

Local Reasoning about Storable Locks and Threads

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- Dijkstra
 - Programs should be insensitive to relative execution speeds
- Brinch Hansen / Hoare
 - Shared variables should be encapsulated and their access controlled
 - Monitors
 - Compiler could check if encapsulation violated for variables
 - Solo operating system written almost entirely with safe primitives
 - But what about the heap? Needed for multi-user OSes
- Owicki & Gries / Jones
 - Limit interference through shared state with predicates / relations
- O'Hearn
 - Concurrent separation logic: encapsulation checking for the heap
 - "Size" of shared state can change
 - "Topology" of access control still fixed

Locks in the heap — Why?

```
typedef struct NODE {  
    int Val;  
    struct NODE* Next;  
} NODE;
```

```
LOCK lock;  
NODE* head;
```

non-empty, last
value is $+\infty$

```
locate_coarse(int e) {  
    NODE *prev, *curr;  
    acquire(lock);  
    prev = head;  
    curr = prev->Next;  
    while (curr->Val < e) {  
        prev = curr;  
        curr = prev->Next;  
    }  
    return (prev, curr);  
}
```

```
typedef struct NODE {  
    LOCK Lock;  
    int Val;  
    struct NODE* Next;  
} NODE;
```

```
NODE* head;
```

```
locate_hand_over_hand(int e) {  
    NODE *prev, *curr;  
    prev = head;  
    acquire(prev);  
    curr = prev->Next;  
    acquire(curr);  
    while (curr->Val < e) {  
        release(prev);  
        prev = curr;  
        curr = prev->Next;  
        acquire(curr);  
    }  
    return (prev, curr);  
}
```

Optimistic / Idealistic

```
resource  $r$   
  
with  $r$  do  
  ⋮  
od
```

- syntactically determined critical regions

(More) Realistic

```
if (flag > 0) {  
  acquire(1);  
}  
⋮  
if (flag > 0) {  
  release(1);  
}
```

- semantically determined critical regions

Locks on the stack vs locks in the heap

Optimistic / Idealistic

```
resource  $r$   
  
with  $r$  do  
  ⋮  
od
```

(More) Realistic

```
 $l$  = new LOCK;  
⋮  
init( $l$ );  
⋮  
acquire( $l$ );  
⋮  
release( $l$ );  
⋮  
finalize( $l$ );  
⋮  
delete  $l$ ;
```

- bounded numbers of resources
- unbounded numbers of locks

Parallel composition vs dynamic thread creation

Optimistic / Idealistic

```
(while  $b$  do ( $P_1 \parallel P_2$ ))  $\parallel P_3$ 
```

- bounded numbers of processes

(More) Realistic

```
for (i = 0; i < n; i++) {  
    t[i] = fork(proc, i);  
}  
:  
for (i = 0; i < n; i++) {  
    join(t[i]);  
}
```

- unbounded numbers of threads

- Program logics for analysis and verification of multithreaded heap-manipulating programs
- Goal: ease static access control
 - Allow unboundedly-many locks and threads
 - That live in the heap (to exploit indirection)
- but also aim to:
 - Retain local reasoning
 - Enable automation in program analysis
 - Treat more realistic programming language constructs

- Logic for storable locks and threads
 - Local reasoning preserved
 - Storable locks as resources
- Not only technical difficulties:
 - Storable locks “make theoreticians wince” (Richard Bornat)
 - Russell’s paradox is lurking nearby:
 - heaps → locks → resource invariants → heaps
 - Analogous to stored procedures: Landin’s “knots in the store”

- First one top-level parallel composition: $C_1 \parallel \dots \parallel C_n$
- Then dynamic thread creation
- Simplification: no shared mutable variables
 - shared mutable heap
 - global pre-initialized constants
 - local variables of threads
- General cases and details:
Technical report MSR-TR-2007-39

