Ants Nearby Treasure Search

Biological Distributed Algorithms - ANTS
Plan

ANTS

- Background / motivation, memory lower bounds, upper bound

Extensions

- Pheromones
- Mobile state machines
Background

Group of agents search for treasure
  ◦ Cooperatively
  ◦ In the $2d$ plane
  ◦ Starting at central location

Goal of the search:
  ◦ Locate nearby treasures as fast as possible
  ◦ At a rate that scales well with number of participants
Motivation

A variety of animals search for food around a central location
  ◦ Nest, sheltered or familiar environment, etc.

Preference for locating nearby food
  ◦ Predation risk
  ◦ Rate of food collection
  ◦ Holding territory
  ◦ Navigating back

Use distributed computing tools to study biological scenarios
ANTS

Feinerman, Korman, Lotker, Sereni, PODC’12
- Collaborative Search on the Plane without Communication

$k$ ants search for food on $2d$ plane
- Start at the nest $(0,0)$ – the source
- Adversary places treasure at some node – the target
- Food item (target) placed at distance $D$ from the nest.

- Goal: minimize $T(k, D)$ – time for $k$ ants to find food in distance $D$
ANTS

$k$ may not be available

Synchronous steps (async is the same)

Goal reached:
  ◦ One of the agents visits the target
ANTS
Mobile Agents

Ants are mobile *agents*
- Probabilistic machines
- Identical
- Cannot communicate

No restriction on internal storage
No restriction on computational power

Trivial lower bound: $\Omega(D + \frac{D^2}{k})$
Optimal Algorithm

\[\text{for } j = 1 \text{ to } \infty\]
\[\quad \text{for } i = 1 \text{ to } j\]
\[\quad \text{\quad } p \leftarrow \text{random} \left(0 \ldots 2^i\right)\]
\[\quad \text{move to } p\]
\[\quad \text{Spiral search } \left(4 \cdot \frac{2^i}{k}\right) \text{ points}\]
\[\quad \text{return to source}\]
Optimal Algorithm

Asymptotically optimal
  ◦ With constant approximation of $k$

Fault-tolerant

Asynchronous

Exact counting

Memory: $\Omega(\log D)$ state bits

Can we avoid these?
Lower Bounds

Ants know constant approximation of $k$
- $T(k, D) = \Omega(D + \frac{D^2}{k})$

Ants have no information regarding $k$
- $T(k, D) = O((D + \frac{D^2}{k}) \cdot \log^{1+\varepsilon} k)$
- For every constant $\varepsilon > 0$
Lower Bounds

Feinerman, Korman, CoRR’13

- Memory Lower Bounds for Randomized Collaborative Search and Applications to Biology

\[ T(k, D) = O\left((D + \frac{D^2}{k}) \cdot \log^{1-\varepsilon} k\right) \]

- Requires \( \Omega(\log \log k) \) memory bits
- For every \( \varepsilon > 0 \)
Lower Bounds

Can we do better?
Pheromones

Lenzen, Radeva, DISC’13
- The Power of Pheromones in Ant Foraging

Not clear whether ants have notion of their number
Double loops behavior unlikely to evolve without necessity

Ants are known to make use of pheromones
Model

Ant marks vertex as *visited* (releases a *pheromone*):
- Can sense whether 4 neighboring vertices have been visited

Asynchronous rounds:
- Round $i$ complete once all ants took at least $i$ steps
- Adversarial scheduler
Model
Pheromones Algorithm

Ant at \((x, y)\).
- Distance \(d(x, y) = \max\{|x|, |y|\}\)

Two types of nodes used:
- \(\text{close}(x, y)\) - distance \(d(x, y)\), clockwise of it
- \(\text{away}(x, y)\) – distance one more than \(d(x, y)\)
Pheromones Algorithm

Precise definition of $away(x, y)$ is irrelevant

- As long as the distance is one larger than $d(x, y)$

**Algorithm 1:** Ant exploration algorithm for an ant located at vertex $(x, y)$

1. **if** clock$(x, y)$ *is not visited* **then** move to clock$(x, y)$ **else** move to away$(x, y)$;
Pheromones Algorithm

Spearhead
- \((x, y)\) is a spearhead, iff \((x, y)\) visited, \(\text{clock}(x, y)\) isn’t

Lemma 1.
- At each time, all spearheads are occupied by ants
Pheromones Algorithm

Lemma 2.

- If an ant is at distance $D$, after $8D$ rounds all vertices at distance at most $D$ have been visited.

