הפיזיקה מתייחסת לעולם כמערכת חישוב.

יש מצב.

ויש כלל התקדמות.

כשמסתכלים על תת-מערכת זה נראה שיש
כוחות שפועלים על גופים, אבל כשמסתכלים על
המערכת כולה, היקום כולו, אז יש מצב
וכלל התקדמות.

איך נראה מצב של העולם?

הנחה: יש ייצוג של היקום כוקטור ממימד סופי
מעל המרוכבים.

למשל: מס. האטומים / חלקיקים / whatever
ביקום סופי, וכל מצב ניתן לתאור
ע"י וקטור באורך סופי מעל המרוכבים.

הכוונה: בכלל, ההתקבצות הזו איזשהו טרנספורמציה
לינארית על המרחב $T: V \to V$.

מערכת כזו היא תאור כולל של היקום,
וממנה נגזרים מסקנות לגבי צופה, שצופה מבפנים
על תת מערכת, או פועל עליה.

מערכת כזו גם מאוד דומה למושגים החישוביים
שאנחנו מכירים ממדע המחשב.

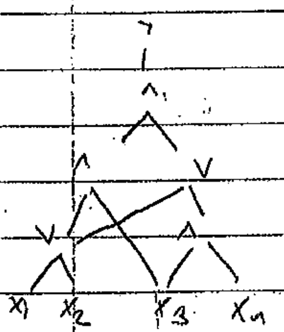
המשך [illegible]

1. חישוב

1.1 חישוב קלסי ע״י מעגלים

שפה: $L \subseteq \{0,1\}^*$

מודל חישוב: מעגל בוליאני



מעגל: גרף מכ״ח עם קודקוד יציאה אחד
ועם שערי $\wedge, \vee, \neg$

גודל מעגל: מס׳ השערים שבו.

נאמר שסדרה $\{C_n\}_{n\in\mathbb{N}}$ מחשבת שפה L
אם לכל $x = x_1 \ldots x_n \in \{0,1\}^n$ מתקיים

$$\underbrace{C_n(x_1, \ldots, x_n)}_{} = 1 \iff x \in L$$

C_n - מעגל על n קלטים

ונניח $1 = True$, $0 = False$

נאמר ש $L \in size(s(n))$ אם יש $\{C_n\}_{n\in\mathbb{N}}$ שמחשבת את L
ומתקיים לכל $n \in \mathbb{N}$ $|C_n| \le s(n)$.

טענה: לכל שפה $L \subseteq \{0,1\}^*$ מתקיים $L \in size(n \cdot 2^n)$.

הגדרה: $Psize = \bigcup_{k\in\mathbb{N}} size(n^k)$

ומבחינתנו מה שניתן לחשב ביעילות זה Psize (ניתן לחשוב יעיל)

מה ניתן לחשב על ביט אחד
ואם עם אנרגיה הפיכה?

4 פונק׳

$f(x) \equiv 0$
$f(x) \equiv 1$
$f(x) \equiv x$
$f(x) \equiv \neg x$

1.2 חישוב קוונטי על רגל אחת

הגדרה = אוסף המצבים האפשריים = $\mathbb{C}^2$ מנורמל לאורך 1

כלומר כל הוקטורים $v\in\mathbb{C}^2$, $\|v\|_2=1$

אין שקול: יש נבחר בסיס כלשהו ל $\mathbb{C}^2$, נאמר

הבסיס הסטנדרטי $e_0=\begin{pmatrix}1\\0\end{pmatrix}$, $e_1=\begin{pmatrix}0\\1\end{pmatrix}$

נסמן $|0\rangle=e_0$, $|1\rangle=e_1$

והמצבים, כל הוקטורים $\alpha|0\rangle+\beta|1\rangle$, $\alpha,\beta\in\mathbb{C}$, $|\alpha|^2+|\beta|^2=1$

	קלאסי	קוונטי
המצבים של ביט	$v\in\{0,1\}$	$v\in\{\alpha\|0\rangle+\beta\|1\rangle \mid \alpha,\beta\in\mathbb{C}, \|\alpha\|^2+\|\beta\|^2=1\}$

פעולה: מעבר של המצב למצב אחר.

יש לנו פעולות מותרות.

כל פעולה צריכה להיות פעולה לינארית

ויתרה מזאת היא אוניטרית (= שומרת זווית)

לינארית $A:\mathbb{C}^2\to\mathbb{C}^2$

כך ש: $u,v\in\mathbb{C}^2$: $\langle u,v\rangle=\langle Au,Av\rangle$

בפרט היא שומרת אורך: $\langle u,u\rangle=1 \Leftarrow \|u\|=1$

$\|Au\|=1 \Leftarrow \langle Au,Au\rangle=1 \Leftarrow$

באופן שקול: $AA^*=I$

<u>בשאלה 1</u>: הפעולה $not: \{0,1\} \to \{0,1\}$

המוגדרת ע"י $not(0)=1,\ not(1)=0$

נתבונן בהרחבה לפעולה אוניטרית:

כאשר נגדיר $not(|0\rangle)=|1\rangle,\ not(|1\rangle)=|0\rangle$

ונרחיב לינארית:

$$not(\alpha|0\rangle+\beta|1\rangle)=\alpha|1\rangle+\beta|0\rangle$$

מקבלים X

$$X=\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

שהיא אוניטרית כי $\underbrace{XX^{T}}_{X^{\dagger}} = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=I$

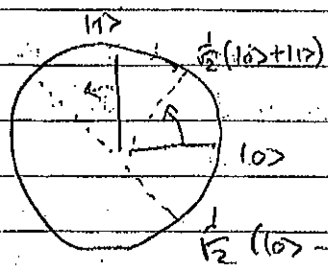
<u>בשאלה 2</u>: $H=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

$$HH^{\dagger}=\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\frac{1}{2}\begin{pmatrix}2&0\\0&2\end{pmatrix}=I$$

מה עושה H?

$$H(|0\rangle)=H\begin{pmatrix}1\\0\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}=\frac{1}{\sqrt{2}}|0\rangle+\frac{1}{\sqrt{2}}|1\rangle$$

$$H(|1\rangle)=H\begin{pmatrix}0\\1\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}=\frac{1}{\sqrt{2}}|0\rangle-\frac{1}{\sqrt{2}}|1\rangle$$



הזזה בזוית $\frac{\pi}{4}$

בסה"כ שינוי של זוית

<u>בשאלה 3</u>, אני סגה $Z=\begin{pmatrix}1&\\&-1\end{pmatrix}$ $\quad \alpha|0\rangle+\beta|1\rangle \Rightarrow \alpha|0\rangle-\beta|1\rangle$

מס' $|0\rangle \perp |1\rangle$ כי $\langle 0,0\rangle = \langle 1,1\rangle = 1$, מה הבעיה ב- $\mathbb{C}^2$

העולם של 2-qubit

~~וקטורים ב- $\mathbb{C}^4$ באורך 1~~ $= \{ v \in \mathbb{C}^4 \mid v = \alpha_0|00\rangle + \alpha_1|01\rangle + \alpha_2|10\rangle + \alpha_3|11\rangle,\ |\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 = 1 \}$