- A Floyd/Hoare-style program logic
- Assertion language: $*$ splits the state into disjoint parts
- Proof system:

$$\frac{\{P\} C \{Q\}}{\{P * R\} C \{Q * R\}}$$

$$\frac{\{P_1\} C_1 \{Q_1\} \quad \{P_2\} C_2 \{Q_2\}}{\{P_1 * P_2\} C_1 || C_2 \{Q_1 * Q_2\}}$$

- Allows for local reasoning
- Processes access shared resources
- Synchronization via conditional critical regions:

`with r when b do C`

to be replaced

- Program state partitioned into (disjoint) substates owned by the different processes and locks
- Processes may access only parts of the state that they own
- Process interaction mediated using resource invariants
- Key in achieving local reasoning:
 - reasoning about each process in isolation
 - using the sequential semantics

```
locate_coarse(int e) {  
    NODE *prev, *curr;  
    acquire(lock);  
    “have (exclusive access to) head list”  
    prev = head;  
    “head has a Next”  
    curr = prev->Next;  
    “curr has a Val”  
    while (curr->Val < e) {  
        prev = curr;  
        “curr has a Next”  
        curr = prev->Next;  
    }  
    return (prev, curr);  
}
```

Need to know this even without
owning curr node:
So ownership of a node comes
with knowledge that the Next
node has a Lock

```
locate_hand_over_hand(int e) {  
    NODE *prev, *curr;  
    prev = head;  
    acquire(prev);  
    “have (exclusive access to) prev node”  
    curr = prev->Next;  
    “curr has a Lock”  
    acquire(curr);  
    “have curr node”  
    while (curr->Val < e) {  
        “prev is locked by this thread”  
        release(prev);  
        “don’t have prev node any more”  
        prev = curr;  
        curr = prev->Next;  
        “curr has a Lock”  
        acquire(curr);  
        “have curr node”  
    }  
    return (prev, curr);  
}
```

- Lock \rightarrow resource invariant
 - lock \rightarrow sort $A(\cdot, \cdot)$
 - sort $A(\cdot, \cdot) \rightarrow$ resource invariant $I_A(\cdot, \cdot)$
 - first parameter – address of the lock

- Example:

```
struct R {  
    LOCK Lock;  
    int Data;  
};
```

$$I_R(l, v) \triangleq l:Data \mapsto v$$

- Knots in the store cut by indirection through $A(\cdot, \cdot)$

- Handles: $A(E, \vec{F})$
 - ensures that the lock at the address E exists and has the sort A and parameters \vec{F}
 - gives permission to acquire the lock
 - can be split among threads:
 - $1A(E, \vec{F}) = \frac{1}{2}A(E, \vec{F}) * \frac{1}{2}A(E, \vec{F})$
 - < 1 – can acquire the lock
 - $= 1$ – can finalize the lock
- Locked-facts: $\text{Locked}_A(E, \vec{F})$
 - lock E is held by the thread owning $\text{Locked}_A(E, \vec{F})$
 - ensures the existence of the lock

$$\{E \mapsto _ \} \text{init}_{A, \vec{F}}(E) \{A(E, \vec{F}) * \text{Locked}_A(E, \vec{F})\}$$

$$\{A(E, \vec{F}) * \text{Locked}_A(E, \vec{F})\} \text{finalize}(E) \{E \mapsto _ \}$$

$$\{\text{Locked}_A(E, \vec{F}) * I_A(E, \vec{F})\} \text{release}(E) \{\text{emp}_h\}$$

$$\{\pi A(E, \vec{F})\} \text{acquire}(E) \{\pi A(E, \vec{F}) * \text{Locked}_A(E, \vec{F}) * I_A(E, \vec{F})\}$$

