

תהי f פונקציה ממספרים ממשיים למספרים ממשיים. הפונקציה
 f קרוי פונקציה, פונקציה

הפונקציה f קרוי פונקציה אם

לכל $x \in \mathbb{R}$ קיים $y \in \mathbb{R}$ כך ש- $f(x) = y$.
 הפונקציה f קרוי פונקציה אם
 לכל $x, y \in \mathbb{R}$ קיים $z \in \mathbb{R}$ כך ש- $f(z) = x$ ו- $f(z) = y$.

הפונקציה f קרוי פונקציה אם
 לכל $x, y \in \mathbb{R}$ קיים $z \in \mathbb{R}$ כך ש- $f(z) = x$ ו- $f(z) = y$.

$Q(N)$	$D(f)$	
1	2	$f: \mathbb{R} \rightarrow \mathbb{R}$ $f(x, y) = x \otimes y$
$Q(N) = \Theta(\sqrt{N})$	$D(f) = N$ $R_2(f) = \Omega(N)$	$OR(x_0, \dots, x_{N-1})$

הפונקציה f קרוי פונקציה אם
 $Q_2(f)$, $D(f)$

הפונקציה f קרוי פונקציה אם $I \subseteq [N]$ ו-

$$x_1, x_2, \dots, x_N$$

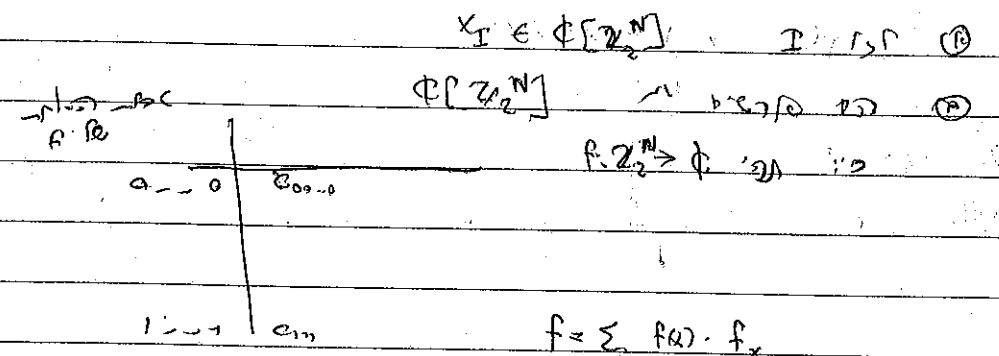
$$j \in I \Rightarrow i \neq j$$

$$j \notin I \Rightarrow i = 0$$

$F_x(y) = \sum_{y=0}^N x_{xy}$
 \Rightarrow is known (in \mathbb{Z}_2^N space) - and ϕ is a map
 $x_s(y) = (-1)^{sy} = \prod_{i=1}^N (-1)^{y_i x_i}$
 is a character of \mathbb{Z}_2^N

X_I is a character of \mathbb{Z}_2^N

$$[X_I(y_1, \dots, y_N) = (-1)^{y \cdot I} = \prod_{i=1}^N (-1)^{y_i x_i}]$$



$$F_x(y) = \prod_{i=1}^N (1 - \prod_{j=1}^N (y_j - x_j))$$

\Rightarrow is a polynomial in \mathbb{Z}_2

is a polynomial
 in \mathbb{Z}_2

$$f = \sum \alpha_x X_I$$

is known f is a polynomial of degree k in \mathbb{Z}_2
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$$|p(x) - f(x)| \leq \frac{1}{3} \quad \mathbb{Z}_2^N \rightarrow x \text{ is a point, } f \text{ is a polynomial of degree } k$$

$$OR(\vec{0}) = 0$$

$$OR(\vec{1}) = 1$$

$$OR: \{0,1\}^N \rightarrow \{0,1\}$$

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$$OR(x_1, \dots, x_n) = \prod_{i=1}^n (1 - x_i)$$

$$\deg(OR) = n$$

$$L: \{0,1\}^N \rightarrow \{0,1\} \text{ s.t. } p \text{ is } \dots : OR$$

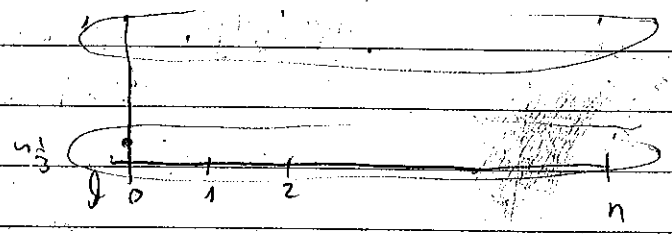
$$L(x) = 0$$

$$L(x) = 1$$

$$P(x_1, \dots, x_n) = L(x)$$

16 2 15 0.1

$\Theta(\sqrt{N})$ 203



$$\deg(OR) = \Theta(\sqrt{N})$$

$$\deg(f) \leq D(f) \leq 2(\deg(f))^4 \text{ if } D: \{0,1\}^N \rightarrow \{0,1\}$$

$$L_{\deg}(f) \leq R_2(f) \leq 216 (\deg(f))^6$$

$$D(f) \geq Q_E(f) \geq \frac{\deg(f)}{2}$$

BP: $\frac{\deg(f)}{2}$

BP: $\frac{\deg(f)}{2}$

$$R_2(f) \geq \frac{\deg(f)}{2}$$

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$$\deg(\alpha_w^{(k+1)}) \leq k$$

loop is (k+1) just to show you can do it

$$\alpha_w^{(k+1)}(x,0) = \begin{cases} x, & 0 \\ 0, & 1 \end{cases} \quad \begin{matrix} \alpha_x = 0 \\ \alpha_x = 1 \end{matrix}$$

$$\alpha_w^{(k+1)}(x,1) = \begin{cases} x, & 1 \\ 0, & 0 \end{cases} \quad \begin{matrix} \alpha_x = 0 \\ \alpha_x = 1 \end{matrix}$$

$$\alpha_w^{(k+1)} =$$

(when you) make a loop, x is the value of w

$$\alpha_w^{(k+1)} = (1 - \alpha_x) \alpha_w^{(k)} + \alpha_x \alpha_w^{(k)}$$

0 over the 1
into 0

$$\deg(\alpha_w^{(k+1)}) \leq \deg(\alpha_w^{(k)}) + 1 \text{ still } \leq k$$

$$\sum_{w \in W} |\alpha_w^{(k)}|^2$$

100 steps, 100

$$2T \geq \text{something}$$

for $\alpha_w = 1$ or $\alpha_w = 0$ it is possible to have a loop

$2T^* \geq \dots$ $0, \dots$ $f(0, \dots)$

$$Q_E(f) \geq \frac{\deg(f)}{2} \quad T \geq \frac{\deg(f)}{2}$$

$\frac{1}{3}$ slope of f

$f(0, \dots) = 1$ or $0, \dots$

$2T \geq \dots$

$$Q_2(f) \geq T \geq \frac{\deg(f)}{2} \quad \deg(f) \geq 2T$$