$\boxed{\text{מה קורה במערכת עם הרבה qubits?}}$

מכפלה טנזורית

יהיו V,W מ"וים

נגדיר $V \otimes W$

$$V \otimes W = \mathrm{span}\{ v \otimes w \mid v \in V,\ w \in W \} \quad \text{מעל } \mathbb{C}$$

עם חיבור, וכפל בסקלר מ- $\mathbb{C}$ ומזהים השוויונות:

$$\sum \alpha_i (v_i \otimes w) = \left(\sum \alpha_i v_i\right) \otimes w$$

$$\sum \alpha_i (v \otimes w_i) = v \otimes \left(\sum \alpha_i w_i\right)$$

טענה: אם V מ"ו ממימד n עם בסיס $E = \{e_1, \dots, e_n\}$

$F = \{f_1, \dots, f_m\}$ " " m " " W

אז $V \otimes W$ " " $n \cdot m$ " " $E \otimes F = \{e_i f_j \mid 1 \le i \le n,\ 1 \le j \le m\}$

הוכחה ① פורש $E \otimes F$

יהיו: $\sum \alpha_i e_i = v \in V$

$\sum \beta_j f_j = w \in W$

אז

$$v \otimes w = \sum \alpha_i \beta_j (e_i \otimes f_j)$$

② $E \otimes F$ הנ"ל

אם c_{ij} לא כולם אפס $\sum c_{ij}(e_i \otimes f_j)$ [illegible]

אז ההצגה זה יחיד [illegible]

[illegible]

[illegible] $\sum a_{ij} \, e_i \otimes f_j =$ [illegible]

הגדרה נוספת: אם V, W מרחבי מכפלה פנימית

אז $V \otimes W$ " " "

מ"פ מוגדרת ע"י: $\langle v_1 \otimes w_1, v_2 \otimes w_2 \rangle = \langle v_1, v_2 \rangle \cdot \langle w_1, w_2 \rangle$

מורחב ע"י: $\langle \sum \alpha_{ij} (v_i^1 \otimes w_j^1) , \sum \beta_{kl} (v_k^2 \otimes w_l^2) \rangle$

$$\sum_{i,j,k,l} \bar{\alpha}_{ij} \, \beta_{kl} \, \langle v_i^1 \otimes w_j^1 , v_k^2 \otimes w_l^2 \rangle$$

$\boxed{\text{למשל שימושי בפיזיקה}}$

בקוונטי: $\mathbb{C}^2 \otimes \mathbb{C}^2$ מימד 4

נסמן ע"י:

$$|0\rangle \otimes |0\rangle \overset{\text{סימון}}{=} |0,0\rangle$$
$$|0\rangle \otimes |1\rangle = |0,1\rangle$$
$$|1\rangle \otimes |0\rangle = |1,0\rangle$$
$$|1\rangle \otimes |1\rangle = |1,1\rangle$$

$$\langle \underbrace{0,0}_{} | \underbrace{0,1}_{} \rangle = \underbrace{\langle 0,0 \rangle}_{1} \cdot \underbrace{\langle 0,1 \rangle}_{0} = 0$$

שני איברי בסיס שונים

$\underbrace{\langle e_0, e_0 \rangle}_{\|e_0\| = 1}$ $\langle e_0, e_1 \rangle = 0$ $e_0 \perp e_1$

$$\begin{array}{ccccc} & |0,0\rangle & |0,1\rangle & |1,0\rangle & |1,1\rangle \\ 0,0 & \begin{pmatrix}1\\0\\0\\0\end{pmatrix} & \begin{pmatrix}0\\1\\0\\0\end{pmatrix} & \begin{pmatrix}0\\0\\1\\0\end{pmatrix} & \begin{pmatrix}0\\0\\0\\1\end{pmatrix} \\ 0,1 & & & & \\ 1,0 & & & & \\ 1,1 & & & & \end{array}$$

הצגה כוקטור

הנ"ל היא המגי הבסיסי של $\mathbb{C}^4$.

סימון: $\mathcal{H}_2$ – מרחב הילברט ממימד 2.

$\mathbb{C}^2$, עם מ"פ, $|0\rangle \perp |1\rangle$

$\mathcal{H}_{2^m}$ – מרחב הילברט ממימד 2^m

מוגדר ע"י

$$\mathcal{H}_{2^m} = \underbrace{\mathcal{H}_2 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_2}_{m \text{ פעמים}}.$$

השוואה של m-qubits הוא: $\mathcal{H}_{2^m}$

	קלסי	קוונטי
השוואה של m ביטים	$v \in \{0,1\}^m = \{0,1\} \times \{0,1\} \times \cdots \times \{0,1\}$	$\|v\|=1,\ v = \sum_{i_1,\dots,i_m \in \{0,1\}} \alpha_{i_1 \dots i_m} \|i_1,\dots,i_m\rangle$ $\mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_2$

בפועל שנתון חישוב על m-qubits:

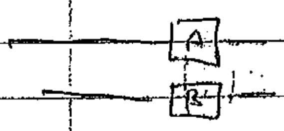
ב. אז $A:\mathcal{H}_{2^m}\to\mathcal{H}_{2^m}$ אוניטרי

	קלסי	קוונטי
פעולות	כל מיפוי $f:\{0,1\}^m\to\{0,1\}^m$	כל אוניטרי $A:\mathcal{H}_{2^m}\to\mathcal{H}_{2^n}$

<u>דוגמא 1</u>: מערכת של 2 qubits:

נפעיל A על qubit הראשון $A:\mathcal{H}_2\to\mathcal{H}_2$

B " " השני



פעולה מקבילית זו נקרא $A\otimes B$

כיון שאנו מגדירים פעולה לינארית, מספיק להגדיר איך מוגדר על וקטורי הבסיס. הבסיס הוא $\{|i\rangle\otimes|j\rangle\}$ כאשר

$|0,0\rangle=|0\rangle\otimes|0\rangle,\ |0\rangle\otimes|1\rangle,\ |1\rangle\otimes|0\rangle,\ |1\rangle\otimes|1\rangle$

$$|i\rangle\otimes|j\rangle\xrightarrow{(A\otimes B)}|A_i\rangle\otimes|B_j\rangle$$

מה המטריצה של $A\otimes B$?

