Amnon Ta-Shma
Dept. of Computer Science
Tel Aviv University

Fall 2013
Quantum Computation

Homework 3

1. Let ψ_1, ψ_2 be two pure states in a Hilbert space $\mathcal{H}$. Prove that

$$\| \, |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2| \, \|_{trace} \;\leq\; 2\sqrt{1 - \mid \langle \psi_1 \mid \psi_2 \, \rangle \mid^2}.$$

2. Write an experiment table for the quantum algorithm for the CHSH game. The entries of the table should be the same as in the PR-box (Popescu-Rohrlich box) but the values are of course different.

   - Check directly that the protocol is non-signalling.
   - A table is *product*, if for each of the four possible questions, the 2×2 table describing the probability of each possible answer, can be expressed as a product of two probability distributions, each describing a probability distribution on answers for a single party. Prove that the table of the quantum protocol cannot be expressed as a convex combination of product tables.

3. Let w be a primitive root of unity of order m. Show an efficient quantum circuit for the transformation $|j,k\rangle \to w^{jk}|j,k\rangle$, where $j,k \in \{0,\ldots,m-1\}$. You may use any one, two or three qubit gates you wish.

4. Let Angle be the following promise problem:

   Input : A pure state $\phi_1 \otimes \phi_2$, $0 \leq \alpha < \beta \leq 1$.

   Yes instances : $|\langle\phi_1|\phi_2\rangle| \leq \alpha$

   No instances : $|\langle\phi_1|\phi_2\rangle| \geq \beta$

   Design a quantum circuit that accepts Yes instances with probability at least p, and No instances with probability at most q for some $q < p$.

5. Run the quantum phase estimation R times as in the attached Figure. Assume we run the algorithm on an eigenvector with eigenvalue w^λ, where $w = e^{2\pi\theta i/T}$.

   - What is the probability to measure $(s_1,\ldots,s_R) \in [T]^R$?
   - Suppose the algorithm outputs the most frequent s in $\{s_1,\ldots,s_R\}$. Let $\zeta(s)$ be the probability the algorithm outputs s. What can you say about the function ζ?