RSA and Digital Signatures

Kurose & Ross, Chapters 8.2-8.3 (5th ed.)

Slides adapted from:

J. Kurose & K. Ross \
Computer Networking: A Top Down Approach (5th ed.)
Addison-Wesley, April 2009.

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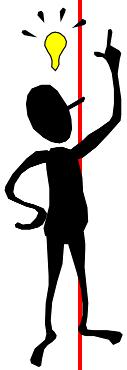
Public Key Cryptography

symmetric key crypto

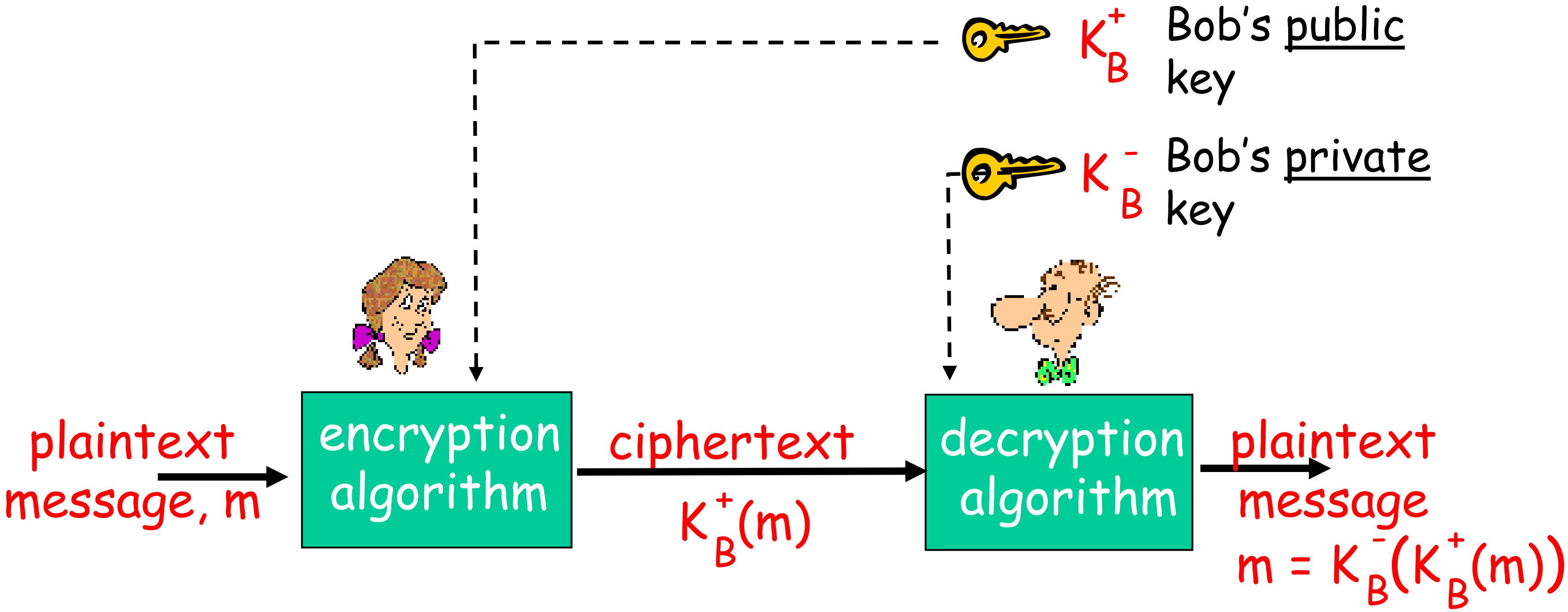
- requires sender, receiver know shared secret key
- Q: how to agree on key in first place (particularly if never "met")?

public key cryptography

- radically different approach [Diffie-Hellman76, RSA78]
- sender, receiver do *not* share secret key
- *public* encryption key known to *all*
- *private* decryption key known only to receiver



Public key cryptography



Public key encryption algorithms

Requirements:

1. need $K_B^+(\cdot)$ and $K_B^-(\cdot)$ such that

$$K_B^-(K_B^+(m)) = m$$

2. given public key K_B^+, it should be impossible to compute private key K_B^-

RSA: Rivest, Shamir, Adelson algorithm

Prerequisite: modular arithmetic

- x mod n = remainder of x when divide by n
- Facts:

  [(a mod n) + (b mod n)] mod n = (a+b) mod n

  [(a mod n) - (b mod n)] mod n = (a-b) mod n

  [(a mod n) * (b mod n)] mod n = (a*b) mod n
- Thus

  $(a \bmod n)^d \bmod n = a^d \bmod n$
- Example: x=14, n=10, d=2:

  $(x \bmod n)^d \bmod n = 4^2 \bmod 10 = 6$

  $x^d = 14^2 = 196 \quad x^d \bmod 10 = 6$

RSA: getting ready

- A message is a bit pattern.
- A bit pattern can be uniquely represented by an integer number.
- Thus encrypting a message is equivalent to encrypting a number.

Example

- m= 10010001 . This message is uniquely represented by the decimal number 145.
- To encrypt m, we encrypt the corresponding number, which gives a new number (the ciphertext).

RSA: Creating public/private key pair

1. Choose two large prime numbers $p, q.$ (e.g., 1024 bits each)

2. Compute $n = pq$, $z = (p-1)(q-1)$

3. Choose e (with $e<n$) that has no common factors with z. (e, z are "relatively prime").

4. Choose d such that $ed-1$ is exactly divisible by z. (in other words: ed mod $z = 1$).

5. *Public* key is (n,e). *Private* key is (n,d).

$\underbrace{(n,e)}_{K_B^+}$ $\underbrace{(n,d)}_{K_B^-}$

RSA: Encryption, decryption

0. Given (n,e) and (n,d) as computed above

1. To encrypt message m $(<n)$, compute

   $c = m^e \bmod n$

2. To decrypt received bit pattern, c, compute

   $m = c^d \bmod n$

Magic happens!

$$m = (\underbrace{m^e \bmod n}_{c})^d \bmod n$$

RSA example:

Bob chooses *p=5, q=7*. Then *n=35, z=24*.

e=5 (so *e, z* relatively prime).

d=29 (so *ed-1* exactly divisible by z).

Encrypting 8-bit messages.

	bit pattern	m	m^e	$c = m^e \bmod n$
encrypt:	0000l000	12	24832	17

	c	c^d	$m = c^d \bmod n$
decrypt:	17	481968572106750915091411825223071697	12

Why does RSA work?

- Must show that c^d mod n = m
  where c = m^e mod n
- Fact: for any x and y: x^y mod n = $x^{(y \bmod z)}$ mod n
  - where n= pq and z = (p-1)(q-1)
- Thus,
  c^d mod n = (m^e mod n)^d mod n

  = m^{ed} mod n

  = $m^{(ed \bmod z)}$ mod n

  = m^1 mod n

  = m

RSA: another important property

The following property will be *very* useful later:

$$K_B^-(K_B^+(m)) = m = K_B^+(K_B^-(m))$$

use public key first, followed by private key

use private key first, followed by public key

Result is the same!

Why $K_B^-(K_B^+(m)) = m = K_B^+(K_B^-(m))$?

Follows directly from modular arithmetic:

$$(m^e \bmod n)^d \bmod n = m^{ed} \bmod n$$
$$= m^{de} \bmod n$$
$$= (m^d \bmod n)^e \bmod n$$

Why is RSA Secure?

- suppose you know Bob's public key (n,e). How hard is it to determine d?
- essentially need to find factors of n without knowing the two factors p and q.
- fact: factoring a big number is hard.

Generating RSA keys

- have to find big primes p and q
- approach: make good guess then apply testing rules (see Kaufman)

Session keys

- Exponentiation is computationally intensive
- DES is at least 100 times faster than RSA

Session key, K_S

- Bob and Alice use RSA to exchange a symmetric key K_S
- Once both have K_S, they use symmetric key cryptography