בבסיס הסטנדרטי $\{|i\rangle\otimes|j\rangle\}$

$$(A\otimes B)_{(i,j),(k,l)}=$$

$$\langle|i\rangle\otimes|j\rangle\,|\,A\otimes B\,|\,|k\rangle\otimes|l\rangle\rangle$$

$$=\langle i\otimes j\,|\,A|k\rangle\otimes B|l\rangle\rangle=\langle i|A|k\rangle\langle j|B|l\rangle=\boxed{A_{ik}B_{jl}}$$

במעגלים:

במטריצה:

$$H\otimes H=\begin{pmatrix}1&1\\1&-1\end{pmatrix}\otimes\begin{pmatrix}1&1\\1&-1\end{pmatrix}=\begin{array}{c|c} & \begin{array}{cccc}00&01&&\end{array}\\ \begin{array}{c}00\\01\\10\\11\end{array} & \begin{pmatrix}1&1&1&1\\1&-1&1&-1\\1&1&-1&-1\\1&-1&-1&1\end{pmatrix}\end{array}$$

$i=0$, $k=0$ — בלוק אם — $\langle i|H|k\rangle=1$ — כלומר מתקיים $B=H$

$i=1$, $k=1$ — בלוק זה — $\langle i|H|k\rangle=-1$ — ולמעשה $-H=-B$

$$(H\otimes H)\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=(H\otimes H)(|0\rangle\otimes|0\rangle)=|H0\rangle\otimes|H0\rangle=\frac{1}{\sqrt2}(|0\rangle+|1\rangle)\otimes\frac{1}{\sqrt2}(|0\rangle+|1\rangle)$$

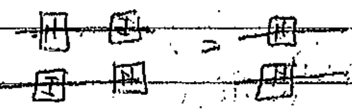
$$=\frac12\left[|00\rangle+|01\rangle+|10\rangle+|11\rangle\right]=\frac12\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$(H\otimes H)\begin{pmatrix}0\\1\\0\\0\end{pmatrix}=H\otimes H\,(|0\rangle\otimes|1\rangle)=|H0\rangle\otimes|H1\rangle=\frac12(|0\rangle+|1\rangle)\otimes(|0\rangle-|1\rangle)$$

$$=\frac12\left[|00\rangle-|01\rangle+|10\rangle-|11\rangle\right]=\frac12\begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}$$

טענה: A,B אוניטריות, אז ~~$A\otimes B$~~ אוניטרית

$$(A\otimes B)^\dagger=A^\dagger\otimes B^\dagger$$



$$(A\otimes B)\cdot(C\otimes D)=AC\otimes BD$$

$(I\otimes H)(H\otimes I)=$ ~~$(\dots)$~~ $=H\otimes H$

הוכחה:

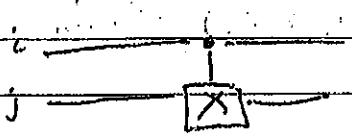
$$(A\otimes B)(A\otimes B)^\dagger=(A\otimes B)(A^\dagger\otimes B^\dagger)=AA^\dagger\otimes BB^\dagger=I\otimes I=I$$

מוגדר ע"י

מסקנה: אם A,B אוניטריות אז $A\otimes B$ אוניטרית.

דוגמא 2: CNOT

$$\mathrm{CNOT}\left(|i\rangle \otimes |j\rangle\right) = \begin{cases} |i\rangle|j\rangle & |i\rangle = |0\rangle \\ |i\rangle \otimes X|j\rangle & |i\rangle = |1\rangle \end{cases}$$



עם מטריצה

$$\begin{array}{c|c} & \begin{matrix} 00 & 01 & & \end{matrix} \\ \begin{matrix} 00 \\ 01 \\ 10 \\ 11 \end{matrix} & \begin{pmatrix} 1 & 0 & & 0 \\ 0 & 1 & & \\ & & 0 & 1 \\ & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$|0,0\rangle \to |0,0\rangle$

$|0,1\rangle \to |0,1\rangle$

$|1,0\rangle \to |1,1\rangle$

$|1,1\rangle \to |1,0\rangle$

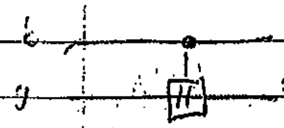
מטריצת פרמוטציה, היא אוניטרית.

הוכחה: Π היא פרמוטציה, אז $\Pi^{\dagger} = (\Pi^{t})^{*} = \Pi^{t} = \Pi^{-1}$

כי זו פרמוטציה ממשית

לכן $\Pi\Pi^{\dagger} = I$, Π אוניטרית.

כמו: Controlled Hadamard - CH



[illegible] שורות

$$\begin{pmatrix} 1 & 0 & & \\ 0 & 1 & & 0 \\ & 0 & 1 & 1 \\ & & 1 & -1 \end{pmatrix}$$

$|0,0\rangle \Rightarrow |0,0\rangle$

$|0,1\rangle \Rightarrow |0,1\rangle$

$|1,0\rangle \to |1,+\rangle$

$|1,1\rangle \Rightarrow |1,-\rangle$

השוואה מכפלה קרטזית מול מכפלה טנזורית

V, W

	מכפלה קרטזית	מכפלה טנזורית
חבור:	קורדינטה קורדינטה $(V,W)=(V,0)+(0,W)$	[illegible] (V,W) [illegible] מלבד שוויון $\sum\alpha_i(V_i,W)=(\sum\alpha_i V_i,W)$ $\sum\alpha_i(V,W_i)=(V,\sum\alpha_i W_i)$
בסיס	$\{(v,0)\}\cup\{(0,w)\}$ ממד $\dim(V)+\dim(W)$	$\{(e_i,f_j)\}$ ממד $\dim(V)\cdot\dim(W)$
[illegible]	$(1,1)+(2,3)=(3,4)$	$1\otimes 1+2\otimes 3$

12

Young Two-Slit Experiment:

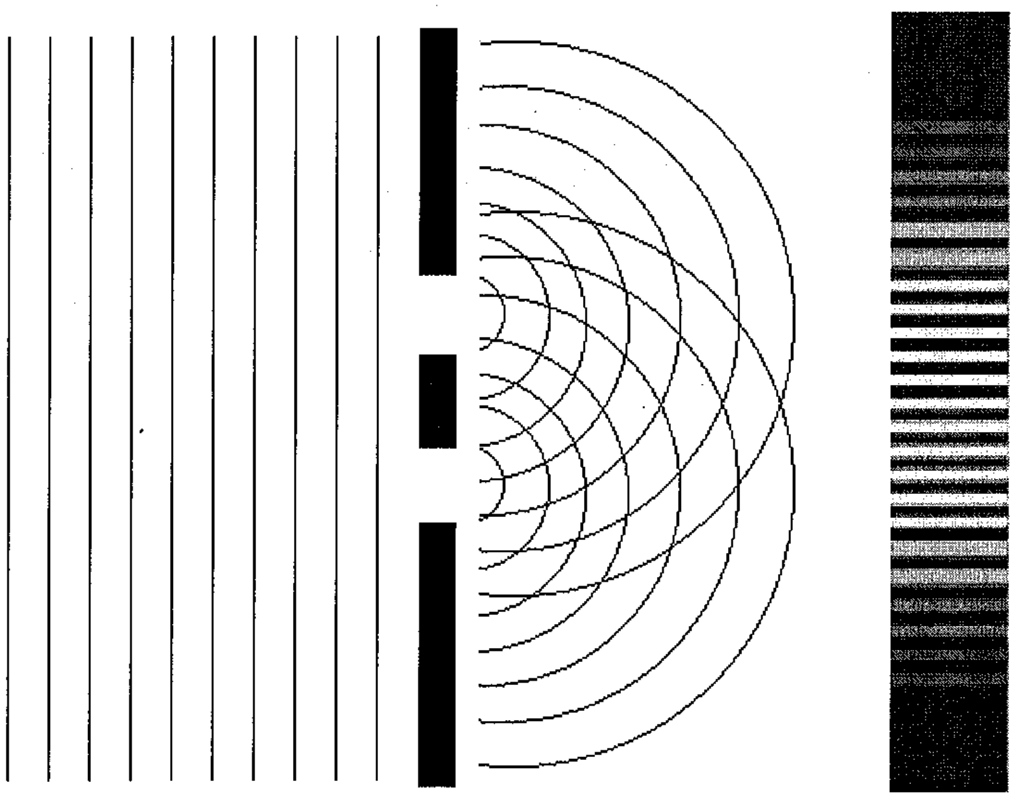
abyss.uoregon.edu/~js/21_century_science/lectures/lec13.html

Readings: two slit experiment Copenhagen Interpretation Quantum Mechanics

- **the two slit experiment is key to understand the microscopic world**

The wave-like properties of light were demonstrated by the famous experiment first performed by Thomas Young in the early nineteenth century. In original experiment, a point source of light illuminates two narrow adjacent slits in a screen, and the image of the light that passes through the slits is observed on a second screen.

