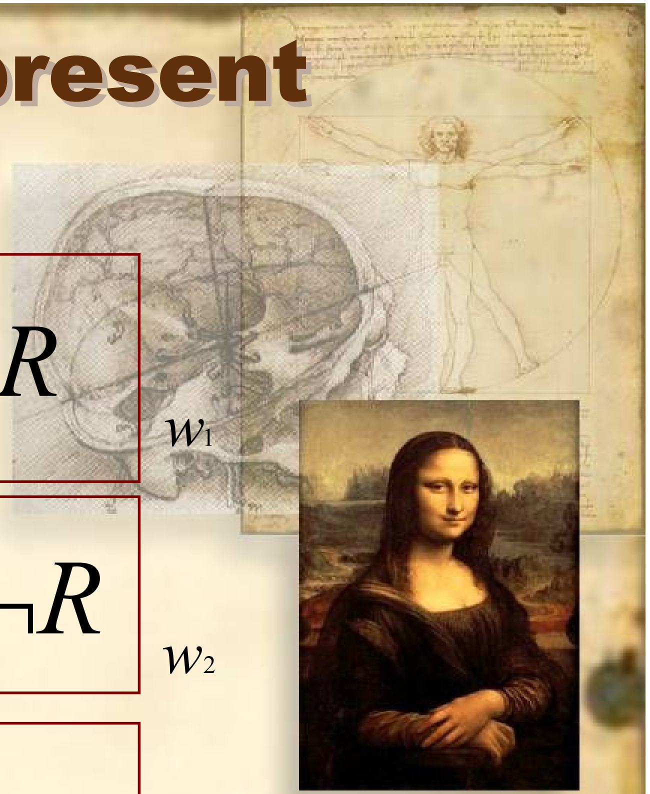
Using Possible Worlds to present Knowledge



- **A** is knower. **p** a proposition.
  - Suppose (in world w_0) that **A** doesn't knows the truth of **p:**
  - In some worlds (associated with **A** in w_0) **p** has true value, and some of them **p** is false.
- if **A** knows **p** to be true (in w_0), then in ***<u>all</u>*** the worlds associated with **A** in w_0, **p** must have a true value.
- Actually, the worlds *associated* with **A** in a world are just those that are accessible for it from the world.

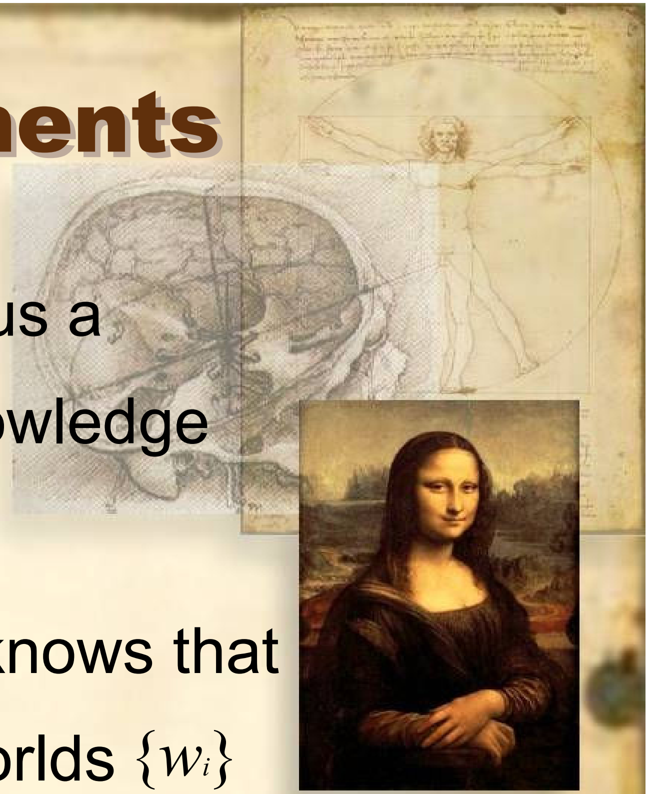
Using Possible Worlds to present Knowledge - Example