- Each unexplored layer contains a spearhead
- Each spearhead has an ant (Lemma 1)
- After one round - $\text{clock}(x, y)$ will be visited
- Hence, in each round – at least one new vertex explored
- At most $8D$ vertices each layer, thus lemma holds
Main Theorem

Theorem 3.

- Terminates in less than $9D + 4D(D + 1)/k$ rounds
- This is optimal up to factor $5.5$

- Initial number of unvisited vertices: $\sum_{d=1}^{D} 8d = 4D \cdot (D + 1)$
- Each round, ant increases distance or explores new vertex
- Thus, after $D + 4D \cdot (D + 1)/k$ rounds – there is an ant at distance $D$
- By Lemma 2, at most $8D$ rounds later, all at distance $D$ are explored.
- Total: $9D + 4D \cdot (D + 1)/k$ rounds
Main Theorem

Theorem 3.
- Terminates in less than $9D + 4D(D + 1)/k$ rounds
- This is optimal up to factor 5.5

- $2D$ rounds to reach $(D, D)$
- $4D \cdot (D + 1)/k$ rounds to explore all up to distance $D$

- If $2D \geq 4D \cdot (D + 1)/k$, then
  - $9D + 4D \cdot (D + 1)/k \leq 11D = 5.5 \cdot 2D$
  - Otherwise, $9D + 4D \cdot (D + 1)/k \leq 5.5 \cdot 4D \cdot (D + 1)/k$
Pheromones Summary

Ants can communicate – with pheromones
Ants are deterministic
Asynchronous rounds
No information of $k$

Running time: $9D + 4D(D + 1)/k$
  ◦ Which is optimal $O(D + D^2/k)$
Mobile Finite State Machines

*Emek, Langner, Uitto, Wattenhofer, CoRR’13*

- Ants: Mobile Finite State Machines

Ants have unbounded memory (i.e., Turing machines)

Need to know $k$ to reach lower bound

Can’t communicate

New model assumes limited capabilities – *randomized FSM*
Model

Synchronous execution
All start at time 0
Ants can distinguish origin (0,0) from other cells

Constant memory (computation is FSM)
Constant communication when in same cell
  ◦ Ant can sense if other ants at cell are in state q
Model

Distance is $l_1$ norm, i.e., $d(x, y) = |x| + |y|$  
Mark directions by compass: $N, S, E, W, P$
  - $P$ – “stay put”

Randomized FSM
  - Transitions are chosen uniformly at random

As before, $k$ and $D$ are unknown
Parallel Rectangle Search

Algorithm depends on an *emission scheme*:
- Divides all ants in origin into teams of *five*
- Emits them continuously from the origin

- No two search teams emitted at the same round
- Ants can distinguish their role (of *five* roles)
- First team knows it is the first

- Described later
Ant Roles

Guide – for $N, S, E, W$
- Travel in some direction
- Until meeting other guide
- Then continue until empty

- Travel outward when meeting explorer

First group’s guides?
- Travel a single vertex
Ant Roles

Explorer

- Bulk of actual search
- Travel with N Guide
- Travel diagonally
- Change when meeting guide
- Stop on N Guide

- Ants proceed outwards
  - Repeat from start
Parallel Rectangle Search
Ant Roles

New Guide
○ Emitted from the origin until becoming a Guide

Guide
○ Waits until meeting an Explorer

Moving Guide
○ Moving outwards (as before) until becoming a Guide
Ant Roles

New Explorer
- Emitted from the origin until becoming an Explorer (as new guide, except moving $W$ before transforming)

Explorer
- Moves diagonally, changes direction according to Guide

Moving Explorer
- Moving outwards (as before) until becoming an Explorer
Parallel Rectangle Search

Observation 1.
- Time to explore a level $d > 0$ is $8d$ rounds

Observation 2.
- Exploring level $d \geq d'$ starts $\geq d - d'$ after exploring level $d'$ starts

Lemma 3.
- No two ants of the same role occupy the same cell
Parallel Rectangle Search

Lemma 4.

- When finished exploring level $d$, all other $MovingExplorer$ ants are at distance $\geq d + 8$ from the origin

- Other $MovingExplorer$ last level: $x < d$
- Level $d$ started $\geq d - x$ rounds later
  - (Observation 2)
- Another $8d$ rounds to explore level $d$
  - (Observation 1)
- Total $\geq (9d - x)$ rounds since $x$ started until $d$ finished
Parallel Rectangle Search

**Lemma 4.**

- When finished exploring level $d$, all other MovingExplorer ants are at distance $\geq d + 8$ from the origin

