Performance of Optimal Beamforming with Partial Channel Knowledge

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***Abstract*—Multiple antenna techniques are used to enhance wireless links and therefore have been studied extensively. Many practical systems that differ from ideal schemes have been discussed in the literature. One example is a system that lacks precise channel information at the transmitter. We evaluate analytically the performance of a multiple input multiple output (MIMO) technique that uses partial channel knowledge. Specifically, we analyze a scheme with M transmit antennas and N receive antennas, employing eigenbeamforming at the transmitter and MRC at the receiver. We assume that only partial channel matrix is known to the transmitter, specifically only P out of N rows of the channel matrix are known. We derive the optimal beamformer for a single stream transmission case and show that it is the singular vector of the known channel submatrix. We show that the diversity order of a transmission scheme where such a precoder is used is $MP + N - P$ and further show that increasing the value of P by one increases the diversity order by $M - 1$. We also derive the array gain for this scheme. All the results are supported by simulations.**

***Index Terms*—MIMO systems, beamforming, partial channel knowledge, WiMAX, LTE.**

I. INTRODUCTION

MULTIPLE input multiple output (MIMO) techniques have been incorporated in all specifications of the recently developed wireless communications systems, including Long Term Evolution (LTE) and Worldwide Interoperability for Microwave Access (WiMAX) technologies. MIMO schemes include Alamouti's space-time coding [1], spatial multiplexing and a few transmit beam-forming methods.

Multiple antennas are employed on both sides of the communications channel in most MIMO systems. In time division duplex (TDD), the same frequency is used by both sides and the same antennas are used for transmission and reception. Thus, the channel matrix can be estimated by each side based on the reception of a known waveform. However, due to cost considerations, early generations of WiMAX and LTE restrict the mobile station (MS) to one transmit antenna although two or more antennas are used for reception. Thus, the transmission from the MS can be used to estimate the channel between one of the MS antennas and all the base station (BS) antennas. The channel between the BS antennas and the other receive antennas of the MS is not known to the BS. As a result, transmit beamforming schemes, which are employed by the BS, are limited to partial channel knowledge. This problem can be solved by allowing the MS to report its channel estimates to the BS [2]. However, this closed loop scheme requires a rather complex receiver at the MS.

Communications that use partial state information have been thoroughly analyzed in the literature. The performance of various scalar and vector quantizers have been discussed in [3]. It is shown that a few feedback bits provide performance which is similar to that of beamforming with perfect channel knowledge. Limited feedback and quantized codebook are discussed in [4]. It is shown that the desired codebooks depend on the number of transmit antennas (but not on the number of receive antennas) and on the number of code words. It is also shown that a sufficient condition for full diversity is a codebook cardinality not less than the number of transmit antennas. The relation between the signal to noise ratio (SNR) and the amount of the required feedback, for the case of MIMO broadcast channels, is analyzed in [5]. It is shown that for zero forcing precoding to achieve full multiplexing gain, the required number of feedback bits per user increases linearly with the SNR. As the SNR increases, multi-user interference becomes more dominant and more feedback bits are required to achieve the multiplexing gain. The conditions for optimal beamforming for two cases of partial channel feedback, the first in which the covariance of the channel is fed back, and the second in which the mean channel is fed back, are discussed in [6], where the analytic derived conditions maximize the average throughput. More analysis of feedback methods and their impact have been thoroughly studied in [7], [8].

The partial knowledge, analyzed in this letter, is different from the partial state information considered so far. We assume here that only part of the channel matrix is perfectly known while the other part is unknown. Specifically, we assume a channel submatrix is perfectly known whereas the rest of the channel matrix is unknown. Our goal is to analyze practical transmission schemes using partial channel knowledge. The analysis in this letter extends the results obtained in [9].

The letter has the following structure: In Section II, we describe a transmission scheme in which the same waveform is transmitted by all the antennas. We evaluate the optimal precoder for this case. In Section III we evaluate the symbol error probability, the diversity order and the array gain of this scheme when using the optimal precoder. Simulation results are presented in Section IV.