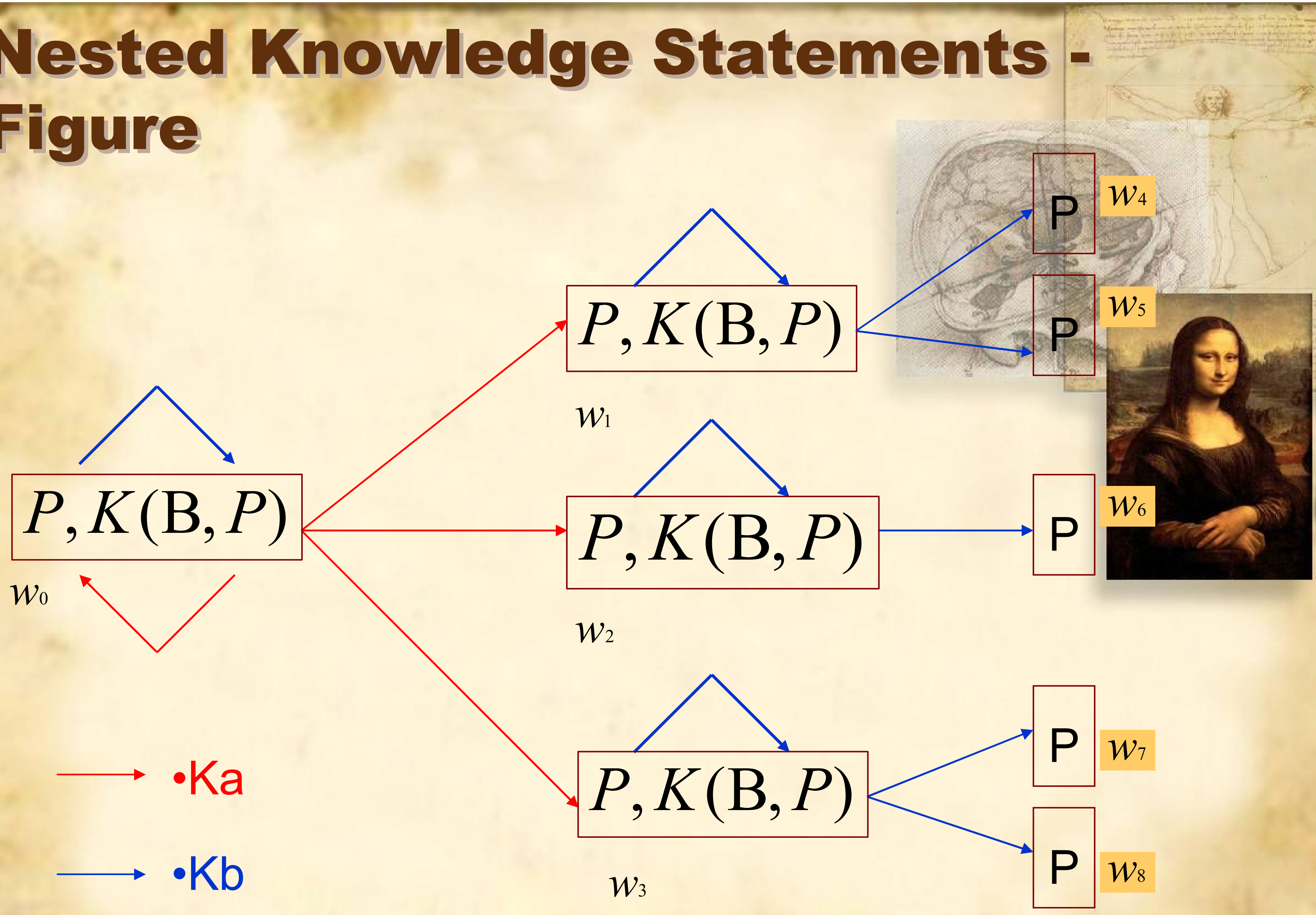
- The worlds w_1, w_2, w_3 are accessible to **A** from w_0
- By the figure, **A** knows **P** and **~R**.
- Knowing **~R** in w_0 means not to know **R** in w_0

Nested Knowledge Statements

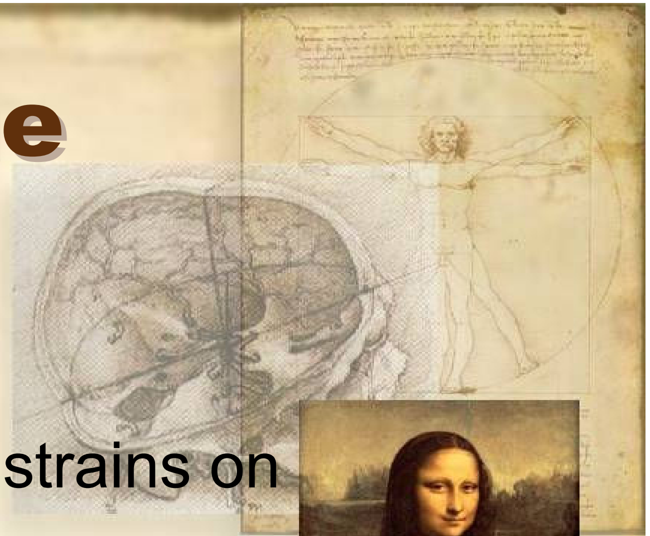


- The possible worlds model also gives us a convenient way to describe nested knowledge statements.
- To say that the agent **A** (in world w_0) knows that agent **B** knows **P** is to say that in all worlds $\{w_i\}$ accessible for **A** from w_0, $K(\mathrm{B}, P)$ has a value true.
- For $K(\mathrm{B}, P)$ to have the value true in any of the worlds $\{w_i\}$, means that in all worlds accessible for **B** in each w_i,**P** has the value true.