Interference



click here to interference movie

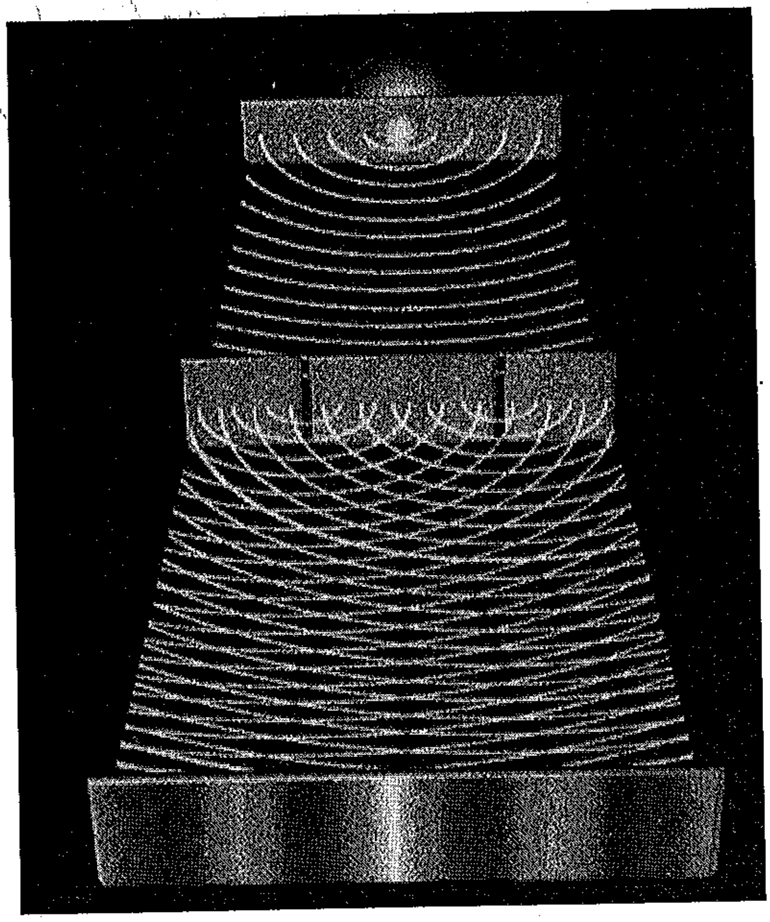
click here to see a wave experiment

- **waves can interfere, for light this will make**

The dark and light regions are called interference fringes, the constructive and destructive interference of light waves. So the question is will matter also produce interference patterns. The answer is yes, tested by firing a stream of electrons.

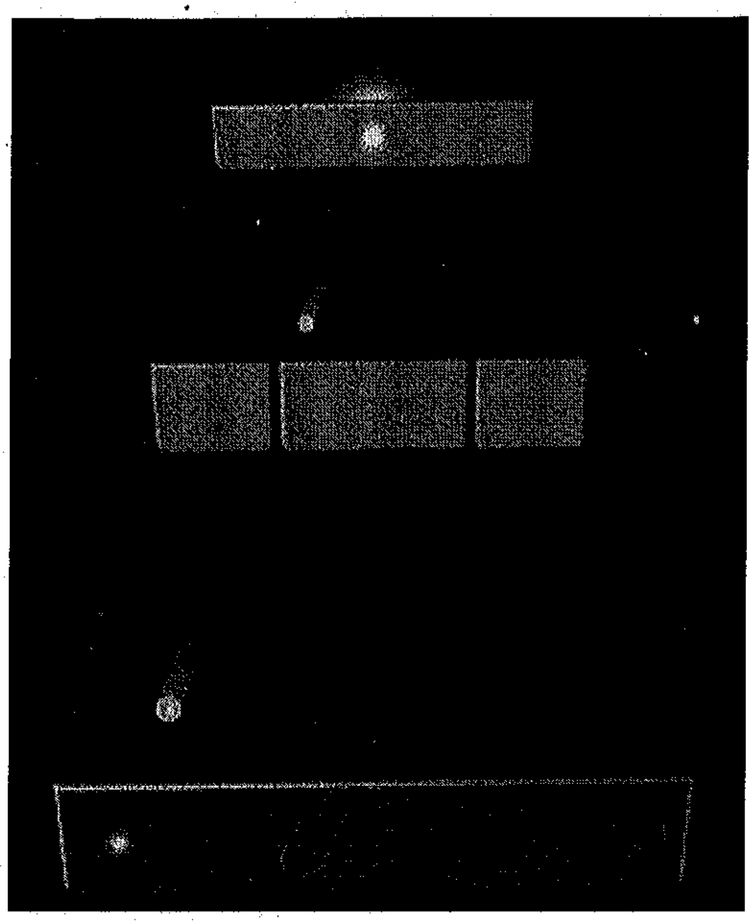
a series of light and dark bands

- matter particles, such as electrons, also produce interference patterns due to their wave-like nature
- so with a high flux of either photons or electrons, the characteristic interference pattern is visible



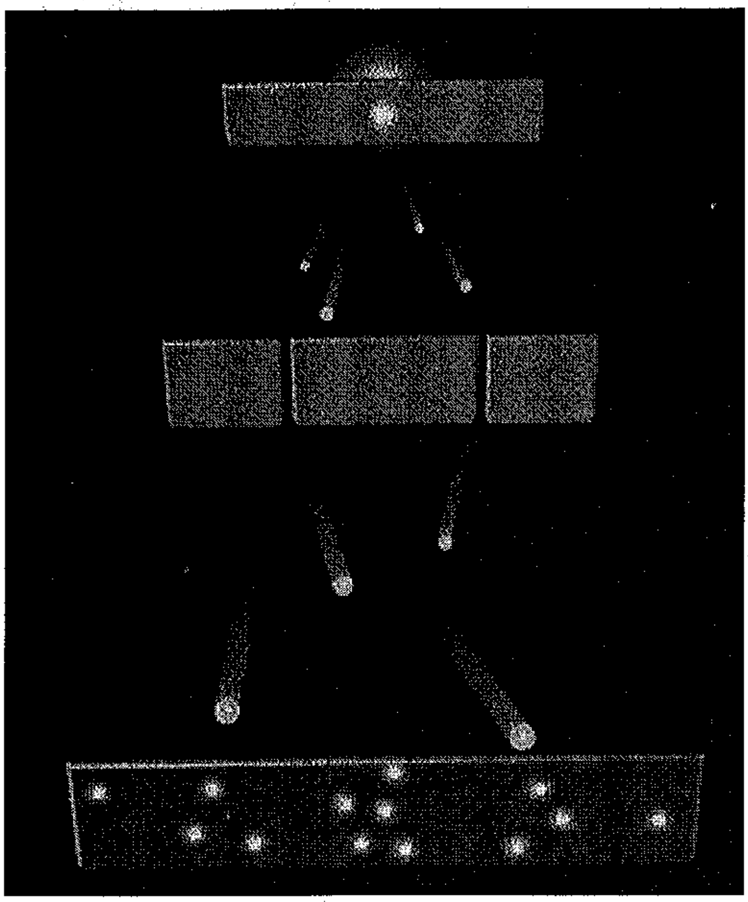
- if we lower the intensity of light, or the flux of electrons (the electric current), we should be able to see each photon strike the screen
- each photon makes a dot on the screen, but where is the interference pattern?