II. ANALYSIS OF A TRANSMISSION SCHEME USING PARTIAL CHANNEL KNOWLEDGE

Assume a BS is equipped with M antennas and a MS is equipped with N antennas. Of the N MS antennas, P antennas are used for transmission and reception while all

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other $N-P$ antennas are used for reception only. This mode is typical in the initial versions of the 4G networks such as LTE [10] and WiMAX [11]. Let us denote the channel between the i-th MS antenna and the j-th BS antenna by $h_{i,j}$, $i \in \{0,1,\dots,N-1\}$, $j \in \{0,1,\dots,M-1\}$. The MS transmits a known sequence (sometimes called Uplink Sounding), which will be used at the BS to estimate the channel by assuming channel reciprocity. This pertains mostly to TDD systems. Since only P antennas are used in the MS for transmission, the sounding signal will be transmitted from these P antennas only. Assume that the channel estimation at the BS is perfect and then channels $\boldsymbol{h}_k^* = [h_{k,0}, h_{k,1}, h_{k,2} \dots h_{k,M-1}]$, $k = 0,1,\dots,P-1$, are known, and $\boldsymbol{h}_j^* = [h_{j,0}, h_{j,1}, h_{j,2} \dots h_{j,M-1}]$, $j = P, P+1, \dots, N-1$, are unknown. We use $[\cdot]^*$ to denote conjugate transpose. We analyze a scheme in which a single stream transmission, which utilizes beamforming with all the M antennas at the BS and maximal ratio combining (MRC) at the MS receiver, is used. The received vector at the N receiver antennas is given by

$$\begin{bmatrix} r_0 \\ r_1 \\ \vdots \\ r_{N-1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h}_0^* \\ \boldsymbol{h}_1^* \\ \vdots \\ \boldsymbol{h}_{N-1}^* \end{bmatrix} \boldsymbol{w}s + \rho\boldsymbol{n} = \boldsymbol{a}s + \rho\boldsymbol{n}, \quad (1)$$

where s is the baseband QPSK signal, $\boldsymbol{w}$ is the precoding vector, $\boldsymbol{n}$ is a zero mean Gaussian vector with covariance equal to the identity $N \times N$ matrix and ρ^2 is the noise variance. The vector $\boldsymbol{a}$ represents the effect of the beamforming and the channel on the signal and its entries can be derived from Eq. (1).

Here we encounter a unique scenario where we have perfect knowledge of the first P rows of the instantaneous realization of the $\boldsymbol{H}$ matrix, where only the statistical model for the other $N-P$ rows is known. Accordingly, we divide $\boldsymbol{H}$ into $\boldsymbol{H}_P$ which contains the first known P rows of $\boldsymbol{H}$, and into $\boldsymbol{H}_N$, which contains the last $N-P$ unknown rows of $\boldsymbol{H}$, such that $\boldsymbol{H} = \begin{bmatrix} \boldsymbol{H}_P^* & \boldsymbol{H}_N^* \end{bmatrix}^*$. Our goal here is to find the optimal precoder $\boldsymbol{w}$ in the sense of the average post processing SNR (averaging over $\boldsymbol{H}_N$) conditioned on the known realization of $\boldsymbol{H}_P$. Specifically, since $\boldsymbol{H}_N$ is unknown, the precoder $\boldsymbol{w}$ depends only on $\boldsymbol{H}_P$ and the statistics of $\boldsymbol{H}_N$. In order to maintain unit transmission power, we consider only precoders for which $\|\boldsymbol{w}\|^2 = 1$. The post-processing SNR, denoted γ, conditioned on $\boldsymbol{H}$ and $\boldsymbol{w}$, is given by

$$\gamma(\boldsymbol{H}, \boldsymbol{w}) = \frac{\|\boldsymbol{H}\boldsymbol{w}\|^2}{\rho^2} = \frac{\|\boldsymbol{H}_P\boldsymbol{w}\|^2 + \|\boldsymbol{H}_N\boldsymbol{w}\|^2}{\rho^2}. \quad (2)$$