Nested Knowledge Statements - Figure

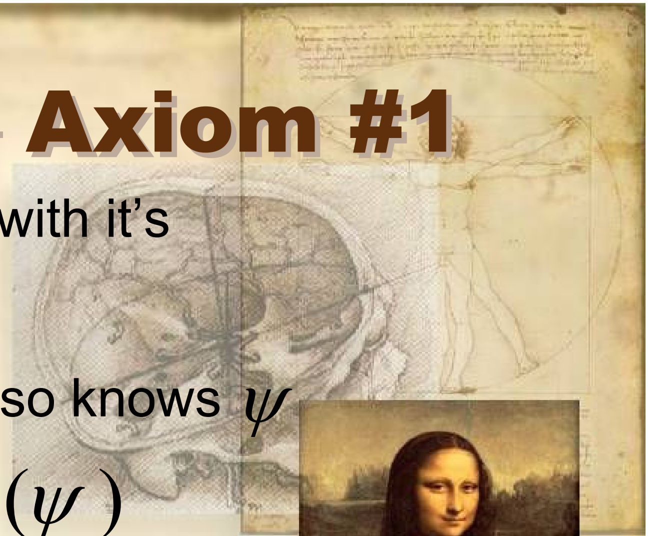


Properties of Knowledge



- **K** & **B** must satisfy several properties.
- Properties are achieved by special constrains on the accessibility relation.
- First characterize Knowledge over belief.
- We shall introduce several axioms and rules helping us to distinguish between them.

Properties of Knowledge – Axiom #1



- Property: An agent ought to be able to reason with it's knowledge.

  if an agent knows ϕ and also $\phi \Rightarrow \psi$, then it also knows ψ

  $$(\boldsymbol{K}_\alpha(\phi) \wedge \boldsymbol{K}_\alpha(\phi \Rightarrow \psi)) \Rightarrow \boldsymbol{K}_\alpha(\psi)$$

- **"Distribution Axiom"**
- Also written as:

  $$\boldsymbol{K}_\alpha(\phi \Rightarrow \psi) \Rightarrow (\boldsymbol{K}_\alpha(\phi) \Rightarrow \boldsymbol{K}_\alpha(\psi))$$

- Already satisfied by the possible worlds semantics with no need to add any constrains
- Reason: Suppose **A** knows ϕ and $\phi \Rightarrow \psi$. So in any world w_0 accessible to **A**, the above hold, so also holds. That is, in every world accessible to **A**, ψ hold. So $\boldsymbol{K}_\alpha(\psi)$

Properties of Knowledge – Axiom #2

- Property: an agent cannot possibly know something that is false. Therefore:

$$K_{\alpha}(\phi) \Rightarrow \phi$$

- **"Knowledge Axiom"**
- Satisfied if the accessibility relation is ***reflexive***: the relation $R(a, w_1, w_1)$ is satisfied; if agent a knows ϕ in w_1, then ϕ must be true in w_1
- Note that this implies that an agent does not know contradictions: $\neg K(\alpha, \mathrm{F})$

Properties of Knowledge – Axiom #3

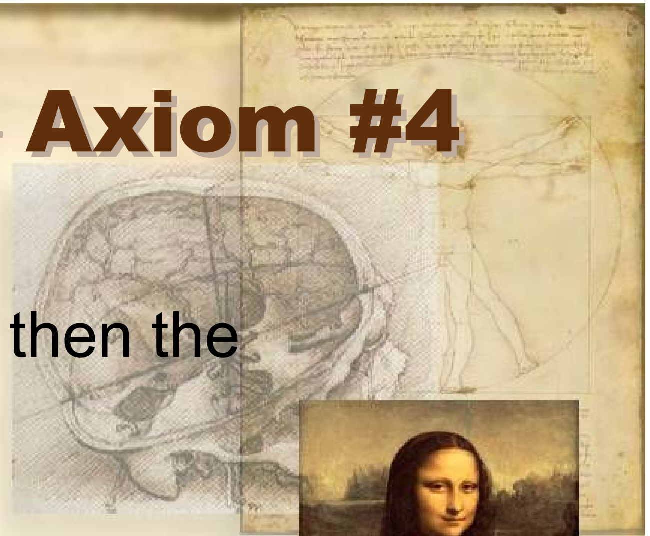
- Property: if an agent knows ϕ, then the agent knows that it knows ϕ :

$$\boldsymbol{K}_{\alpha}(\phi) \Rightarrow \boldsymbol{K}_{\alpha}(\boldsymbol{K}_{\alpha}(\phi))$$

- **"Positive-introspection"**
- Satisfied if the accessibility relation is ***transitive***:

  i.e. $R(a, w_{1,} w_{2})$ and $R(a, w_{2,} w_{3})$ imply $R(a, w_{1,} w_{3})$

Properties of Knowledge – Axiom #4



- Property: if an agent doesn't knows ϕ, then the agent knows that it doesn't knows ϕ :

  $$\neg K_\alpha(\phi) \Rightarrow K_\alpha(\neg K_\alpha(\phi))$$

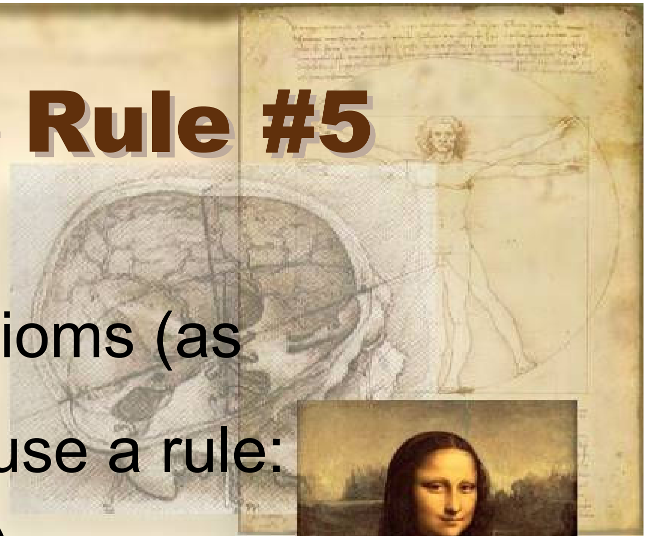
- **"Negative-introspection"**
- Satisfied if the accessibility relation is ***Euclidean***:

  i.e. $R(a, w_1, w_2)$ and $R(a, w_1, w_3)$ imply $R(a, w_2, w_3)$
- What about **symmetry**? Symmetry leads to:

  $$\neg K_\alpha(\neg K_\alpha(\phi)) \Rightarrow \phi$$ **"Brouin Axiom"**

  But this can be shown by the previous axioms.

Properties of Knowledge – Rule #5



- Property: any agent knows all these axioms (as well as logical axioms). We'll this time use a rule:

$$\text{from } \vdash \phi \text{ infer } \boldsymbol{K}_\alpha(\phi)$$

- **"Epistemic Necessitation"**
- This inference rule follow the possible worlds semantics: if ϕ is a logical axiom, then ϕ is true at any possible world. In particular, the worlds accessible from some agent α 's world. So $\boldsymbol{K}_\alpha(\phi)$
- Note the ϕ is a logical axiom, not a proper axiom!

Properties of Knowledge – Rule #6 & #7

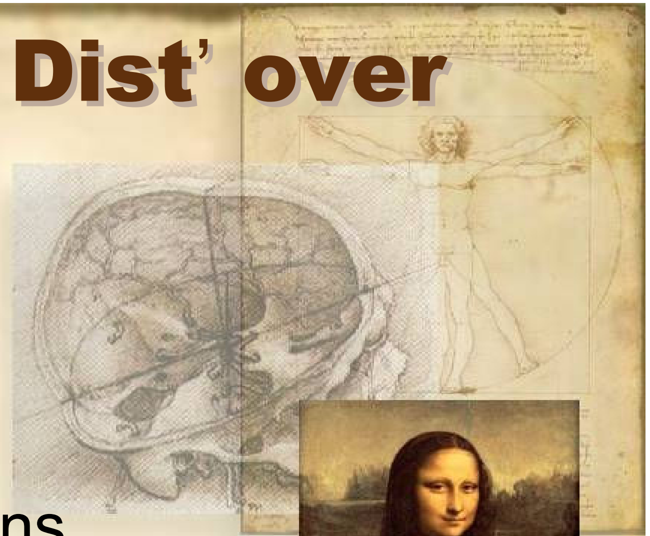
- From Axiom #1 and Rule #5 - ***logically omniscient:***

$$\text{from } \phi \vdash \psi \text{ and from } \boldsymbol{K}_{\alpha}(\phi) \text{ infer } \boldsymbol{K}_{\alpha}(\psi)$$

$$\text{from } \vdash \phi \Rightarrow \psi \text{ infer } \boldsymbol{K}_{\alpha}(\phi) \Rightarrow \boldsymbol{K}_{\alpha}(\psi)$$

- Seems unrealistic for a finite agents. Can an agent derive *all* the consequences from his knowledge?
- *Platonic* view of knowledge
  - an agent knows all the consequences from his knowledge, even though he might not necessarily explicitly believe them.

Properties of Knowledge – Dist' over Conjunctions



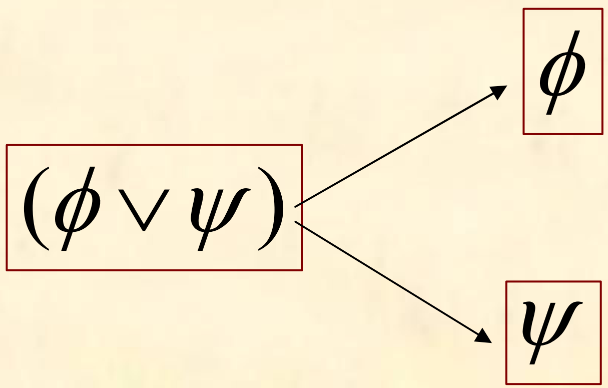
- From rule #6 we can derive:

$$\boldsymbol{K}_{\alpha}(\phi) \wedge \boldsymbol{K}_{\alpha}(\psi) \Leftrightarrow \boldsymbol{K}_{\alpha}(\phi \wedge \psi)$$