A simple example

```
struct R {
  LOCK Lock;
  int Data;
} *x;
//  $I_R(l) \triangleq l:Data \rightarrow \_$ 
```

```
initialize() {
  {emph}
  x = new R;
  { $x \mapsto \_ * x:Data \mapsto \_$ }
  initR(x);
  { $x:Data \mapsto \_ * R(x)$ 
   * LockedR(x)}
  x->Data = 0;
  { $x:Data \mapsto 0 * R(x)$ 
   * LockedR(x)}
  release(x);
  {R(x)}
}
```

```
thread() {
  { $\frac{1}{2}R(x)$ }
  acquire(x);
  { $x:Data \mapsto \_ * \frac{1}{2}R(x)$ 
   * LockedR(x)}
  x->Data++;
  { $x:Data \mapsto \_ * \frac{1}{2}R(x)$ 
   * LockedR(x)}
  release(x);
  { $\frac{1}{2}R(x)$ }
}
```

```
cleanup() {
  {R(x)}
  acquire(x);
  { $x:Data \mapsto \_ * R(x)$ 
   * LockedR(x)}
  finalize(x);
  { $x \mapsto \_ * x:Data \mapsto \_$ }
  delete x;
  {emph}
}
```


Assertion language model

- Semantic domains:

$$\text{Stacks} = \text{Vars} \rightarrow_{\text{fin}} \text{Values}$$

$$\text{Heaps} = \text{Locations} \rightarrow_{\text{fin}}$$

$$(\mathbf{Cell}(\text{Values}) \cup \mathbf{Lock}(\text{Sorts} \times \text{LockValues} \times \text{LockPerms}))$$

$$\{U, 0, 1, \dots, n\}$$

$$[0, 1]$$

- each program proof associates each sort with an invariant:

$$I_A(\vec{E}) : \text{Sorts} \rightarrow \text{Values}^+ \rightarrow \mathcal{P}(\text{Stacks} \times \text{Heaps})$$

- Satisfaction relation : $(s, h) \models_k \Phi$

$$(s, h) \models_k E \mapsto F \quad \Leftrightarrow \quad h = [[E]]_s : \mathbf{Cell}([F]_s)$$

$$(s, h) \models_k \pi A(E) \quad \Leftrightarrow \quad h = [[E]]_s : \mathbf{Lock}(A, U, [\pi]_s) \wedge [\pi]_s > 0$$

$$(s, h) \models_k \text{Locked}_A(E) \quad \Leftrightarrow \quad h = [[E]]_s : \mathbf{Lock}(A, k, 0)$$

- * adds up permissions for locks and their values:

$$U * k = k, \quad U * U = U, \quad k * j \text{ undefined}$$

Semantics of programs

- $pc \in \{1, \dots, n\} \rightarrow \text{ProgPoint}$
- $F \subseteq \text{ProgPoint} \times \text{Command} \times \text{ProgPoint}$
- \rightarrow_S is the least relation satisfying:

$$\frac{(v, C, v') \in F \quad k \in \{1, \dots, n\} \quad C, (s, h) \rightsquigarrow_k q}{pc[k : v], (s, h) \rightarrow_S pc[k : v'], q}$$