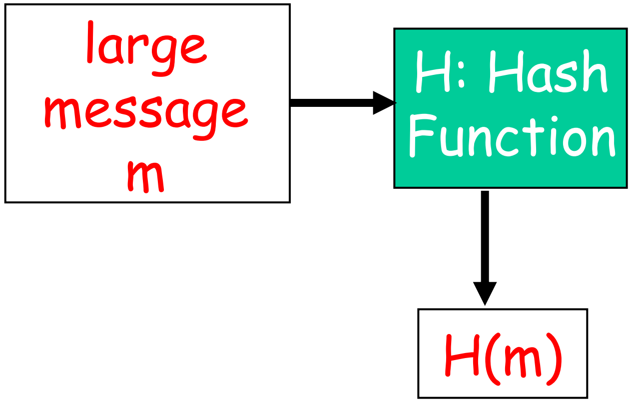
Chapter 8 roadmap

8.1 What is network security?
8.2 Principles of cryptography
8.3 Message integrity
8.4 Securing e-mail
8.5 Securing TCP connections: SSL
8.6 Network layer security: IPsec
8.7 Securing wireless LANs
8.8 Operational security: firewalls and IDS

Message Integrity

- allows communicating parties to verify that received messages are authentic.
  - Content of message has not been altered
  - Source of message is who/what you think it is
  - Message has not been replayed
  - Sequence of messages is maintained
- let's first talk about message digests

Message Digests



- function H( ) that takes as input an arbitrary length message and outputs a fixed-length string: "message signature"
- note that H( ) is a many-to-1 function
- H( ) is often called a "hash function"

desirable properties:
- easy to calculate
- irreversibility: Can't determine m from H(m)
- collision resistance: computationally difficult to produce m and m' such that H(m) = H(m')
- seemingly random output

Internet checksum: poor message digest

Internet checksum has some properties of hash function:

- ✓ produces fixed length digest (16-bit sum) of input
- ✓ is many-to-one

- but given message with given hash value, it is easy to find another message with same hash value.
  - e.g.,: simplified checksum: add 4-byte chunks at a time:

message	ASCII format
I O U 1	49 4F 55 31
0 0 . 9	30 30 2E 39
9 B O B	39 42 D2 42
	B2 C1 D2 AC

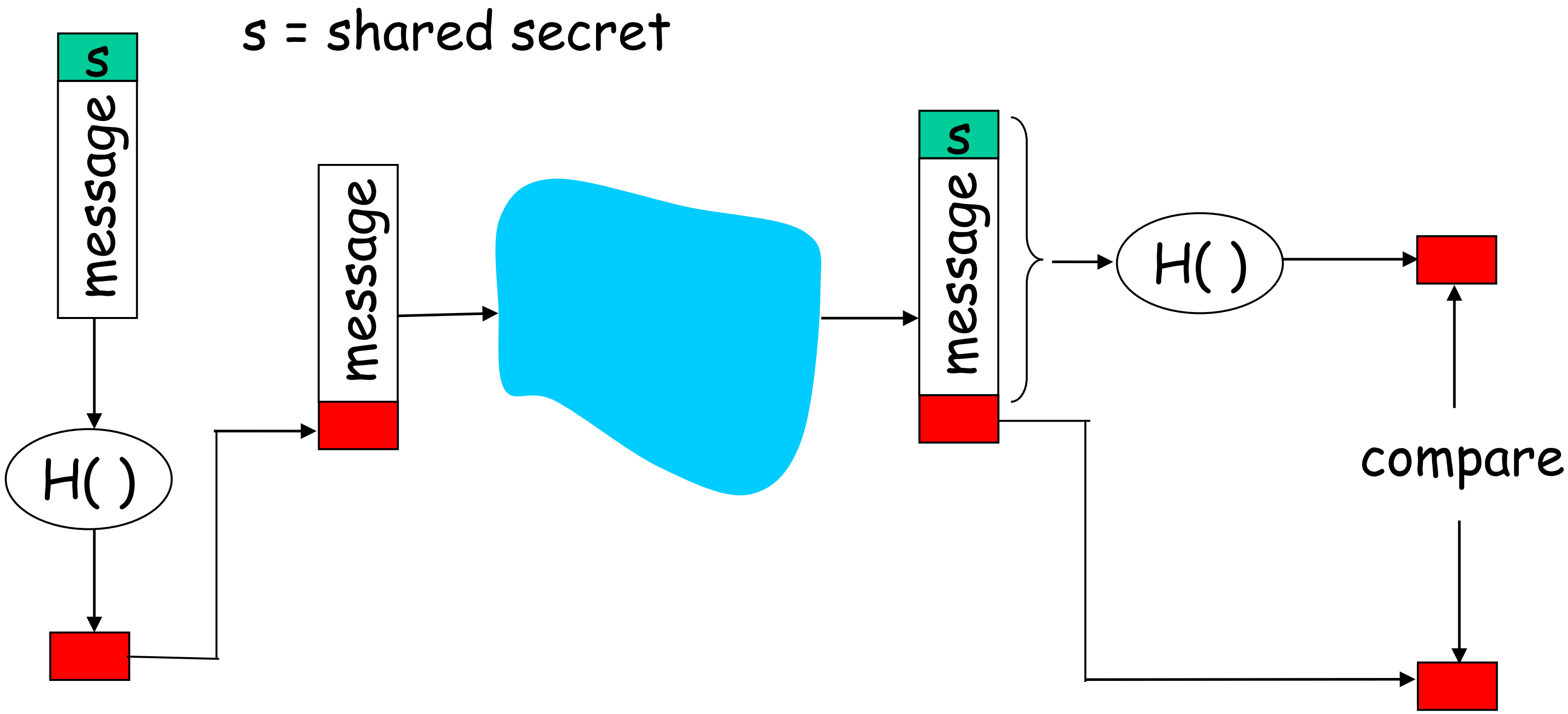
message	ASCII format
I O U 9	49 4F 55 39
0 0 . 1	30 30 2E 31
9 B O B	39 42 D2 42
	B2 C1 D2 AC