The average post-processing SNR over $\boldsymbol{H}_N$ and conditioned on $\boldsymbol{H}_P$ and $\boldsymbol{w}$ is

$$\begin{aligned} \gamma(\boldsymbol{H}_P, \boldsymbol{w}) &= E_{\boldsymbol{H}_N}\{\gamma(\boldsymbol{H}, \boldsymbol{w})\} \\ &= \frac{\boldsymbol{w}^* \left[\boldsymbol{H}_P^*\boldsymbol{H}_P + (N-P)\boldsymbol{I}_{M\times M}\right]\boldsymbol{w}}{\rho^2}, \end{aligned} \quad (3)$$

so the optimal precoder $\boldsymbol{w}$ is the eigenvector that corresponds to the largest eigenvalue of $\boldsymbol{H}_P^*\boldsymbol{H}_P + (N-P)\boldsymbol{I}_{M\times M}$. Since the eigenvectors of $\boldsymbol{H}_P^*\boldsymbol{H}_P + (N-P)\boldsymbol{I}_{M\times M}$ are identical to the eigenvectors of $\boldsymbol{H}_P^*\boldsymbol{H}_P$, it follows that the optimal precoder is an eigenvector of $\boldsymbol{H}_P^*\boldsymbol{H}_P$.

The analysis in the sequel is done under the assumptions that different channels experience independent Rayleigh fading and quadrature phase shift keying (QPSK) modulation is used by the BS. The shift in performance resulting from the choice of a different modulation, such as 16QAM, is easily computed using the corresponding minimum distance within the constellation, known as d_{min}, and is displayed in the performance plots presented in Section IV.

III. Performance of a Single Stream Transmission using the Optimal Precoder

The optimal precoding vector $\boldsymbol{v}_P$ is the principal eigenvector of $\boldsymbol{H}_P{}^*\boldsymbol{H}_P$. Assume again that the MS knows the channel. The transmitted vector is then $\boldsymbol{v}_P s$. Assume the MS knows the channel and therefore knows $\boldsymbol{a}$, then Eq. (1) is left multiplied by the pseudo inverse of $\boldsymbol{a}$. This processing is known as MRC and the result is

$$\hat{s} = s + \rho\left(\lambda + \sum_{j=P}^{N-1} |\boldsymbol{h}_j^*\boldsymbol{v}_P|^2\right)^{-1} \begin{bmatrix} \boldsymbol{v}_P^*\boldsymbol{h}_0, \dots, \boldsymbol{v}_P^*\boldsymbol{h}_{N-1} \end{bmatrix} \boldsymbol{n},$$

where λ is the largest eigenvalue of $\boldsymbol{H}_P{}^*\boldsymbol{H}_P$. The first order expansion of λ is given by [12] to be

$$p(\lambda) \approx MP\lambda^{MP-1} \frac{\prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!}. \quad (4)$$

After the application of MRC to the receiver, the post processing SNR is given by $\gamma = \rho^{-2}(\lambda + \sum_{j=P}^{N-1} |\boldsymbol{h}_j^*\boldsymbol{v}_P|^2)$. The post processing SNR is composed of two components: the first is a result of the optimal combination of the known P antennas, whereas the second is the result from the other $N-P$ antennas. The error probability, given the channel matrix $\boldsymbol{H} = [\boldsymbol{h}_0, \boldsymbol{h}_1, \dots \boldsymbol{h}_{N-1}]^*$, is

$$\begin{aligned} \Pr\{\text{error}|\boldsymbol{H}\} &= 2Q\left(\sqrt{\gamma}\right) - Q^2\left(\sqrt{\gamma}\right) \\ &\approx \frac{1}{6}\exp\left(-\frac{\gamma}{2}\right) + \frac{1}{2}\exp\left(-\frac{2\gamma}{3}\right). \end{aligned} \quad (5)$$