However, notice that electrons do act as particles, as do photons. For example, they make a single strike on a cathode ray tube screen. So if we lower the number of electrons in the beam to, say, one per second. Does the interference pattern disappear?



- the interference pattern is still there, it simply takes some time for enough photons, or electrons, to strike the screen to build up a recognizable pattern
- interference, or a wave phenomenon, is still occurring even if we only let the photons, or electrons, through one at a time
- so what are the individual particles interfering with? apparently, themselves

The answer is no, we do see the individual electrons (and photons) strike the screen, and with time the interference pattern builds up. Notice that with such a slow rate, each photon (or electron) is not interacting with other photons to produce the interference pattern. In fact, the photons are interacting with themselves, within their own wave packets to produce interference.



- **in order for a particle to interfere with itself, it must pass through both slits**
- **this forces us to give up the common sense notion of location**

But wait, what if we do this so slow that only one electron or one photon passes through the slits at a time, then what is interfering with what? i.e. there are not two waves to destructively and constructively interfere. It appears, in some strange way, that each photon or electron is interfering with itself. That its wave nature is interfering with its own wave (!).

click here to see a particle experiment

The formation of the interference pattern requires the existence of two slits, but how can a single photon passing through one slit 'know' about the existence of the other slit? We are stuck going back to thinking of each photon as a wave that hits both slits. Or we have to think of the photon as splitting and going through each slit separately (but how does the photon know a pair of slits is coming?). The only solution is to give up the idea of a photon or an electron having location. The location of a subatomic particle is not defined until it is observed (such as striking a screen).

Role of the Observer:

- **since the quantum world can not be observed directly, we**

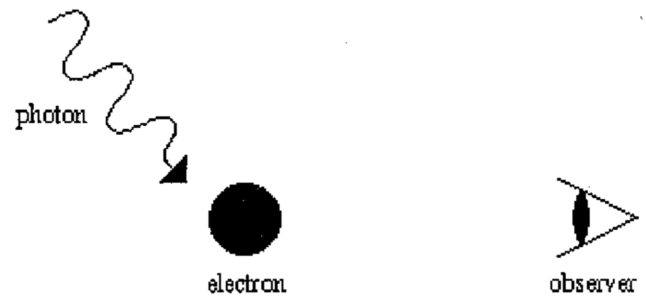
The quantum world can be not be perceived directly, but rather through the use of instruments. And, so, there is a problem with the fact that the act of measuring disturbs the energy and position of subatomic particles. This is called the

- **are forced to use instruments as extensions of our senses**
- **however, quantum entities are so small that even contact with one photon changes their position and momentum = measurement problem**
- **1st hint that the observer is an important piece of any quantum experiment, can not isolate the observer or their effects**

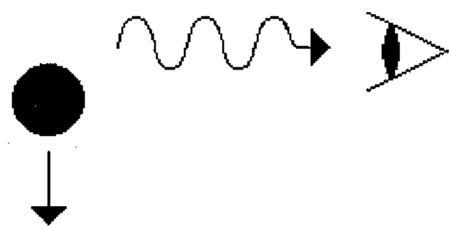
measurement problem.

Measurement Problem in Quantum Mechanics

before observation



after observation



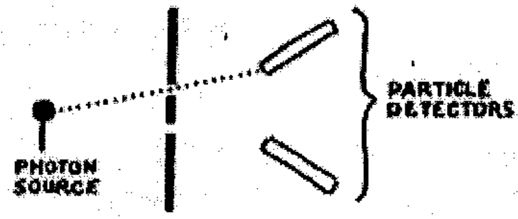
the act of observing effects the position and energy of electron

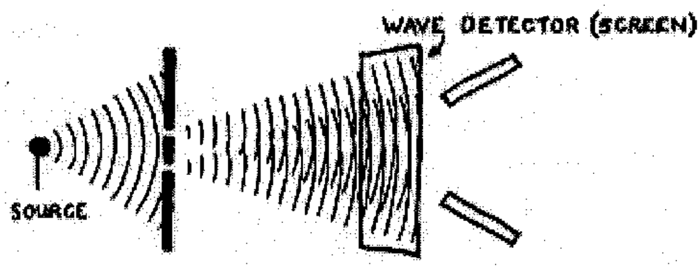
- **the two slit experiment is a good test of the role of the observer in the quantum realm**
- **any experimental design that attempts to determine which slit a photon has passed through (test for its particle nature) destroys the interference pattern (its wavelike nature)**
- **this is a breakdown of objective reality**
- **each quantum entity**

Thus, we begin to see a strong coupling of the properties of an quantum object and and the act of measuring those properties. The question of the reality of quantum properties remains unsolved. All quantum mechanical principles must reduce to Newtonian principles at the macroscopic level (there is a continuity between quantum and Newtonian mechanics).

How does the role of the observer effect the wave and particle nature of the quantum world? One test is to return to the two slit experiment and try to determine count which slit the photon goes through. If the photon is a particle, then it has to go through one or the other slit. Doing this experiment results in wiping out the interference pattern. The wave nature of the light is eliminated, only the particle nature remains and particles cannot make interference patterns. Clearly the two slit experiments, for the first time in physics, indicates that there is a much deeper relationship between the observer and the phenomenon, at least at the subatomic level. This is an extreme break from the idea of an objective reality or one where the laws of Nature have a special, Platonic existence.

has dual potential properties, which become an actual characteristic if and when it is observed





If the physicist looks for a particle (uses particle detectors), then a particle is found. If the physicist looks for a wave (uses a wave detector), then a wave pattern is found. A quantum entity has a dual *potential* nature, but its *actual* (observed) nature is one or the other.