different messages but identical checksums!

Hash Function Algorithms

- MD5 hash function widely used (RFC 1321)
  - computes 128-bit message digest in 4-step process.
- SHA-1 is also used.
  - US standard [NIST, FIPS PUB 180-1]
  - 160-bit message digest

Message Authentication Code (MAC)

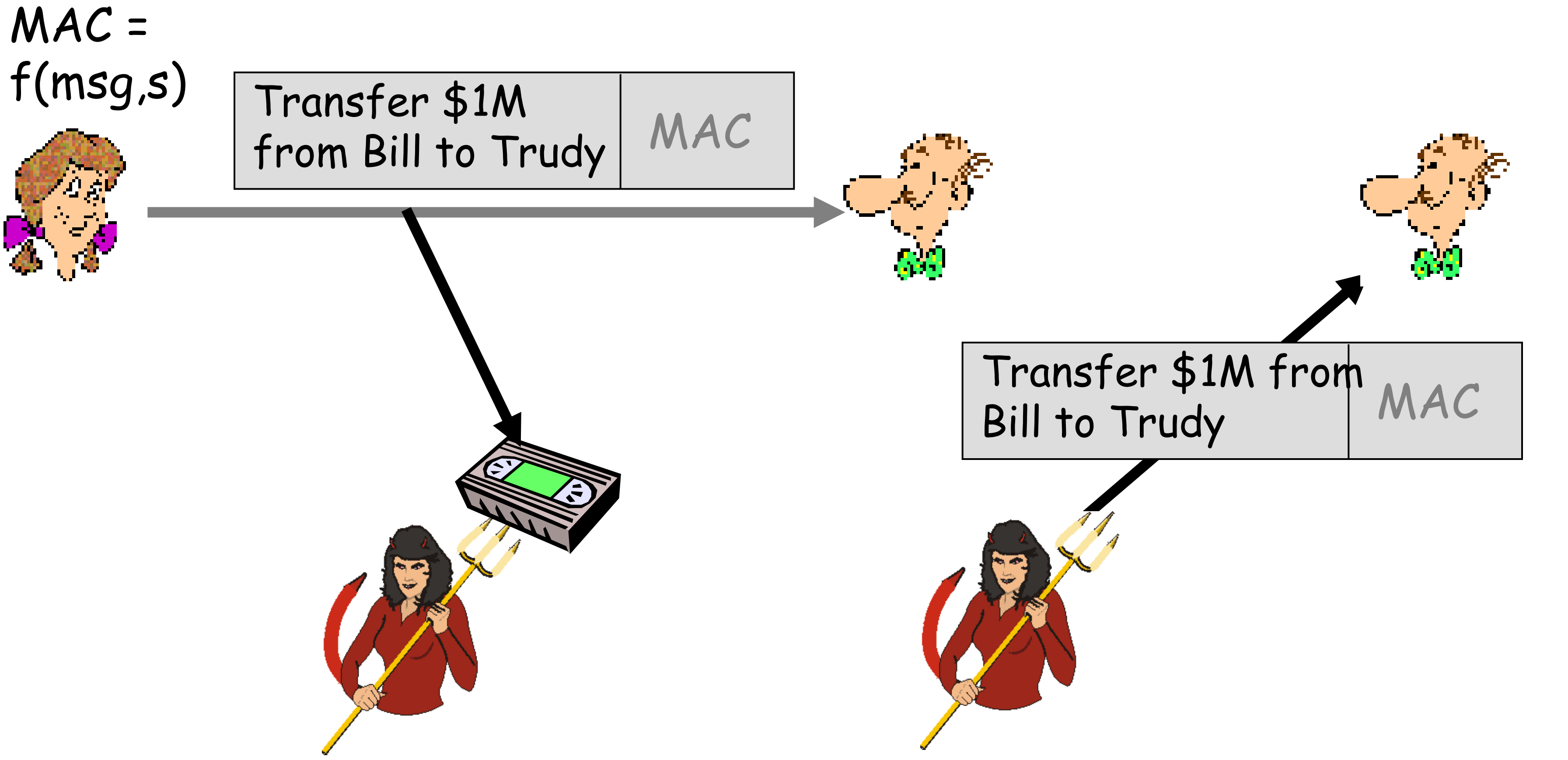


- ***Authenticates sender***
- ***Verifies message integrity***
- No encryption !
- Also called "keyed hash"
- Notation: $MD_m = H(s||m)$; send $m||MD_m$

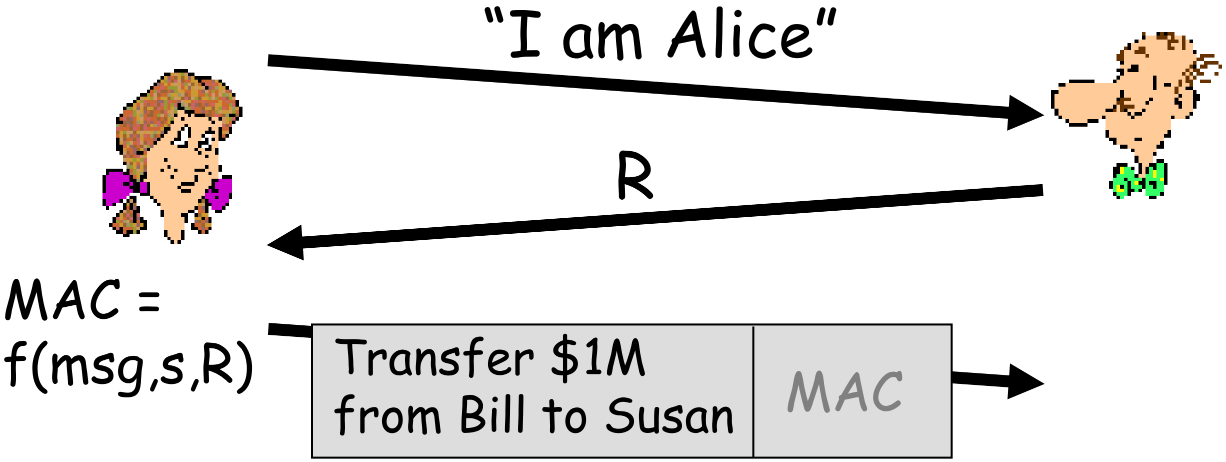
End-point authentication

- want to be sure of the originator of the message – *end-point authentication*
- assuming Alice and Bob have a shared secret, will MAC provide end-point authentication?
  - we do know that Alice created message.
  - … but did she send it?

