

# Communication Networks (0368-3030) / Spring 2011

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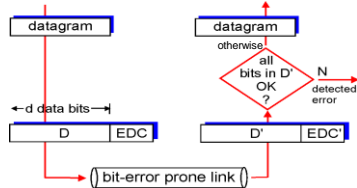
## Error Detection and Correction

Kurose & Ross, Chapter 5.2 (5<sup>th</sup> ed.)

### Error Detection

EDC= Error Detection and Correction bits (redundancy)  
D = Data protected by error checking, may include header fields

- Error detection not 100% reliable!
  - protocol may miss some errors, but rarely
  - larger EDC field yields better detection and correction

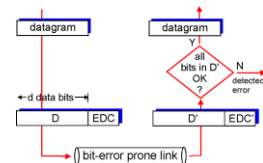


Data Link Layer 5-3



### Example – Parity bit

- Assume D has  $d$  bits.
- The EDC is one bit s.t. the number of 1's in the  $d+1$  bits (D and the EDC) is even.
- Receiver can detect an error inverting an odd number of bits
- Example:
  - D = 11101
  - Sender sends 111010
  - Receiver gets 101010
  - Illegal parity – an error has occurred
  - Receiver cannot correct the error



Error detection vs.  
Error correction



### Some theory

- Assume all messages are of size  $d$ 
  - We have  $2^d$  possible messages, all of them valid
  - When some bits flip, the receiver still gets a valid message
  - It cannot know there was an error
- The proposed solution:
  - Add  $r$  bits of *redundancy*.
  - Now, we have  $2^{d+r}$  possible messages, but only  $2^d$  of them are valid (these are called *codewords*).
  - "Small errors" are likely to transform the valid message into an invalid one, so that the receiver knows an error has occurred.

### Some theory (cont.)

- **Definition:** The *Hamming-distance* of two strings  $x$  and  $y$  is the number of bits in which they differ, denoted  $dH(x,y)$ .
  - For instance:  $x = 110010$   $y = 111000$ .  $dH(x,y) = 2$
- **Definition:** The *Hamming-distance of an error-correction scheme* ( = code) is the minimal Hamming-distance between two valid messages ( = codewords).  
 $dH(C) = \min \{ dH(x,y) : x, y \in C \}$

## Parity bit revisited

- Assume all messages have  $d$  bits.
- The valid messages (= codewords) are all the  $d+1$  messages s.t their total number of 1's is even.
- The Hamming-distance of this scheme is 2.
  - No two valid codewords  $x,y$  s.t.  $dH(x,y) = 1$ 
    - If  $dH(x,y) = 1$  then either  $x$  or  $y$  has an odd number of 1's.
  - There are two valid codewords with distance 2:
    - For instance, for  $d = 6$   
 $x = 111100$   $y = 111111 \rightarrow dH(x,y) = 2$

## Why is Hamming-distance important?

- Theorem 1: If a code  $C$  has  $dH(C) = k+1$ , then it can **detect** all errors of  $k$  bits or less.
  - Such errors necessarily produce an invalid codeword
- Theorem 2 : If a code  $C$  has  $dH(C) = 2k+1$ , then it can **correct** all errors of  $k$  bits or less.
  - Think why
- And indeed: parity bit can detect all single-bit errors, but cannot correct any.

## CRC – Cyclic Redundancy Check

- Bits represent polynomials over  $GF(2)$ 
  - Example:  $100101 = x^5 + x^2 + 1$
  - Addition and subtraction are actually XOR (no carry)
  - Example:  $1101 + 0111 = 1010$   
 $(x^3 + x^2 + 1) + (x^2 + x + 1) = x^3 + x$
- Sender & receiver agree in advance on a **generating polynomial**  $G$  of degree  $r$
- When sender wish to send  $D$ , it calculates  $R$  s.t.  $DR$  is divisible by  $G$ .
- When the receiver gets  $D'R'$  it divides it by  $G$ . If the remainder is not 0 – an error has occurred.

## Calculating $R$

- $DR = x^r \cdot D + R$
- We want:  $DR = x^r \cdot D + R = n \cdot G$ 
  - But addition and subtraction are just XOR – they are interchangeable
- Equivalently, then, we want:  $x^r \cdot D = n \cdot G + R$
- Namely,  $R$  is the remainder of  $x^r \cdot D$  divided by  $G$ !
- Observation:  $R$ 's degree is at most  $r-1$ .
  - It is the remainder of dividing by  $G$ , and  $deg(G) = r$

## CRC Example

- $D = 101110 = x^5 + x^3 + x^2 + x$
- $G = 1001 = x^3 + 1$ 
  - $r = deg(G) = 3$
- Shift  $D$   $r$  bits to the left:
  - $x^r \cdot D = x^8 + x^6 + x^5 + x^4$
  - $x^r \cdot D = 101110000$
- Now we can divide  $x^r \cdot D$  by  $G$ :
  - on board