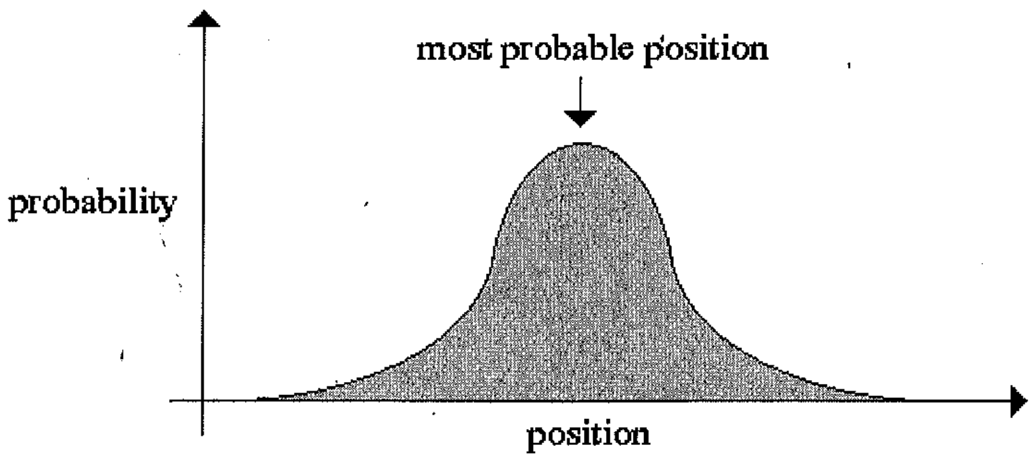
Quantum Wave Function:

- **a wave packet interpretation for particles means there is an intrinsic fuzziness assign to them**
- **the wave function is the mathematical tool to describe quantum entities**

The wave nature of the microscopic world makes the concept of 'position' difficult for subatomic particles. Even a wave packet has some 'fuzziness' associated with it. An electron in orbit has no position to speak of, other than it is somewhere in its orbit.

To deal with this problem, quantum physics developed the tool of the quantum wave function as a mathematical description of the superpositions associated with a quantum entity at any particular moment.

Quantum Wave Function



- **wave function express likelihood *until* a measurement is made**

The key point to the wave function is that the position of a particle is only expressed as a likelihood or probability *until* a measurement is made. For example, striking an electron with a photon results in a position measurement and we say that the wave function has `collapsed' (i.e. the wave nature of the electron converted to a particle nature).

<u>Superposition</u>:

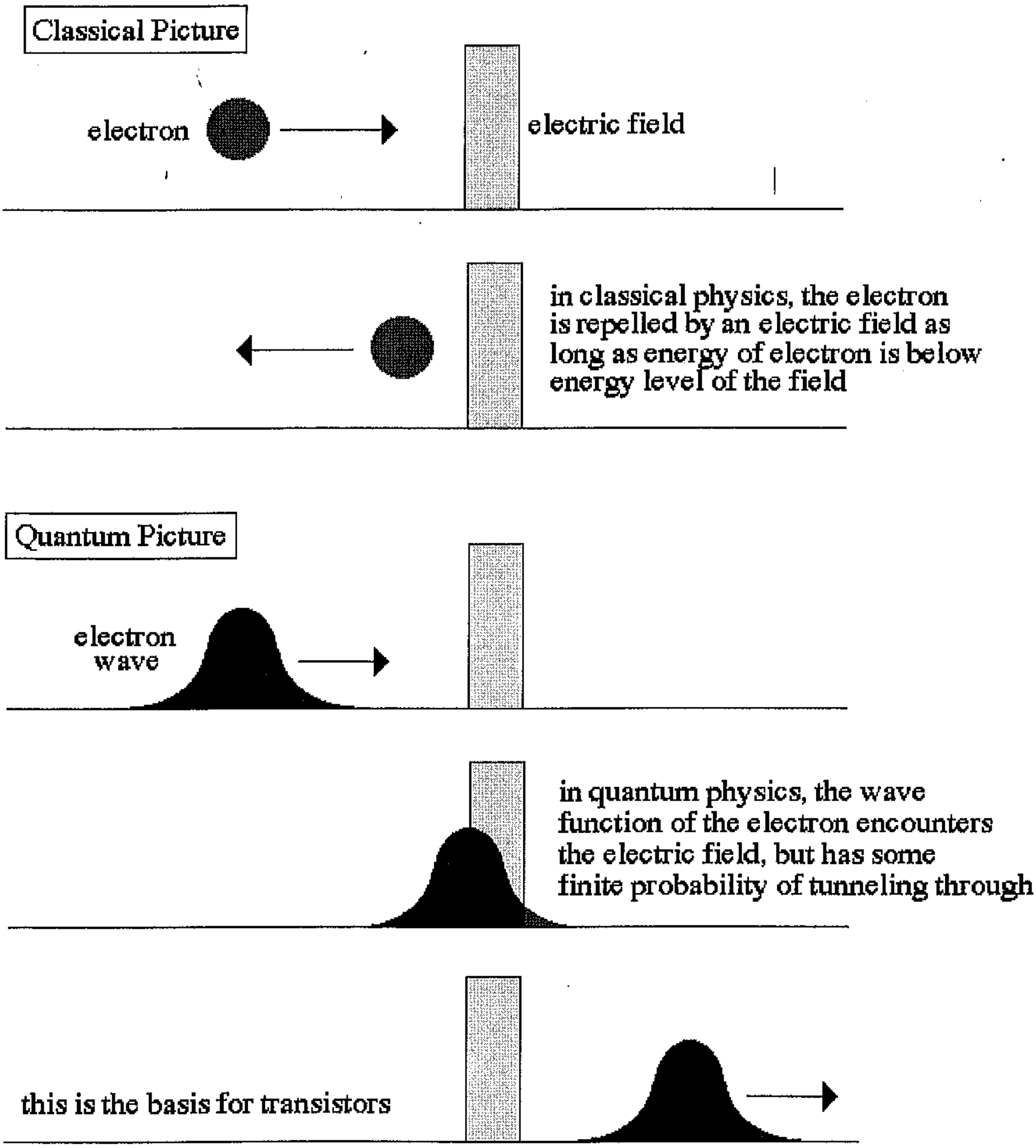
- **quantum physics is a science of possibilities rather than exactness of Newtonian physics**
- **quantum objects and quantities becomes actual when observed**
- **key proof of quantum superpositions is the phenomenon of quantum tunneling**

The fact that quantum systems, such as electrons and protons, have indeterminate aspects means they exist as possibilities rather than actualities. This gives them the property of being things that might be or might happen, rather than things that are. This is in sharp contrast to Newtonian physics where things are or are not, there is no uncertainty except those imposed by poor data or limitations of the data gathering equipment.

Further experimentation showed that reality at the quantum (microscopic) level consists of two kinds of reality, actual and potential. The actual is what we get when we see or measure a quantum entity, the potential is the state in which the object existed before it was measured. The result is that a quantum entity (a photon, electron, neutron, etc) exists in multiple possibilities of realities known as superpositions.

The superposition of possible positions for an electron can be demonstrated by the observed phenomenon called <u>quantum tunneling</u>.

Quantum Tunneling



- **the position of the electron, the wave function, is truly spread out, not uncertain**
- **observation causes**

Notice that the only explanation for quantum tunneling is if the position of the electron is truly spread out, not just hidden or unmeasured. It raw uncertainty allows for the wave function to penetrate the barrier. This is genuine indeterminism, not simply an unknown quantity until someone measures it.