- $x < d \Rightarrow 8x + 8 \leq 8d$

- Thus: $9d - x \geq 8x + (d - x) + 8$

- Time to finish exploring $x$ (8x), reach level $d$ ($d - x$), and take 8 more steps
Parallel Rectangle Search

**Corollary 5.**
- Distance between any two $\text{MovingExplorer}$ ants $\geq 8$

$t_0$ - first time there are no $\text{NewExplorer}$ ants anymore

**Observation 6.**
- After $t_0$, role of $\geq 7/8$ of exploring ants is $\text{Explorer}$

**Corollary 7.**
- After $t_0$, $\Omega(k)$ new cells are being explored
Main Theorem

Theorem 8.
\begin{itemize}
  \item Emitting a team each round, treasure is found in time:
    \[ O(D + D^2/k) \]
\end{itemize}

If \( D \leq k \)
\begin{itemize}
  \item Level \( d \) exploring starts after \( \leq 2D \) rounds
  \item Finishes \( 8D \) rounds later
  \item Treasure found in \( O(D) \) rounds
\end{itemize}
Main Theorem

Theorem 8.
- Emitting a team each round, treasure is found in time:
  \[ O(D + D^2/k) \]

If \( D > k \)
- \textit{Guides} occupy contiguous segment of cells
- No cell in level larger than \( D + k/5 < 2D \) explored
- First \( 2D \) levels contain \( O(D^2) \) cells
- All cells explored after \( t_0 + O(D^2) \)
- Treasure found in \( O(D^2) \) rounds
Emission Scheme

Takes $c \cdot \log k$ rounds w.h.p.

Team is emitted every constant number of rounds

By main theorem, almost optimal run-time of

$$O(D + D^2/k + \log k)$$
Emission Scheme

Each round, ant moves east with probability $\frac{1}{2}$
- If all decide to move – none moves
- Ant **alone** marked as **ready** and stops moving
- When encountering **ready** ant – move east

When finished
- All ants spread at ray R (to east)
- Each cell (up to $k$) contains a **single** ant
Emission Scheme

Ant ready at distance $d \equiv 1 \ (mod \ 5)$ is an Explorer
  ◦ First Explorer notifies others

Move east to collect 4 ready ants, marked as Guide
  ◦ Wait for ready ant in each cell

Move one further to notify next Explorer to collect
  ◦ Ants know role according to distance

All 5 team ants move to origin, then emitted
Emission Scheme
Mobile FSM Summary

Ants can communicate – constant size
Ants store no memory (only constant-sized state)
Synchronous rounds
No information of $k$

Running time: $O(D + D^2/k + \log k)$
  ◦ Optimal with a “better” emission scheme
<table>
<thead>
<tr>
<th>Mobile FSM</th>
<th>Pheromones</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ants can communicate – constant-size state</td>
<td>Ants can communicate – with pheromones</td>
</tr>
<tr>
<td>Ants are <strong>randomized</strong> FSM</td>
<td>Ants are <strong>deterministic</strong></td>
</tr>
<tr>
<td>Synchronous rounds</td>
<td>Asynchronous rounds</td>
</tr>
<tr>
<td>Ants store no memory (only state)</td>
<td>Memory not specified</td>
</tr>
<tr>
<td><strong>No information of</strong> $k$</td>
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</tr>
<tr>
<td>Running time: $O(D + D^2/k + \log k)$ Reaching optimal with $O(1)$ emission scheme</td>
<td>Running time: $9D + 4D(D + 1)/k$ Which is optimal $O(D + D^2/k)$</td>
</tr>
</tbody>
</table>
## Results

<table>
<thead>
<tr>
<th></th>
<th>know $k$</th>
<th>don’t know $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No communication</td>
<td>$O(D + D^2 / k)$ \newline $\Omega(\log \log k)$ memory bits</td>
<td>$O((D + D^2 / k) \cdot \log^{1+\varepsilon} k)$ \newline $\Omega(\log \log k)$ memory bits</td>
</tr>
<tr>
<td>Pheromones</td>
<td>Irrelevant</td>
<td>$O(D + D^2 / k)$ \newline Memory vague - $O(\log D)$</td>
</tr>
<tr>
<td>FSM communication</td>
<td>Irrelevant</td>
<td>$O(D + D^2 / k)$ \newline Constant memory</td>
</tr>
</tbody>
</table>
Open Questions

**Pheromones**
- Ants don’t use pheromones that way
  - Cost producing pheromones, danger of alerting enemies
- Are pheromones efficient in other areas?
  - Other animals, robots sweeping an area

**Mobile FSM**
- $O(1)$ emission scheme
- Asynchronous environment

**General**
- Fault tolerance
- Find way back to nest? (finder / all others)