## CRC Example

Want:

$$D \cdot 2^r \text{ XOR } R = nG$$

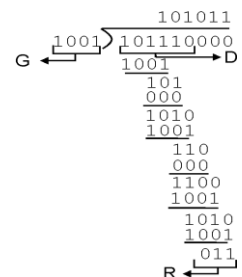
equivalently:

$$D \cdot 2^r = nG \text{ XOR } R$$

equivalently:

if we divide  $D \cdot 2^r$  by  $G$ , want remainder  $R$

$$R = \text{remainder} \left[ \frac{D \cdot 2^r}{G} \right]$$



## CRC Example (cont.)

- $D = 101110 = x^5 + x^3 + x^2 + x$
- $G = 1001 = x^3 + 1$ 
  - $r = \deg(G) = 3$
- Now we can divide  $x^r \cdot D$  by  $G$ :
  - we get:  $R = x + 1 = 011$
- Sender sends:
  - $DR = x^r \cdot D + R = 101110011$

## Ethernet's CSMA/CD

Kurose & Ross, Chapter 5.5.2 (5<sup>th</sup> ed.)

## The algorithm

When a the network layer generates a new frame:

1. If the adapter senses the channel to idle (that is no signal detected for 96 bit times) – start transmitting the frame
2. Otherwise (channel is busy) – wait until you sense no signal energy (plus 96 bit times) and then start transmitting
3. While transmitting – listen for signal coming from other adapters. If the adapter transmitted the entire frame without detecting signal energy – it is done with the frame.
4. If signal energy is detected while transmitting – stop transmitting the frame.
  - i. Transmit a 48 bit jam-signal
  - ii. Exponential backoff: after experiencing the  $n$ -th collision is a row for the current frame choose  $K$  randomly from  $\{0, 1, \dots, 2^n - 1\}$  with  $m = \min\{n, 10\}$ . Wait  $K \cdot 512$  bit times and return to step 1.

## The jam signal

- The jam-signal makes sure all other transmitting adapters are aware of the collision
  - Suppose that A starts to transmit
  - Just before A's signal reaches B, B begins to transmit
  - B senses A's signal and aborts.
  - B transmitted just a few bits before aborting. These bits propagate to A but might not constitute enough energy for A to detect the collision!
  - To make sure A detects the collision, B transmits the 48-bit jam signal

## Exponential backoff

- When an adapter first detects collision, it cannot know how many adapters are involved in the collision,
- Exponential backoff dynamically adapts the waiting-time before reattempting transmission to the number of adapters involved in the collision
  - Few adapters involved: Choose  $K$  from a small set, so that no one waits unnecessarily
  - Many adapters involved: Choose  $K$  from a large set, so everyone is likely to choose a different time to transmit, and the collision will be resolved.

## Exponential backoff example

- Assume A and B both have a new frame to transmit. They both begin to transmit exactly on the same time and collide.
- They both choose  $K$  from  $\{0, 1\}$
- The possible outcomes:

### Exponential backoff example (cont.)

Case	A chooses	B chooses	Probability	Outcome
(a)	0	0	0.25	another collision on round 2
(b)	0	1	0.25	A successful on round 2, B successful on round 3
(c)	1	0	0.25	B successful on round 2, A successful on round 3
(d)	1	1	0.25	another collision on round 3

### תרגיל – אורך מסגרת מינימלי ב - CSMA/CD

נתונה רשת CSMA/CD עם התכונות הבאות:

- כבל קואקסיאלי באורך 250 מטר.
- קצב שידור 100Mbit/sec.
- מהירות סיגנל 200,000 ק"מ לשניה.

מהו אורך המסגרת המינימלי?

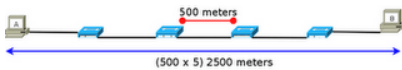
- $L$ : minimal frame length
- $T$ : the time it takes to transmit a frame of length  $L$
- Require:  $T \geq RTT$

$$T \geq 2 \cdot \frac{250}{200,000 \cdot 10^3} = 2.5 \cdot 10^{-6} \text{ s}$$

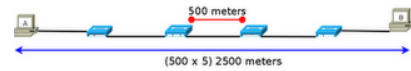
- $L \geq (100 \cdot 10^6) \cdot (2.5 \cdot 10^{-6}) = 250 \text{ bit}$

### Example: Ethernet's minimal frame length

- Assume an Ethernet network with the following properties (this is quite an old network):
  - Transmission rate 10Mbps
  - Built of coaxial cables of length up to 500m
  - Up to 4 repeaters allowed
  - Signal speed on coaxial cable 200,000 km/sec
  - Every repeater adds a delay of  $3\mu\text{s}$
- We show why the minimal frame length in this network is 64 bytes.



### Example: Ethernet's minimal frame length



- 4 repeaters = 5 network segments
- $RTT = 2 \left( \frac{5 \cdot 500}{200,000 \cdot 10^3} + 4 \cdot 3 \cdot 10^{-6} \right) = 4.9 \cdot 10^{-5} \text{ s}$   
=  $49\mu\text{s}$
- $L \geq (10 \cdot 10^6) \cdot (4.9 \cdot 10^{-5}) = 490 \text{ bits}$
- Take the next 8-power (which is both convenient and adds a margin of safety):  
 $L = 512 \text{ bits} = 64 \text{ bytes}$