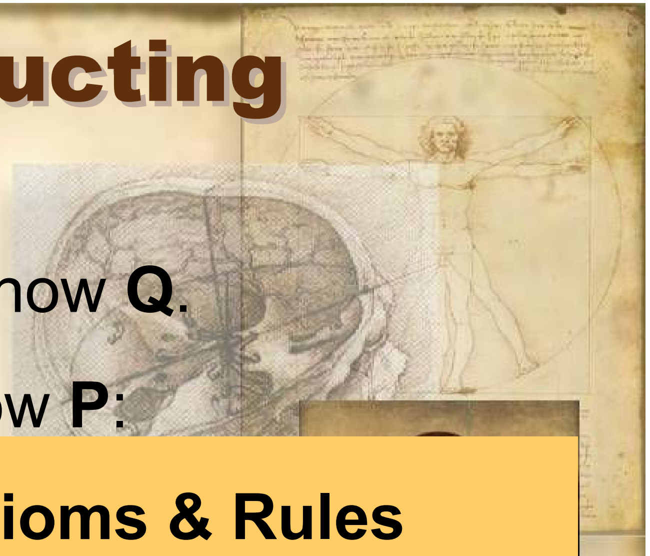
- **K** operator ***distributes*** over conjunctions.
- Does the following exist?

$$\boldsymbol{K}_{\alpha}(\phi \vee \psi) \Rightarrow \boldsymbol{K}_{\alpha}(\phi) \vee \boldsymbol{K}_{\alpha}(\psi)$$

- **NO**:



Using the axioms – Constructing a proof



- Recall: Nora knows $P \Rightarrow Q$ but doesn't know **Q**.

  We want to prove that Nora doesn't know **P**:

1. $\boldsymbol{K}_{Nora}(P \Rightarrow Q)$ given
2. $\boldsymbol{K}_{Nora}(P) \Rightarrow \boldsymbol{K}_{Nora}(Q)$ Axiom 1
3. $\neg \boldsymbol{K}_{Nora}(Q) \Rightarrow \neg \boldsymbol{K}_{Nora}(P)$ c"p of 2
4. $\neg \boldsymbol{K}_{Nora}(Q)$ given
5. $\neg \boldsymbol{K}_{Nora}(P)$ 3,4, MP

Axioms & Rules

1. $\boldsymbol{K}_\alpha(\phi \Rightarrow \psi) \Rightarrow (\boldsymbol{K}_\alpha(\phi) \Rightarrow \boldsymbol{K}_\alpha(\psi))$
2. $\boldsymbol{K}_\alpha(\phi) \Rightarrow \phi$
3. $\boldsymbol{K}_\alpha(\phi) \Rightarrow \boldsymbol{K}_\alpha(\boldsymbol{K}_\alpha(\phi))$
4. $\neg \boldsymbol{K}_\alpha(\phi) \Rightarrow \boldsymbol{K}_\alpha(\neg \boldsymbol{K}_\alpha(\phi))$
5. from $\phi \vdash$ infer $\boldsymbol{K}_\alpha(\phi)$
6. from $\phi \vdash \psi$ and from $\boldsymbol{K}_\alpha(\phi)$ infer $\boldsymbol{K}_\alpha(\psi)$
7. from $\vdash \phi \Rightarrow \psi$ infer $\boldsymbol{K}_\alpha(\phi) \Rightarrow \boldsymbol{K}_\alpha(\psi)$

Substitution problem under Possible Worlds semantics

- Recall the substitution problem we encountered earlier.
- Let us see how Possible Worlds Semantics blocks this problem:

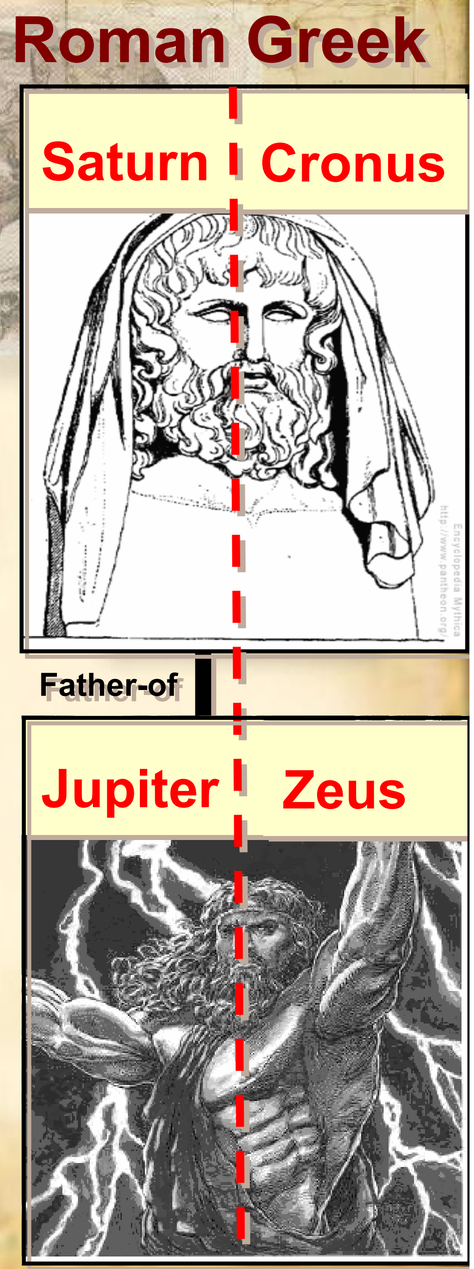
  Given: $K_{\alpha}(Father-of(Zeus, Chronus))$

  $K_{\alpha}(Father-of(Jupiter, Saturn))$
- These propositions have the value TRUE in the actual world w_0.
- Using the knowledge Axiom, we get that:

  $Father-of(Zeus, Chronus)$

  $Father-of(Jupiter, Saturn)$

  are true in w_0 .