Playback attack



Defending against playback attack: nonce



Digital Signatures

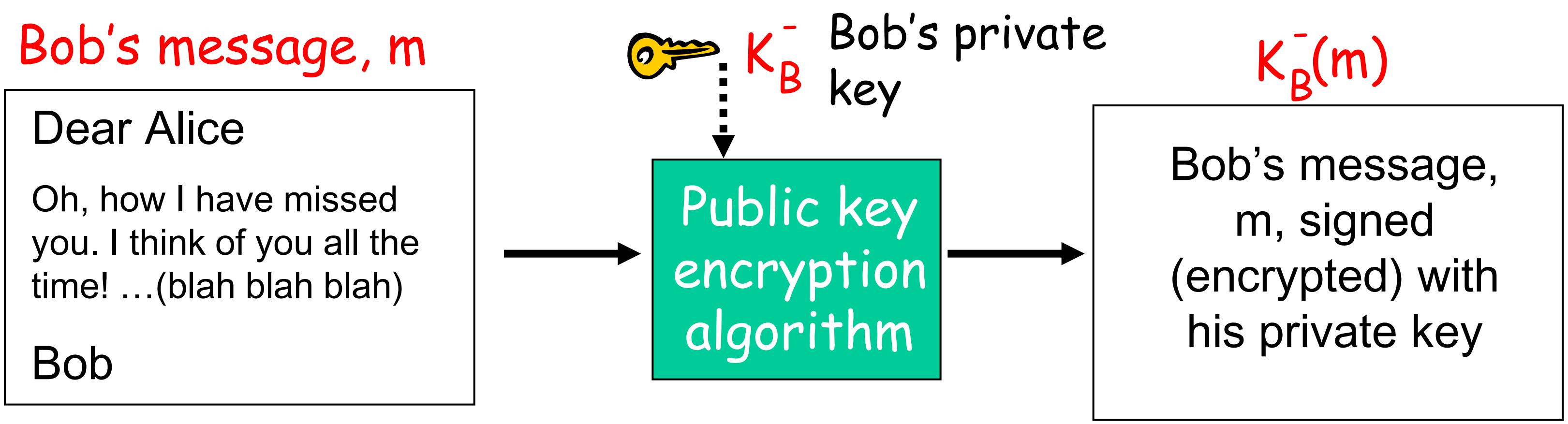
cryptographic technique analogous to hand-written signatures.

- sender (Bob) digitally signs document, establishing he is document owner/creator.
- goal is similar to that of MAC, except now use public-key cryptography