We used the approximation given in [13] for the evaluation of the Q-Function. Note that $|\boldsymbol{h}_j{}^*\boldsymbol{v}_P|^2$ can be written as $|\boldsymbol{h}_j{}^*\boldsymbol{v}_P|^2 = \|\boldsymbol{h}_j\|^2\cos^2(\alpha_j) = \cos^2(\alpha_j)\sum_{i=0}^{M-1}|h_{ji}|^2$ where α_j, which is the angle between the vectors $\boldsymbol{h}_j$ and $\boldsymbol{v}_P$, is defined by $\cos\alpha_j = |\boldsymbol{h}_j^*\boldsymbol{v}_P|(\|\boldsymbol{h}_j\|)^{-1}$. Denote $\psi_{ji} = |h_{ji}|$. By substituting $|\boldsymbol{h}_j{}^*\boldsymbol{v}_P|^2$ into Eq. (5), the error probability for any given $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \dots \alpha_{N-1}]$ becomes

$$\begin{aligned} \Pr\{\text{error}|\boldsymbol{\alpha}\} \approx &\int_0^\infty \cdots \int_0^\infty \left[\frac{1}{6}\exp\left\{-\frac{1}{2\rho^2} \times \left(\lambda + \sum_{j=P}^{N-1}\sum_{i=0}^{M-1}\psi_{ji}^2\cos^2\alpha_j\right)\right\}\right. \\ &\left. + \frac{1}{2}\exp\left\{-\frac{2}{3\rho^2}\left(\lambda + \sum_{j=P}^{N-1}\sum_{i=0}^{M-1}\psi_{ji}^2\cos^2\alpha_j\right)\right\}\right] \\ &\times p(\lambda)d\lambda 2\psi_{P,0}\exp(-\psi_{P,0}^2)d\psi_{P,0}\dots 2\psi_{N-1,M-1} \\ &\times \exp(-\psi_{N-1,M-1}^2)d\psi_{N-1,M-1}. \end{aligned} \quad (6)$$

Since $\boldsymbol{h}_j$ are i.i.d. circularly symmetric complex normal vectors, it follows that $\boldsymbol{\alpha}$ is statistically independent of λ and

ψ. We can then solve the integral in Eq. (6) separately for each component of the post processing SNR. Afterwards we combine these components to get the overall error probability.

A. First Component of the Post Processing SNR

The derivation here uses the asymptotic analysis given in [14] for the solution of Eq. (6). Using this approach, the asymptotic (high SNR) diversity order and array gain depend only on the Taylor expansion (around the origin) of the pdf of the post processing SNR. Following this approach, we use the Taylor expansion of the pdf of the largest eigenvalue given in Eq. (4). The kernel integral to be solved for the first component of the post processing SNR is

$$I_1 = \int_0^\infty \exp\left(-\frac{\lambda}{2\rho^2}\right) p(\lambda) d\lambda. \quad (7)$$

By substituting Eq. (4) into Eq. (7), I_1 becomes

$$I_1 \approx MP \frac{\prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!} \int_0^\infty \lambda^{MP-1} \times \exp\left(-\frac{\lambda}{2\rho^2}\right) d\lambda. \quad (8)$$

The integral on the right is a form of the Laplace Integral, which can be solved for high SNR. Denote $f(\lambda) = \lambda^{MP-1}$, then by integration by parts we get

$$\int_0^\infty \lambda^{MP-1} \exp\left(-\frac{\lambda}{2\rho^2}\right) d\lambda \approx \sum_{n=0}^{MP-1} \frac{f^{(n)}(\lambda=0)}{(1/2\rho^2)^{MP}} = (2\rho^2)^{MP}(MP-1)!. \quad (9)$$