$x = E, (s[x : (u, 1)], h)$	$\rightsquigarrow_k (s[x : ([E]_{s[x:(u,1)}], 1)], h)$
$x = [E], (s[x : (u, 1)], h[e : \text{Cell}(u)])$	$\rightsquigarrow_k (s[x : (u, 1)], h[e : \text{Cell}(u)]), e = [E]_{s[x:(u,1)]}$
$[E] = F, (s, h[[E]_s : \text{Cell}(u)])$	$\rightsquigarrow_k (s, h[[E]_s : \text{Cell}([F]_s)])$
$x = \text{new}, (s[x : (u, 1)], h)$	$\rightsquigarrow_k (s[x : (v, 1)], h[v : \text{Cell}(w)]), \text{ if } h(v) \uparrow$
$\text{delete } E, (s, h[[E]_s : \text{Cell}(u)])$	$\rightsquigarrow_k (s, h)$
$\text{init}_A(E), (s, h[[E]_s : \text{Cell}(u)])$	$\rightsquigarrow_k (s, h[[E]_s : \text{Lock}(A, k, 1)])$
$\text{finalize}(E), (s, h[[E]_s : \text{Lock}(A, k, 1)])$	$\rightsquigarrow_k (s, h[[E]_s : \text{Cell}(u)])$
$\text{assume}(G), (s, h)$	$\rightsquigarrow_k (s, h), \text{ if } [G]_s = \text{true}$
$\text{assume}(G), (s, h)$	$\not\rightsquigarrow_k \quad \text{if } [G]_s = \text{false}$
$\text{acquire}(E), (s, h[[E]_s : \text{Lock}(A, 0, \pi)])$	$\rightsquigarrow_k (s, h[[E]_s : \text{Lock}(A, k, \pi)])$
$\text{acquire}(E), (s, h[[E]_s : \text{Lock}(A, j, \pi)])$	$\not\rightsquigarrow_k \quad \text{if } j > 0$
$\text{release}(E), (s, h[[E]_s : \text{Lock}(A, k, \pi)])$	$\rightsquigarrow_k (s, h[[E]_s : \text{Lock}(A, 0, \pi)])$

Flies in the ointment

- Consider invariants: $I_A(x, y) \triangleq B(y, x)$ $I_B(x, y) \triangleq A(y, x)$
- with code:


```

      {x ↦ _ * y ↦ _}
      initA,y(x);
      initB,x(y);
      {A(x, y) * LockedA(x, y) * B(y, x) * LockedB(y, x)}
      release(x);
      {A(x, y) * LockedB(y, x)}
      release(y);
      {emph}
      
```
- Postcondition has forgotten that locks x and y exist!
- Logic may not detect a memory leak
- Formulating soundness becomes non-trivial

- Usual interleaving-based operational semantics
- Program $C_1 \parallel \dots \parallel C_n$
- $\vdash \{P_k\} C_k \{Q_k\}$
- Resource invariants are precise
 - Unambiguously pick out an area of the heap

$$\llbracket \Phi \rrbracket^k = \{(s, h) : (s, h) \models_k \Phi\}$$

- Theorem:

If $\sigma_0 \in \left(\bigotimes_{k=1}^n \llbracket P_k \rrbracket^k \right) * (\bigotimes \{\text{invariants for free locks in } \sigma_0\})$,
then the program is “safe”

and $\sigma_f \in \left(\bigotimes_{k=1}^n \llbracket Q_k \rrbracket^k \right) * (\bigotimes \{\text{invariants for free locks in } \sigma_f\})$

- Cheat: statement about σ_o/σ_f uses information about free locks in σ_o/σ_f

- How can we find all free locks allocated in a state from a set p ?
 - Take $\sigma \in p$
 - Conjoin to σ resource invariants for all locks with value U in σ
 - and set the value of these locks to 0
 - Do the same for every state obtained in this way...
- Definition:

The resulting states without locks with value U form the closure of p : $\langle p \rangle$
- Example: $\langle R(x) \rangle$ where $I_R(l) = (l:\text{Data} \mapsto -)$
- Example: $\langle B(y,x) \rangle$ where $I_B(x,y) = A(y,x)$ and $I_A(x,y) = B(y,x)$
- Are we guaranteed to add invariants for **all** free locks in this way?
- **No! – Due to self-contained sets of locks**

- Admissibility disallows self-contained sets of locks
- If resource invariants are admissible, closure finds all free locks
- Definition:

Resource invariants for lock sorts \mathcal{L} are admissible if there do not exist:

 - a non-empty set L of lock sorts from \mathcal{L} with parameters
 - a state $\sigma \in \otimes\{\text{invariants for all locks in } L\}$

such that the permission associated with the every lock from L in σ is 1
- Examples:
 - $\{I_R(l) \triangleq l:Data \rightarrow -\}$ is admissible
 - $\{I_A(x, y) \triangleq B(y, x), I_B(x, y) \triangleq A(y, x)\}$ is not