Substitution under Possible Worlds semantics-Cont'd

- For: $K_\alpha(Father-of(Zeus, Saturn))$

  to have a true value in the actual world w_0,

  $Father-of(Zeus, Saturn)$ must be true in

  all the worlds accessible to α from w_0.

- This cannot be the case unless

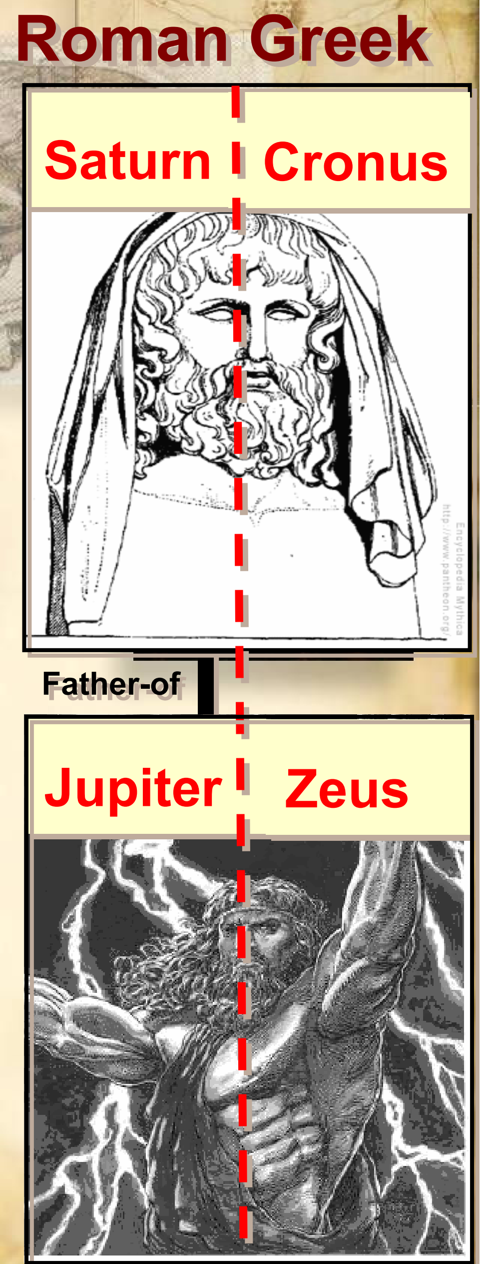
  $(Zeus = Jupiter)$

  $(Cronus = Saturn)$

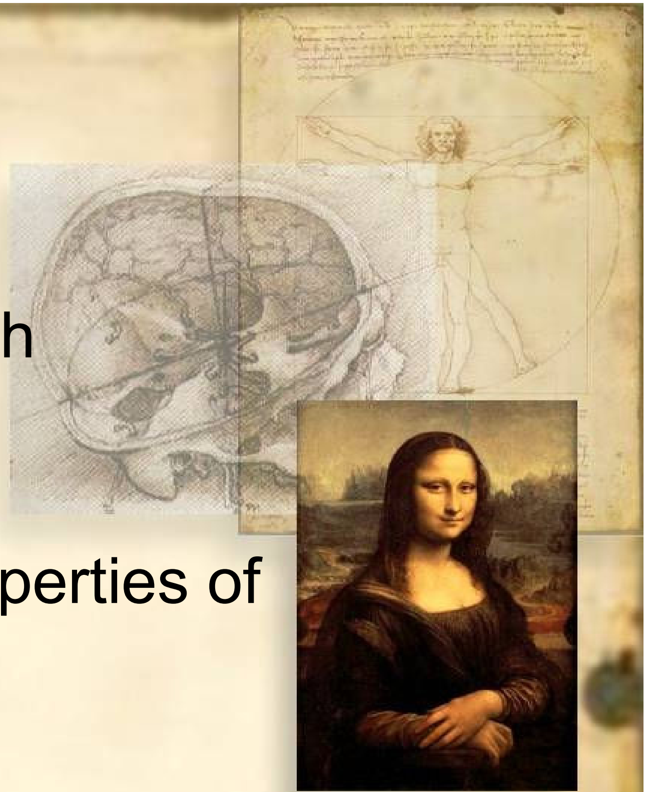
  each have the value true in all the worlds

  accessible to α from w_0. But it'll be true

  only if α knows it!

