1. (Continued fractions) For positive numbers \(a_0, \ldots, a_k\) denote \([a_0] = a_0, [a_0, \ldots, a_k] = a_0 + \frac{1}{[a_1, \ldots, a_k]}\) for \(k > 0\). Assume \([a_0, \ldots, a_n] = \frac{p_n}{q_n}\). The following is a recursive expression for \(p_n, q_n\):

\[
\begin{align*}
 p_0 &= a_0, \quad q_0 = 1, \\
 p_1 &= a_0a_1 + 1, \quad q_1 = a_1, \\
 p_n &= a_n p_{n-1} + p_{n-2}, \quad q_n = a_n q_{n-1} + q_{n-2}
\end{align*}
\]

- Write the continued fraction of \(\alpha = 179/32\).
- Prove the above recursion. (You may use the simple observation that \([a_0, \ldots, a_n] = [a_0, \ldots, a_{n-1} + 1/a_n]\).)
- Show how to find the continued fraction expansion of a rational input \(\alpha = \frac{a}{b}\) in time polynomial in the input length.

2. Let

\[
A = \frac{1}{2} \begin{pmatrix}
1 & \frac{7}{25} \\
\frac{7}{25} & 1
\end{pmatrix}
\]

Prove that \(A\) is positive and find \(\sqrt{A}\) and \(\log(A)\).

3. An automorphism of a graph \(G\) is an isomorphism from \(G\) to itself. The graph automorphism problem, GAUT, is given a graph determine wether it has a non-trivial automorphisms or not. Reduce GAUT to a HSP on \(S_n\).

4. Let \(w\) be a known root of unity. Show an efficient quantum circuit for the transformation \(|j, k\rangle \rightarrow w^{jk}|j, k\rangle\). You may use any one, two or three qubit gates you wish.

5. Let Angle be the following promise problem:

**Input**: Two pure states \(\phi_1, \phi_2, 0 \leq \alpha < \beta \leq 1\).

**Yes instances**: \(|\langle \phi_1 | \phi_2 \rangle| \leq \alpha\)

**No instances**: \(|\langle \phi_1 | \phi_2 \rangle| \geq \beta\)

Design a quantum circuit that accepts Yes instances with probability at least \(p\), and No instances with probability at most \(q\) for some \(q < p\).

6. Show that \(Q^{\downarrow}(EQ) = O(\log n)\).

Hint: Use an error correcting code together with the previous question.