By using Eq. (9), Eq. (8) can be rewritten as

$$I_1 \approx \frac{\prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!} \left(\frac{\overline{\gamma}}{2}\right)^{-MP} (MP)! \quad (10)$$

where $\overline{\gamma} = \rho^{-2}$. According to Eq. (10), the diversity order, defined as $\lim_{\overline{\gamma}\to\infty} \frac{-\log_e \Pr\{\text{error}\}}{\log_e \overline{\gamma}}$, is MP. The array gain, defined as the shift in symbol error rate (SER) as $\overline{\gamma} \to \infty$, is given by

$$\frac{2MP}{\left(2^{MP} \frac{\prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!}(MP)!\right)^{1/MP}}.$$

B. Second Component of the Post Processing SNR

The solution for the second component of the post processing SNR in Eq. 6 is similar to what was derived in [15], where the sum is performed over $N-P$ antennas instead of N. Therefore, the kernel integral to be solved for the second component becomes

$$I_2(\boldsymbol{\alpha}) \approx \prod_{j=P}^{N-1} \left(1 + \frac{\overline{\gamma}\cos^2\alpha_j}{2}\right)^{-M}. \quad (11)$$

The average I_2 is found by integrating Eq. 11 over all possible values of $\boldsymbol{\alpha}$, which requires knowledge of the pdf of α_j. The pdf of α_j for complex valued vectors of length n is given in [16] by $p(\alpha_j) = 2(n-1)\sin(\alpha_j)^{2n-3}\cos(\alpha_j)$, which yields the unconditional error probability $I_2 \approx \left(1 + \frac{\overline{\gamma}}{2}\right)^{-(N-P)}$. The diversity order and the array gain generated by the second component of the post processing SNR are then $N-P$.

C. Combined Error Probability

The total error probability is derived by combining I_1 and I_2 which yields (12) on the next page. Equation (12) can be modified in order to handle low SNR values, resulting in

$$\Pr\{\text{error}\} \approx x\left[\frac{1}{6}\left(x^{\frac{1}{MP+N-P}} + \frac{\overline{\gamma}}{2}\right)^{-(MP+N-P)} + \frac{1}{2}\left(x^{\frac{1}{MP+N-P}} + \frac{2\overline{\gamma}}{3}\right)^{-(MP+N-P)}\right] \quad (13)$$

where $x = \frac{(MP)!\prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!}$. The diversity order for this scheme is $MP+N-P$. When $P=N$, which means that full channel knowledge exists at the transmitter, the diversity order becomes MN as expected. An increase of P by one results in a diversity order increase by $M-1$. The combined array gain is given by $\frac{2MP}{\left(2^{MP}\frac{\prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!}(MP)!\right)^{1/MP}} + N - P$. When $P=1$, both the diversity order and the array gain equal $M+N-1$ which complies with the obtained results in [15], for which the error probability was shown to be

$$\Pr\{\text{error}\} \approx \frac{1}{6}\left(1 + \frac{\overline{\gamma}}{2}\right)^{-(M+N-1)} + \frac{1}{2}\left(1 + \frac{2\overline{\gamma}}{3}\right)^{-(M+N-1)}. \quad (14)$$

IV. Simulation Results

A comparison between the performances of this technique for $M=4$ and $N=4$ for all values of P in uncorrelated Rayleigh fading channels is given in Fig. 1. Monte-Carlo simulations were carried out in order to compare with theoretical results. The results are upper bounded by MRC using all receive antennas, assuming no channel knowledge at the transmitter, and are lower bounded by full channel knowledge at the transmitter. As seen, the theoretical and the simulation plots are closely matched for all the P values except $P=4$ that has a gap of approximately 0.5dB. This is explained by the fact that the approximation used in Eq. (9) assumed high SNR, whereas as the values of M and P increase, the SNR working point decreases. It is also seen that the diversity order and the array gain increase with P. For every increase of P by 1, the diversity order increases by $M-1=3$. The array gain, on the other hand, increases slightly with an increase of P by 1. Whereas for $P=2$ the array gain is approximately 8.9dB, for $P=4$ (full channel knowledge) the array gain is approximately 9.3dB. We see that the gap between full knowledge of the channel matrix ($P=N$) and the partial knowledge ($P<N$) is not very large at the range of SNR where wireless systems usually operate. For example, at SER=10^{-5}, the gap between $P=4$ (full channel knowledge)