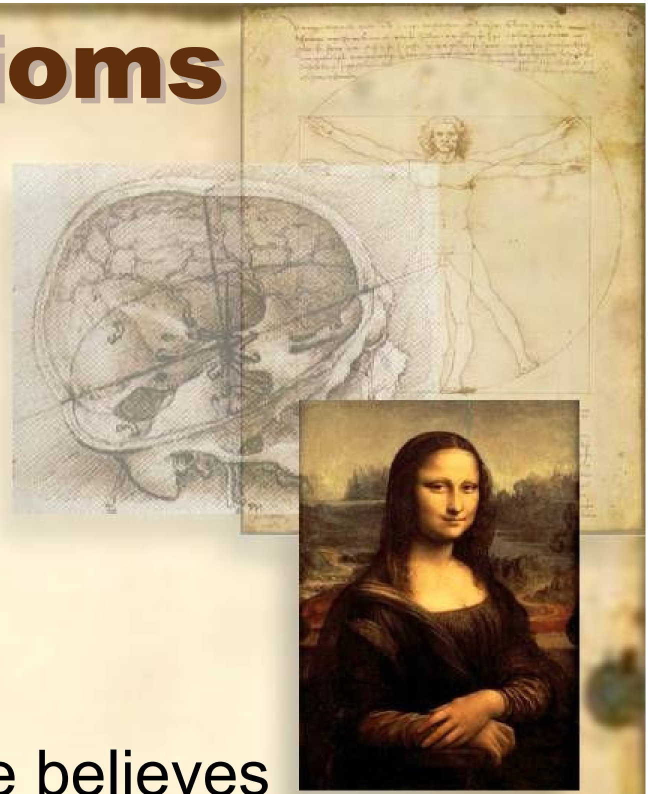


Properties of Belief



- Possible Worlds semantic is not enough appropriate for belief
- But we can still state some desired properties of belief as axioms.
- Essential difference from knowledge: **agent cannot have false knowledge, but can have false belief.**
- Axiom #2, Knowledge axiom, is canceled in this case. $K_{\alpha}(\phi) \Rightarrow \phi$

Properties of Belief–Axioms #8,#9,#10



- Property: An agent doesn't believe in contradictions:

$$\neg B_{\alpha}(F)$$

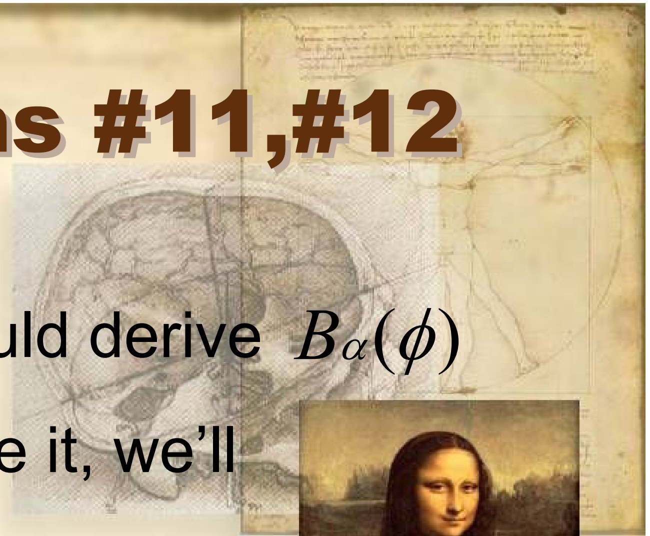
- Property: positive-introspection:
  if an agent believes something, then he believes that he believes it:

$$B_{\alpha}(\phi) \Rightarrow B_{\alpha}(B_{\alpha}(\phi))$$

- Safe enough to assume that:

$$B_{\alpha}(\phi) \Rightarrow K_{\alpha}(B_{\alpha}(\phi))$$

Properties of Belief–Axioms #11,#12



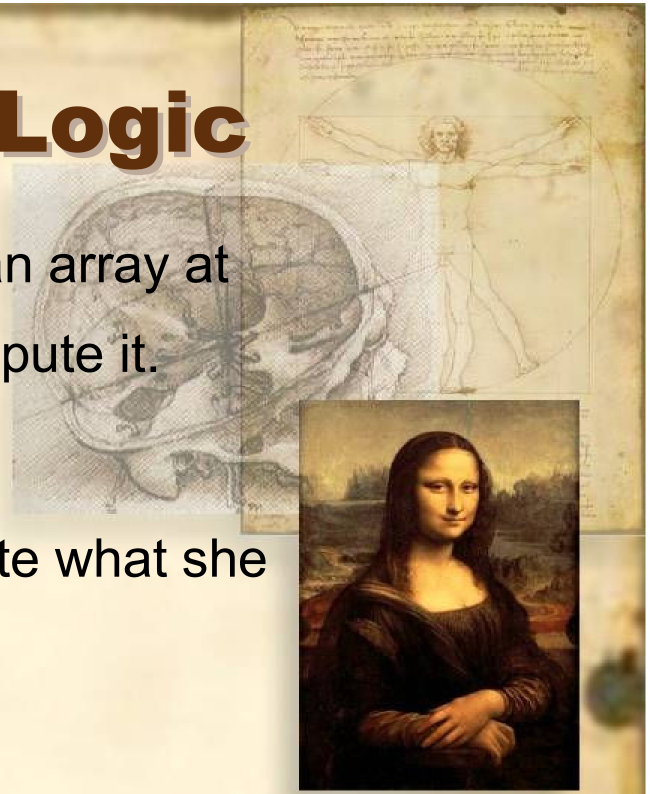
- If we had the knowledge axiom, we could derive $B_\alpha(\phi)$ from $B_\alpha(B_\alpha(\phi))$. Since we don't have it, we'll add it as an axiom:

$$B_\alpha(B_\alpha(\phi)) \Rightarrow B_\alpha(\phi)$$

- If an agent has confidence in the beliefs of other agents, we also might want to say that an agent believes ϕ if it believes that some other agent believes ϕ :

