## Bases and Spectral Representations

## Exercise 1

November 7, 2012

This exercise deals with Matrix Approximation and Completion

- 1. This question deals with Fischer's inequality: Let  $\mathbf{A} \in \mathbb{R}^{n \times n}$  be symmetric and positive definite (all the eigenvalues of  $\mathbf{A}$  are non-negative). Prove that  $\det(A) \leq \prod_{i=1}^{n} a_{ii}$ .
- 2. Let **A** be a square matrix  $(\mathbf{A} \in \mathbb{C}^{n \times n})$  whose singular values are  $\{\sigma_i\}_{i=1}^n$ . Prove that  $\sum_{i=1}^n e^{\sigma_i} \ge \sum_{i=1}^n e^{|a_{ii}|}$
- 3. Given a matrix **M**, find its best approximation under Frobenius norm **X**, that satisfied  $\|\mathbf{X}\|_{(2)} \leq \lambda$ , where  $\|\cdot\|_{(2)}$  indicates the Ky-Fan norm with k = 2. This question is similar to what was shown in class. Leave your answer in terms of semidefinite programming.
- 4. Matlab exercise: Take any natural image of your choice, convert it to graylevel and plot a graph of the mean squared error (MSE) between the original image and its best rank k approximation,  $k = 1, ..., \min(m, n)$ , where  $m \times n$  is the size of the image.

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