- *verifiable, nonforgeable*: recipient (Alice) can prove to someone that Bob, and no one else (including Alice), must have signed document

Digital Signatures

simple digital signature for message m:

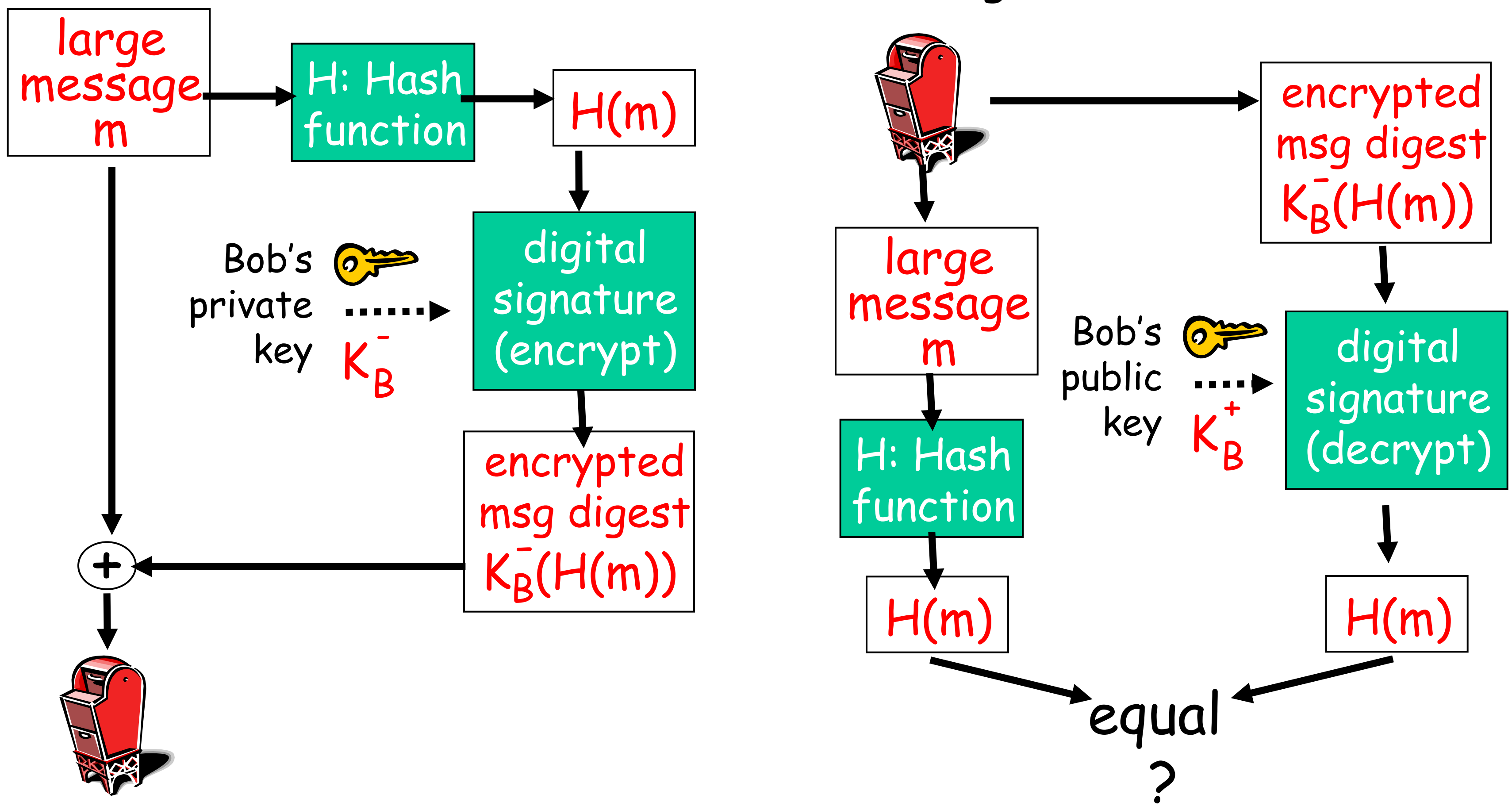
- Bob signs m by encrypting with his private key K_B^-, creating "signed" message, $K_B^-(m)$