$$\Pr\{\text{error}\} \approx \frac{(MP)! \prod_{i=1}^{P}(P-i)!}{\prod_{i=1}^{P}(M+P-i)!} \left[\frac{1}{6}\left(1+\frac{\overline{\gamma}}{2}\right)^{-(N-P)} \left(\frac{\overline{\gamma}}{2}\right)^{-MP} + \frac{1}{2}\left(1+\frac{2\overline{\gamma}}{3}\right)^{-(N-P)} \left(\frac{2\overline{\gamma}}{3}\right)^{-(MP)} \right] \tag{12}$$

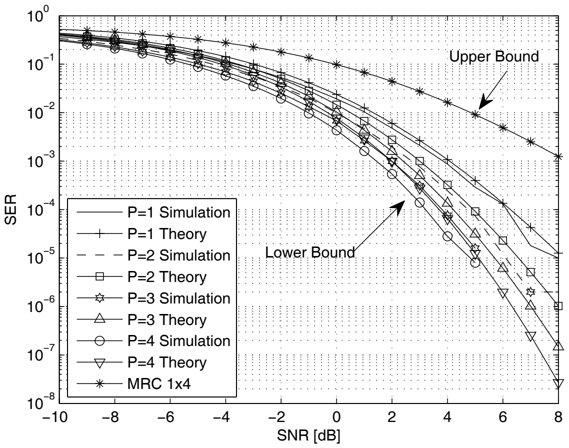

Fig. 1. Performance comparison for $M = 4$ and $N = 4$ between various values of P in uncorrelated Rayleigh fading channels.

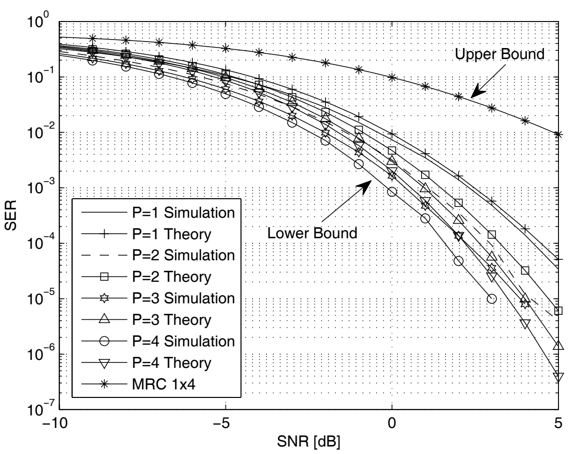

Fig. 2. Performance comparison between various values of P in uncorrelated Rayleigh fading channels for $M = 6$ and $N = 4$.

and $P = 2$ is only 1dB. At SER=10^{-6}, this gap grows to 1.5dB. Therefore, in this example, half the transmit chains may be used at the MS side, which is a significantly simpler and cheaper design than when using all transmit chains.

A comparison in a larger antenna array system, where $M = 6$ and $N = 4$ for various values of P in uncorrelated Rayleigh fading channels, is given in Fig. 2. In this case, the theoretical and the simulation plots are closely matched for all the values of P except $P = 4$ where there is a gap of approximately 0.6dB. In this case, the gap between full knowledge ($P = 4$) and partial knowledge with $P = 2$ at SER=10^{-5} is approximately 1.6dB. The increase in the array gain as P increases is again small. When $P = 2$, the array gain is approximately 10dB whereas when $P = 4$ the array gain is approximately 10.4dB.