- Usual interleaving-based operational semantics
- Program $C_1 \parallel \dots \parallel C_n$
- $\vdash \{P_k\} C_k \{Q_k\}$
- Resource invariants are precise

- Theorem:

Suppose that

- either resource invariants are admissible
- or one of Q_k is intuitionistic (does not notice heap extension)

If $\sigma_0 \in \left\langle \bigotimes_{k=1}^n \llbracket P_k \rrbracket^k \right\rangle$, then the program is “safe”
 and $\sigma_f \in \left\langle \bigotimes_{k=1}^n \llbracket Q_k \rrbracket^k \right\rangle$

- Programs: $\text{let } f_1() = C_1, \dots, f_n() = C_n \text{ in } C$
- Two new commands: $x = \text{fork}(f)$ and $\text{join}(E)$
- Assertion language: thread handles $\text{tid}_f(E)$
 - thread running f with identifier E exists
 - gives permission to join it
 - only one thread can join any given thread
- Satisfaction relation: $(s, h, t) \models_k \Phi$
 - t – thread pool

- Need to give up the precondition of the thread at fork:

$$\Gamma, \{P\} f() \{Q\} \vdash \{P\} \mathbf{x} = \mathbf{fork}(f) \{ \mathbf{emp}_h \wedge \mathbf{tid}_f(x) \}$$

- and receive the postcondition at join:

$$\Gamma, \{P\} f() \{Q\} \vdash \{ \mathbf{emp}_h \wedge \mathbf{tid}_f(E) \} \mathbf{join}(E) \{Q\}$$

where $\mathbf{fv}(\{P, Q\}) \subseteq \mathbf{GlobalConsts}$

- Other axioms adjusted accordingly

- Proof of the program `let $f_1() = C_1, \dots, f_n() = C_n$ in C` :

$$\Gamma \vdash \{P_1\} C_1 \{Q_1\}$$

$$\vdots$$

$$\Gamma \vdash \{P_n\} C_n \{Q_n\}$$

where

$$\Gamma = \{P_1\} f() \{Q_1\}, \dots, \{P_n\} f() \{Q_n\}$$

$$\Gamma \vdash \{P\} C \{Q\}$$

- Technical issues:
 - Soundness conditions:
 - P_k are precise
 - P_k and Q_k have an empty lockset (no lock in a state satisfying them has a value other than U)
 - Same circularity problem as with locks: $\text{tid}_f \rightarrow Q_f \rightarrow \text{tid}_f$
 - Admissibility, closure, and soundness can be generalized

Compared to concurrent separation logic

- Original concurrent separation logic can reason about storable locks:
 - represent them as cells storing the identifier of the thread owning the lock
 - build a global invariant of memory as a whole
- Drawbacks:
 - lots of auxiliary state \Rightarrow horrible proofs
 - reasoning is not modular
 - automation is infeasible

- RGSep – Combination of Jones' rely-guarantee and separation logic
 - Locks not treated natively
 - Uses rely-guarantee to simplify reasoning about the global invariant
 - (+) Reasoning about complex fined-grained concurrency algorithms
 - (–) Awkward reasoning about programs that allocate and deallocate many simple data structures
- One fancy pre-allocated data structure vs many dynamically allocated simpler ones
- We'd like both at once

- Proposed a Floyd/Hoare-style program logic for
 - concurrent, heap-manipulating programs that:
 - allows local reasoning about unboundedly-many storable locks and threads
 - i.e., more realistic concurrent programming primitives
 - is strong enough to prove some examples published as challenges
 - piece of multicasting code
 - lock-coupling list operations
 - is set up to found a program analysis
 - thread-local fixed-point semantics is an analysis scheme
 - is sound via a reasonably lightweight mechanism for cutting recursive knots in the heap
 - using only a simple semantics
- Want a semantic analysis of admissibility of resource invariants