It is important to note that the superposition of possibilities only occurs before the entity is observed. Once an observation is made (a position is measured, a mass is determined, a velocity is detected) then the superposition converts to an actual. Or, in

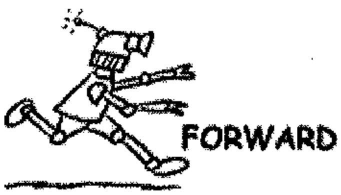
the wave function to collapse to an actual

quantum language, we say the wave function has collapsed.

- **quantum existence is tied to the environment, opposite to the independence of macroscopic objects**

The collapse of the wave function by observation is a transition from the many to the one, from possibility to actuality. The identity and existence of a quantum entities are bound up with its overall environment (this is called contextualism). Like homonyms, words that depend on the context in which they are used, quantum reality shifts its nature according to its surroundings.

In the macroscopic world ruled by classical physics, things are what they are. In the microscopic world ruled by quantum physics, there is an existential dialogue among the particle, its surroundings and the person studying it.



מסקנה

איך מקבלים תוצאות בחישוב קוונטי?

מדידה: חלוקה של המרחב $\mathcal{H}_{2^m}$ לתתי אורתוגונליים | מדידות כללית

$$\mathcal{H}_{2^m} = W_1 \oplus W_2 \ \ldots \ \oplus W_k$$

כש W_i תתי אורתוגונלים אחד לשני שסכומם הישר הוא $\mathcal{H}_{2^m}$

תוצאות המדידה: ~~...~~ $\forall v \in \mathcal{H}_{2^m}$

נסמן E_i ההטלה של $\mathcal{H}_{2^m}$ על W_i

E_i הם אופרטורים $(\sum E_i = I \ , \ E_i \geq 0)$

אזי כאן במצב $v \in \mathcal{H}_{2^m}$, מקבלים תוצאה i בהסתברות $\|E_i v\|$

ובהנחה שקיבלנו תוצאה i נעבור למצב $\frac{E_i v}{\|E_i v\|}$

דוגמא: מודדים יפה: ה-qubit הראשון

$W_0 = \text{span}\{|0\rangle \otimes \mathcal{H}_{2^{m-1}}\}$ נקבל 2 ת"מ

$W_1 = \text{span}\{|1\rangle \otimes \mathcal{H}_{2^{m-1}}\}$

ת"מ אורתוגונליים שסכומם פורשים את $\mathcal{H}_{2^m}$.

א"כ

$\alpha_{i_1,\ldots,i_m} \in \mathbb{C}$ $\quad V = \sum \alpha_{i_1\ldots i_m} |i_1,\ldots,i_m\rangle$

$\sum_{i_1,\ldots,i_m} |\alpha_{i_1\ldots i_m}|^2 = 1$ $\quad i_1,\ldots,i_m \in \{0,1\}$

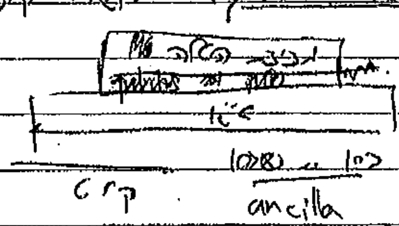
עם הסתברות $\sum_{i_2\ldots i_m} |\alpha_{0,i_2\ldots i_m}|^2$ נקבל 0

ונפל למצב $|0\rangle \otimes \sum_{i_2\ldots i_m} \alpha_{0,i_2\ldots i_m} |i_2,\ldots,i_m\rangle$ מנורמל

ובהסתברות $\sum_{i_2\ldots i_m} |\alpha_{1,i_2\ldots i_m}|^2$ נקבל 1

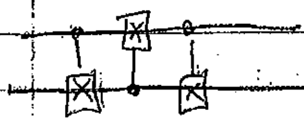
ונפל למצב $|1\rangle \otimes \sum_{i_2\ldots i_m} \alpha_{1,i_2\ldots i_m} |i_2,\ldots,i_m\rangle$ מנורמל

הערה: משפט שנוכיח בהמשך: מספיק קיוביט אחד.



דוגמא: מצב $\frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |10\rangle)$ מודדים qubit ראשון

בהסתב' $\frac{1}{3}$ נקבל 0, ונפל ל $|00\rangle$, בהסתב' $\frac{2}{3}$ נקבל 1 ונפל ל $\frac{1}{\sqrt{2}}(|10\rangle + |11\rangle)$



$$|a,b\rangle \longrightarrow |a, a\oplus b\rangle$$

$$\longrightarrow |a\oplus a\oplus b,\ a\oplus b\rangle = |b, a\oplus b\rangle$$

$$\longrightarrow |b, a\oplus b\oplus b\rangle = |b,a\rangle$$

Super dense Coding   entanglement

מצב $\psi \in H_A \otimes H_B$ נקרא שזור אם $\psi \neq v \otimes w$
עבור $v \in H_A$, $w \in H_B$, אחרת הוא לא entangled.

אפשר לומר "כמה" הוא "entangled" אבל לא נעסוק בזה כעת.

דוגמה: $\varphi = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ הוא לא מכפלה

הוכחה: נניח $v = \alpha|0\rangle + \beta|1\rangle$
$w = \gamma|0\rangle + \delta|1\rangle$

$$v \otimes w = \alpha\gamma|00\rangle + \alpha\delta|01\rangle + \beta\gamma|10\rangle + \beta\delta|11\rangle$$

לכן אם $\varphi = v \otimes w$
אז

$$\alpha\gamma = \beta\delta = \frac{1}{\sqrt{2}} \quad (1)$$

$$\alpha\delta = \beta\gamma = 0$$

לכן $\alpha \neq 0$ וגם $\delta \neq 0$ סתירה ל-(1)

$\frac{1}{\sqrt{2}}[|00\rangle + |11\rangle]$ נקרא EPR-pair על שם איינשטיין-פודולסקי-רוזן

No cloning thm

משפט: אין A על $\mathcal{H}_{2^2}$ כך שלכל $\psi\in\mathcal{H}_2$

$$A\big(|\psi\rangle\otimes|0\rangle\big) = |\psi\rangle\otimes|\psi\rangle \quad (*)$$

הוכחה: יהיו $e_0, e_1 \in \mathcal{H}_2$ שמקיימים את הדרישה (*)

$$A\big(|e_i\rangle\otimes|0\rangle\big) = |e_i\rangle\otimes|e_i\rangle$$

תזכורת: [illegible]

$$A\big(|e_0\rangle\otimes|0\rangle\big) = |e_0\rangle\otimes|e_0\rangle$$
$$A\big(|e_1\rangle\otimes|0\rangle\big) = |e_1\rangle\otimes|e_1\rangle$$