After the completion of the analysis and simulation of the suboptimal schemes in classical uncoded Rayleigh fading channels, we analyze the performance of these schemes in benchmark WiMAX channels. These results provide us with an insight into the performance in real situations. It will also corroborate the results obtained earlier by showing similar characteristics.

Several channel models were defined by the International Telecommunications Union (ITU) [17]. Each assumes a different number of taps, delays, powers and Doppler spectrum. Two of the most popular channel models, which are used within the WiMAX community as a benchmark to validate results, are the Pedestrian B and the Vehicular A channel models. We compare between the results of the schemes in these channel models when all the channels are uncorrelated and forward error correction is not used. Then, we compare between them by utilizing error correction. In order to be consistent with the flat fading case, orthogonal frequency division multiplexing (OFDM) modulation was employed and sufficiently long guard interval was assumed. For simplicity, results derived from the use of the encoder are shown in terms of bit error rate and not by symbol error rate. Results for the uncoded case are given in Fig. 3. We see that again, the gap between full knowledge of the channel matrix ($P = N$) and partial knowledge ($P < N$) is not very large at the range of SNR where wireless systems usually operate. For example, at BER=10^{-5}, the gap between $P = 4$ (full channel knowledge) and $P = 1$ is only approximately 2dB. Results for the coded case with QPSK modulation and coding rates 1/2 and 3/4 are given in Fig. 4. We see that although the SNR range shifted due to the use of error correction, the gap between full knowledge of the channel matrix ($P = 4$) and partial knowledge ($P = 1$) remains approximately 2dB at BER=10^{-4}. Results for the coded case with 16QAM modulation and coding rates 1/2 and 3/4 are given in Fig. 5. Here the gaps between full and partial channel knowledge remain similar to those achieved with QPSK modulation. At BER=10^{-4}, the gap between full knowledge of the channel ($P = 4$) and partial knowledge ($P = 1$) is approximately 2.5dB. Results for the coded case in the Vehicular A channel model, with QPSK modulation and coding rates 1/2 and 3/4, are given in Fig. 6. In this case, the gap between different values of P is small, resulting from the channel characteristics of the Veh-A channel model, in which the fades are not as deep. It is important to note that in this channel model, where a high velocity is used, a much higher update rate is required to accommodate the shorter coherence time, an issue which we did not address here.

V. Conclusions

In this letter, we derived the optimal precoder for a MIMO scheme in which perfect knowledge of a channel submatrix exists at the transmitter. Specifically, when the channel matrix is of size $N \times M$, only $P < N$ rows are known at the

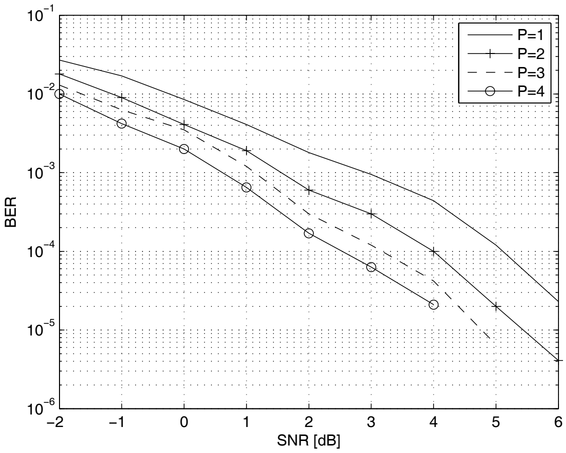

Fig. 3. Performance comparison between various values of P in uncorrelated Ped-B 3km/h fading channels for $M = 4$ and $N = 4$ without channel coding.

