1. In class (March 10) we saw a simple algorithm for Broadcast and Echo (in which one initiator sends a message to all other nodes in the network and receives a signal after all nodes are known to have received a copy of the message). In that algorithm we used two types of messages, M and ACK (where M is the original message that is being disseminated).

(a) Here you are required to describe the algorithm again and provide its code (pseudo code, which is more than what we did in class) in such a way that one type of messages are being used, the message M. You are required to describe the code of the algorithm that results from the simple Broadcast algorithm combined with the Termination detection of diffusing computation. The complexity of the new algorithm should be very close to the complexity of the algorithm that was presented in class. What is the exact complexity (messages and time) of the new algorithm.

(b) Modify the code so at the end the root of the broadcast tree will know the depth (= height) of the tree (The longest distance in the tree from the root to any leaf node). The message M may now have a variable as an argument carried with the message.

2. Here we modify the flood algorithm presented in the first class as follows: The flood algorithm is the phase of the broadcast and echo algorithm where only message M is being forwarded down to the nodes (without the echo). We want to modify the code so at the end of the algorithm the tree that is defined by the flood (as we defined in class) is a BFS tree. I.e., shortest path spanning tree from the initiator to the rest of the network. To do that each message M is being tagged with the number of hops its sender is from the initiator. That is messages are now M(i), where 0 ≤ i ≤ n. Each time a node receives a message M(j) where j is smaller than the minimum value it has received so far, that node sends again a new message M(j + 1) to all its neighbors (or to all its neighbors except the one from which it got M(j)).
(a) Provide the code of the new algorithm.
(b) What is the message complexity of the new algorithm.

3. There are $n$, $n$ even, identical finite automata connected by $n$ unidirectional links (can communicate or pass signals only in one direction), to form a unidirectional ring, $n > 1$. The same automaton works for any size ring. Exactly one automaton (one node), called the king, receives a WAKE signal from the environment.

Design an algorithm (pseudo code of the finite automaton) which informs the automaton located across from the king that it is across of the king. Show that your algorithm has asymptotically optimal bit complexity (i.e., prove a lower bound on the total number of bits exchanged between all the nodes by any such solution.)

(a) Asynchronous: Assume the automata and the ring are Asynchronous. Give the upper and lower bounds.
(b) Synchronous: Assume the automata and the ring are Synchronous. Give the upper and lower bounds.
(c) Only Synchronous: Design an algorithm on top of the previous so that the two nodes, the king and the one across fire a flare (ZIKOOK) together at the same round.

4. Consider an anonymous ring where each processes starts with a binary input.

(a) Present a (non-uniform, i.e., they do know $n$) asynchronous algorithm for computing logical AND; the algorithm should send $O(n^2)$ messages in the worst case. Justify shortly why the algorithm is correct.
(b) Prove that $\Omega(n^2)$ is a lower bound on the message complexity of any asynchronous algorithm that computes logical AND.
(c) Present a (non-uniform) synchronous algorithm for computing logical AND; the algorithm should send $O(n)$ messages in the worst case. Justify shortly why the algorithm is correct.