A אוניטרי, לכן: $\langle x,y\rangle = \langle Ax, Ay\rangle$

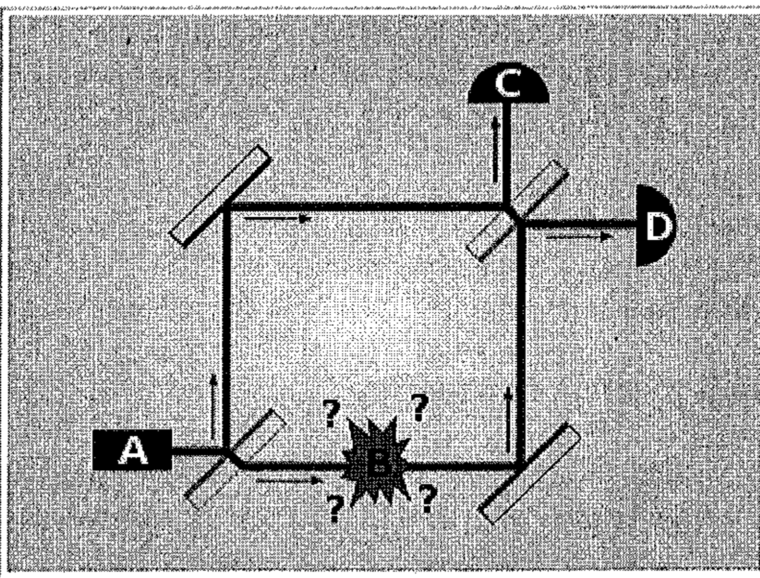
$$\langle e_0,e_1\rangle = \langle e_0,0 \mid e_1,0\rangle = \langle e_0,e_0 \mid e_1,e_1\rangle = \langle e_0,e_1\rangle\langle e_0,e_1\rangle$$

לכן $\langle e_0,e_1\rangle\in\{0,1\}$, כלומר או $e_0=e_1$ או $e_0\perp e_1$. □

Elitzur–Vaidman bomb tester

From Wikipedia, the free encyclopedia

In physics, the **Elitzur–Vaidman bomb-testing problem** is a thought experiment in quantum mechanics, first proposed by Avshalom Elitzur and Lev Vaidman in 1993.[1] An actual experiment demonstrating the solution was constructed and successfully tested by Anton Zeilinger, Paul Kwiat, Harald Weinfurter, and Thomas Herzog From the University of Innsbruck, Austria and Mark A. Kasevich of Stanford University in 1994.[2] It employs a Mach–Zehnder interferometer for ascertaining whether a measurement has taken place.



Bomb-testing problem diagram. A - photon emitter, B - bomb to be tested, C,D - photon detectors. Mirrors in the lower left and upper right corners are half-silvered.

Contents

Problem

Consider a collection of bombs, of which some, but not all, are duds. Suppose that these bombs possess certain properties: usable (non-dud) bombs have a photon-triggered sensor, which will absorb an incident photon and detonate the bomb. Dud bombs have a malfunctioning sensor, which will not interact with the photons in any way.[3] Thus, the dud bomb will not detect the photon and will not detonate. The problem is how to separate at least some of the usable bombs from the duds. A bomb sorter could accumulate dud bombs by attempting to detonate each one. Unfortunately, this naive process destroys all the usable bombs.

Solution

A solution is for the sorter to use a mode of observation known as counterfactual measurement, which relies on properties of quantum mechanics.[4]

Start with a Mach–Zehnder interferometer and a light source which emits single photons. When a photon emitted by the light source reaches a half-silvered plane mirror, it has equal chances of passing through or reflecting.[5] On one path, place a bomb (B) for the photon to encounter. If the bomb is working, then the photon is absorbed and triggers the bomb. If the bomb is non-functional, the photon will pass through the dud bomb unaffected.

When a photon's state is non-deterministically altered, such as interacting with a half-silvered mirror where it non-deterministically passes through or is reflected, the photon undergoes quantum superposition, whereby it takes on all possible states and can interact with itself. This phenomenon continues until an 'observer' (detector) interacts with it, causing the wave function to collapse and returning the photon to a deterministic state.

Step-by-step explanation

- After being emitted, the photon 'probability wave' will *both* pass through the 1st half-silvered mirror (take the lower-route) *and* be reflected (take the upper-route).

If the bomb is a dud:

- The bomb will not absorb a photon, and so the wave continues along the lower route to the second half silvered mirror (where it will encounter the upper wave and cause self-interference).
- The system reduces to the basic Mach–Zehnder apparatus with no sample bomb, in which constructive interference occurs along the path horizontally exiting towards (D) and destructive interference occurs along the path vertically exiting towards (C).
- Therefore, the detector at (D) will detect a photon, and the detector at (C) will not.

If the bomb is usable:

- Upon meeting the observer (the bomb), the wave function collapses and the photon must be either on the lower route or on the upper route, but not both.
- **If the photon is measured on the lower route**:
  - Because the bomb is usable, the photon is absorbed and triggers the bomb which explodes.
- **If the photon is measured on the upper route**:
  - It will not encounter the bomb - but since the lower route can not have been taken, there will be no interference effect at the 2nd half-silvered mirror.
  - The photon on the upper route now both (i) passes through the 2nd half-silvered mirror and (ii) is reflected.
  - Upon meeting further observers (detector C and D), the wave function collapses again and the photon must be either at detector C or at detector D, but not both.

Thus we can state that if any photons are detected at (C), there must have been a working detector at (B) – the bomb position.

With this process, 25% of the usable bombs can be identified as usable without being consumed.[1] whilst 50% of the usable bombs will be consumed and 25% remain 'unknown'. By repeating the process with the 'unknowns', the ratio of surviving, identified, usable bombs approaches 33% of the initial population of usable bombs. See Experiments section below for a modified experiment that can identify the usable bombs with a yield rate approaching 100%.

This phenomenon may be seen as a form of weak coupling between worlds in the many-worlds interpretation.[*citation needed*]

Experiments

In 1994, Anton Zeilinger, Paul Kwiat, Harald Weinfurter, and Thomas Herzog actually performed an equivalent of the above experiment, proving interaction-free measurements are indeed possible.[2]

In 1996, Kwiat et al. devised a method, using a sequence of polarising devices, that efficiently increases the yield rate to a level arbitrarily close to one. The key idea is to split a fraction of the photon beam into a large number of beams of very small amplitude, and reflect all of them off the mirror, recombining them with the original beam afterwards.[6] (See also http://www.nature.com/nature/journal/v439/n7079/full/nature04523.html#B1 .) It can also be argued that this revised construction is simply equivalent to a resonant cavity and the result looks much less shocking in this language. See Watanabe and Inoue (2000).

This experiment is philosophically significant because it determines the answer to a counterfactual question: "What would happen were the photon to go into the bomb sensor?". The answer is either: "the bomb would work, the photon would be observed, and the bomb would explode", or "the bomb is a dud, the photon would not be observed and it would pass through unimpeded". If we were to directly perform the measurement, any working bomb would actually explode. But here the answer to the question "what would happen" can be determined *without* a working bomb necessarily going off. This provides an example of an experimental method to answer a counterfactual question.

See also

- Counterfactual definiteness
- Interaction-free measurement
- Mach–Zehnder interferometer
- Renninger negative-result experiment

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1. ^ *a b* Elitzur & Vaidman 1993
2. ^ *a b* Experimental realization of "interaction-free" measurements (http://www.univie.ac.at/qfp/publications3/pdffiles/1994-08.pdf), Paul Kwiat 1994
3. ^ Can Schrodinger's Cat Collapse the Wavefunction? (http://nonlocal.com/quantum-d/v2/kbowden_03-15-97.html), Keith Bowden 1997
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5. ^ David Harrison
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