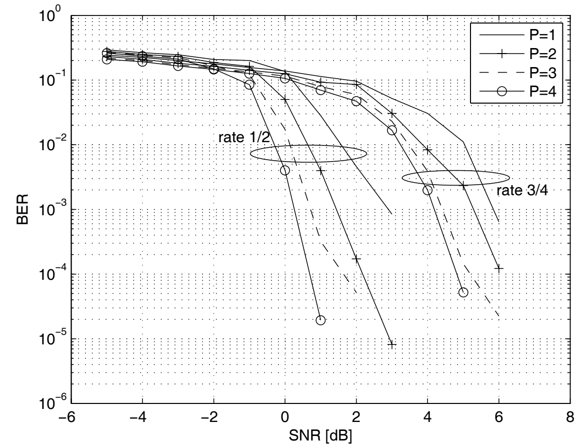

Fig. 5. Performance comparison between various values of P in uncorrelated Ped-B 3km/h fading channels for $M = 4$ and $N = 4$ with 16QAM modulation and rates 1/2 and 3/4 channel coding.

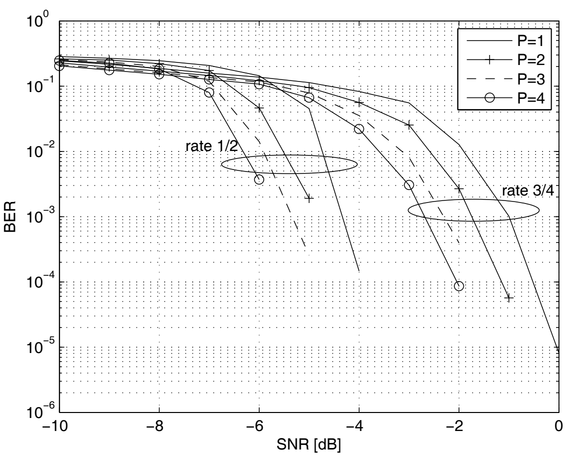

Fig. 4. Performance comparison between various values of P in uncorrelated Ped-B 3km/h fading channels for $M = 4$ and $N = 4$ with QPSK modulation and rates 1/2 and 3/4 channel coding.

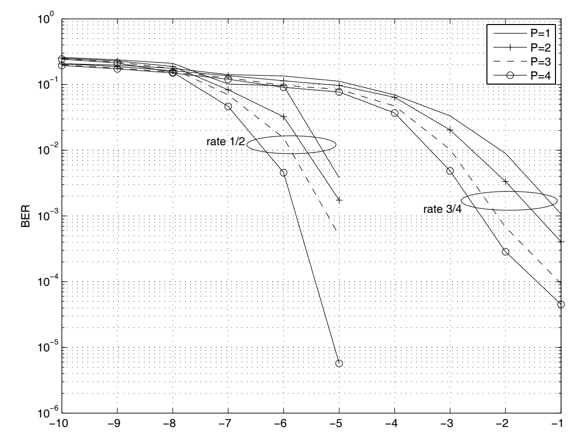

Fig. 6. Performance comparison between various values of P in uncorrelated Veh-A 60km/h fading channels for $M = 4$ and $N = 4$ with QPSK modulation and rates 1/2 and 3/4 channel coding.

transmitter. These cases are common in 4G systems such as WiMAX and LTE. We then analyzed the performance of the system when this optimal precoder was used. The eigen-beamforming scheme, for which $P = M$ and where full channel is known at the transmitter, acted as a reference for performance comparisons of the analyzed scheme in terms of error probability, diversity order and array gain. The results from these comparisons are surprising - the array gain of the analyzed partial CSI based schemes is similar to the array gain of the full CSI based scheme. Although the diversity order of these partial CSI schemes is lower than the high diversity order of the full CSI scheme ($MP+N-P$ and MN, respectively), in many cases, it is still significantly high, resulting in low error rates.

The full CSI based scheme discussed here requires the MS to use all antennas for transmission, which increases the cost and complexity of an MS. Furthermore, it requires a large singular value decomposition calculation at the transmitter, which adds to the complexity of the implementation. The performance analysis and the comparisons in this letter show that low complexity partial CSI based schemes such as the scheme discussed in this letter, can replace high complexity full CSI schemes without causing a major performance degradation. A forthcoming paper will extend these results to the case of multiple streams via spatial multiplexing.

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