Digital signature = signed message digest

Bob sends digitally signed message:

Alice verifies signature and integrity of digitally signed message:



Digital Signatures (more)

- suppose Alice receives msg m, digital signature $K_B^-(m)$
- Alice verifies m signed by Bob by applying Bob's public key K_B^+ to $K_B^-(m)$ then checks $K_B^+(K_B^-(m)) = m$.
- if $K_B^+(K_B^-(m)) = m$, whoever signed m must have used Bob's private key.

Alice thus verifies that:

✓ Bob signed m.

✓ no one else signed m.

✓ Bob signed m and not m'.

Non-repudiation:

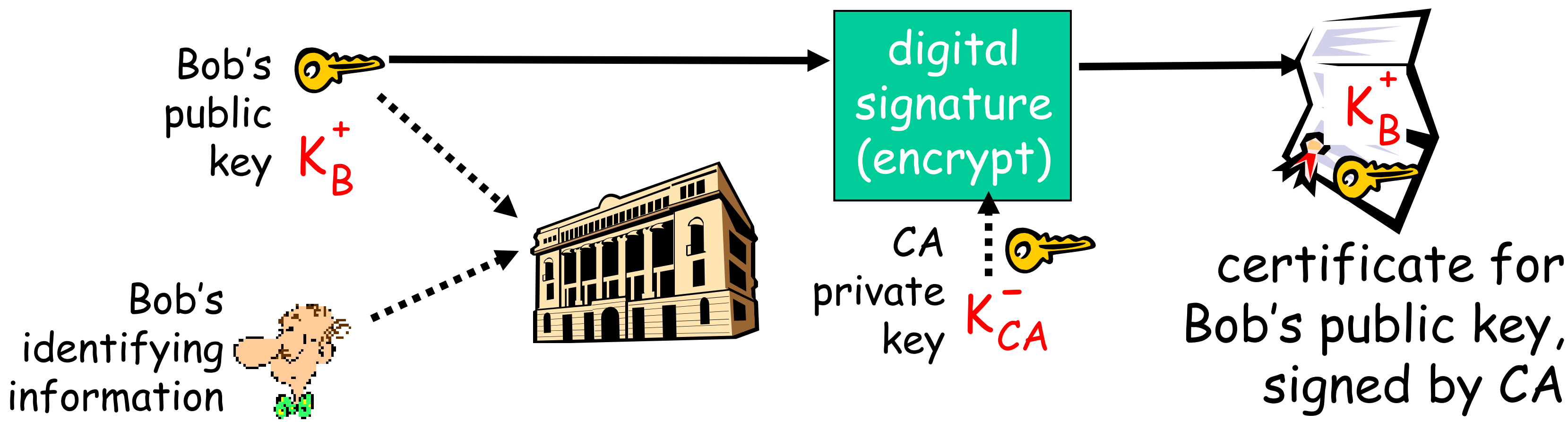
✓ Alice can take m, and signature $K_B^-(m)$ to court and prove that Bob signed m.