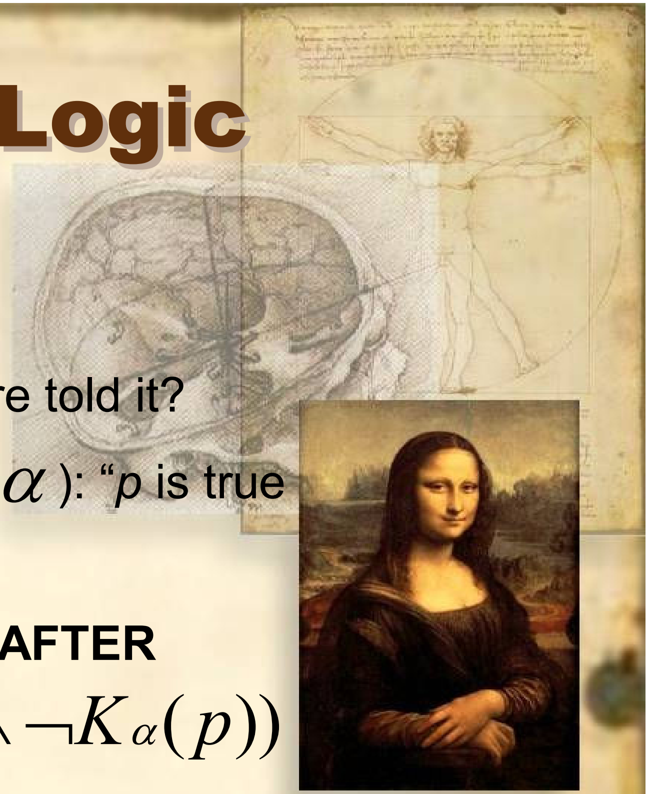
$$B_{\alpha 1}(B_{\alpha 2}(\phi)) \Rightarrow (B_{\alpha 1}(\phi))$$

Applications of Epistemic Logic



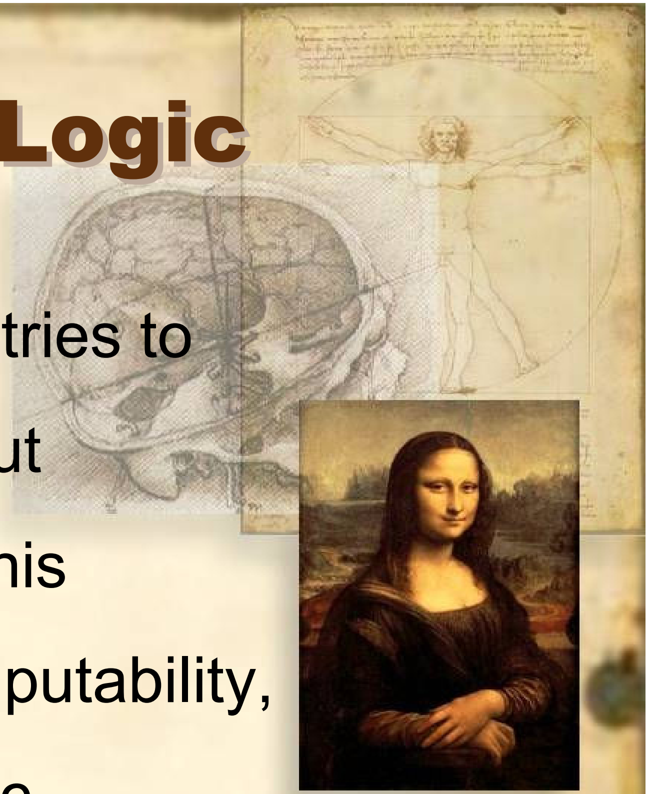
- When a computer program needs a length of an array at some state, the programmer must tell it to compute it.
- ***"Knowledge-Base Programming"***
  - "*what she wants"* instead of "how to compute what she wants".
- Knowledge & Action
  - For a planning program, knowledge is necessary to perform an action and new knowledge is gained as a result of performing action.
  - Under Possible Worlds semantics, a logic was constructed to face the problem of automatically generating deduction within logic.

Applications of Epistemic Logic



- Knowledge & Communication:
  - Do you necessarily know something after you are told it?
  - Consider someone is telling you (“you” = agent α): “*p* is true but you don’t know it”: $p \wedge \neg K_{\alpha}(p)$
  - \- **when** said, this sentence is perfectly true. But **AFTER** being said, the following doesn’t hold: $K_{\alpha}(p \wedge \neg K_{\alpha}(p))$
  - **It is actually inconsistent!**
- The solution is using a multi-agent system, and describe knowledge of an agent as being acquired, rather than a static set of formulas.
- Levesque tested a knowledge base that interacted with its domain using TELL & ASK queries.
- This model can be used to perform the solution above, in which a TELL operation is another agent

Applications of Epistemic Logic



- ***"Zero-knowledge proofs"*** - a prover tries to convince a verifier a certain fact, without revealing any additional information. This requires notions of probability and computability, that can be formalize in Epistemic Logic combined with *logic of probability and knowledge.*
- ***Agent oriented programming***