Public-key certification

- motivation: Trudy plays pizza prank on Bob
  - Trudy creates e-mail order:
    Dear Pizza Store, Please deliver to me four pepperoni pizzas. Thank you, Bob
  - Trudy signs order with her private key
  - Trudy sends order to Pizza Store
  - Trudy sends to Pizza Store her public key, but says it's Bob's public key.
  - Pizza Store verifies signature; then delivers four pizzas to Bob.
  - Bob doesn't even like Pepperoni

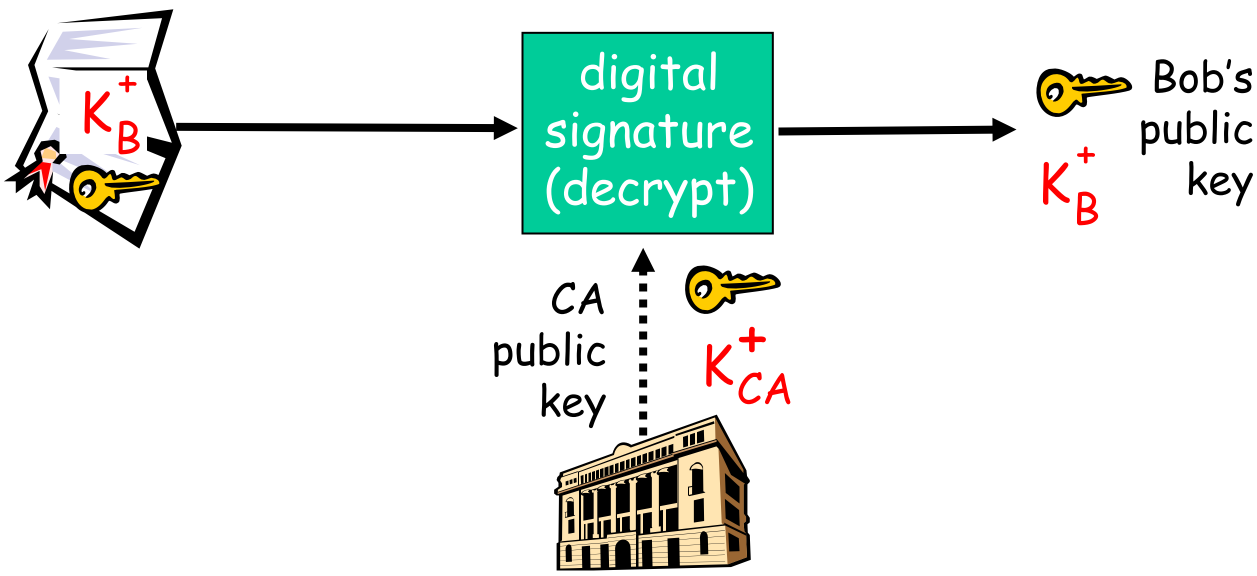
Certification Authorities

- Certification authority (CA): binds public key to particular entity, E.
- E (person, router) registers its public key with CA.
  - E provides "proof of identity" to CA.
  - CA creates certificate binding E to its public key.
  - certificate containing E's public key digitally signed by CA – CA says "this is E's public key"



Certification Authorities

- when Alice wants Bob's public key:
  - gets Bob's certificate (Bob or elsewhere).
  - apply CA's public key to Bob's certificate, get Bob's public key



Certificates: summary

- primary standard X.509 (RFC 2459)
- certificate contains:
  - issuer name
  - entity name, address, domain name, etc.
  - entity's public key
  - digital signature (signed with issuer's private key)
- Public-Key Infrastructure (PKI)
  - certificates, certification authorities
  - often considered "heavy"

Why study computer networks?

- An interface between theory (algorithms, mathematics) and practice
- Understanding the design principles of a truly complex system
- Industry-relevant knowledge
- Fun!

- Challenges in teaching computer networks
- Students' feedback