ULTIMATELY INCREMENTAL SAT

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SAT'14, Vienna, Austria

Introduction

- Incremental SAT is at the core of a variety of applications
- Assumptions are widely used in Incremental SAT
- Preprocessing is essential for performance

Recall: SatELite Preprocessing

1. Variable Elimination

   $\phi = \phi \cup Res_v(\phi_v, \phi_{\neg v}) \backslash (\phi_v \cup \phi_{\neg v})$

2. Subsumption

   $c_1=(a \vee b)$ $c_2=(a \vee b \vee c)$ ➔ c_1 subsumes c_2

3. Self-subsuming resolution

   $c_1=(a \vee b)$ $c_2=(a \vee \neg b \vee c)$ ➔ $c_2=(a \vee c)$

Recall: Clause Database Simplification

1. Propagation of unit clauses
2. Elimination of satisfied clauses
3. Removal of falsified literals from clauses

Incremental SAT under Assumptions

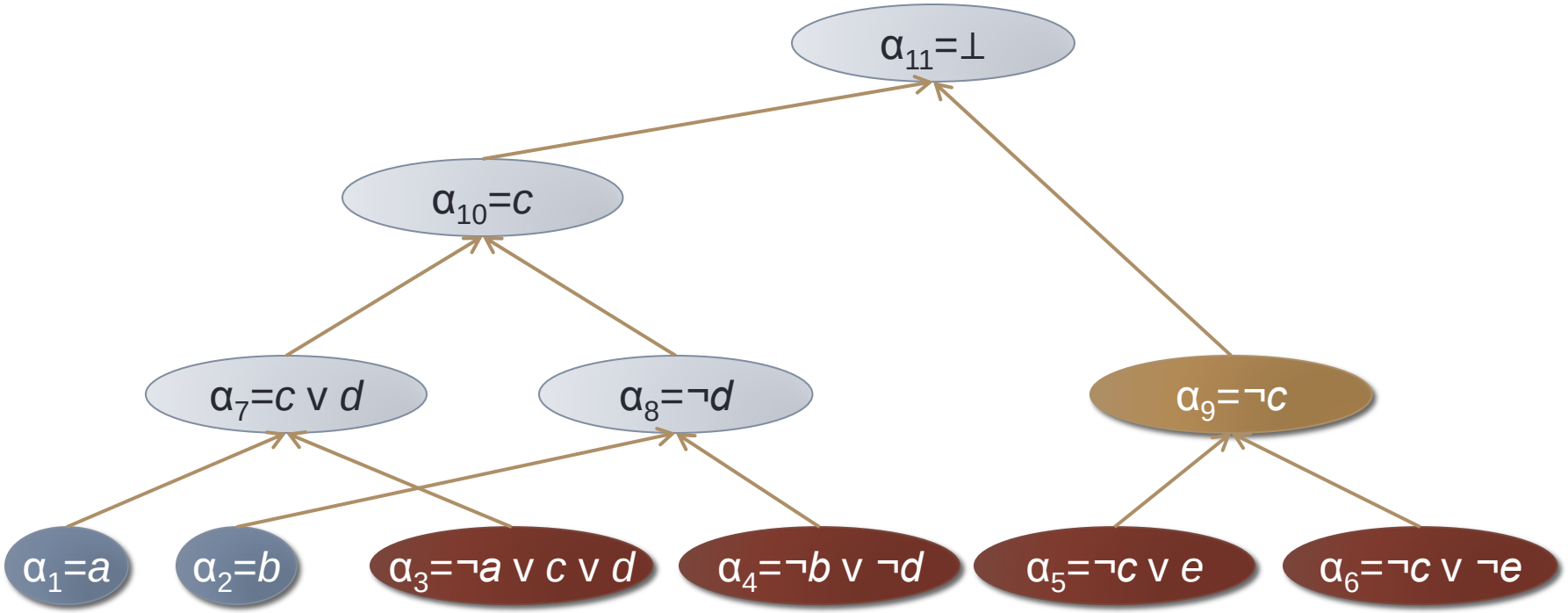
- Initial Formula + Assumptions: $(\phi_0 = \Delta_0) \wedge A_0$
- 1st incremental Call: $(\phi_1 = \phi_0 + \Delta_1) \wedge A_1$

- Nth incremental call: $(\phi_n = \phi_{n-1} + \Delta_n) \wedge A_n$

Temporary vs. Pervasive clauses

- We say that a clause is *temporary* if it is either an assumption or was derived from one or more assumptions, and *pervasive* otherwise.

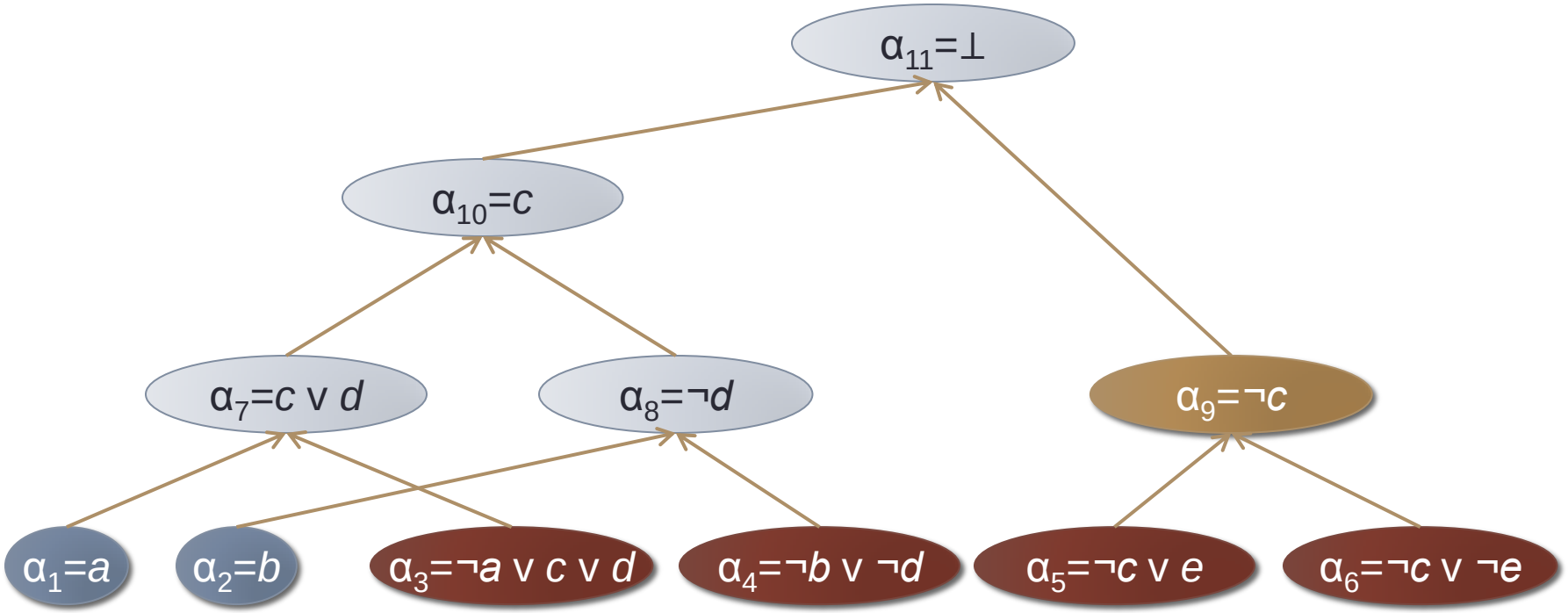
Temporary vs. Pervasive



Legend:

Input clauses	Pervasive conflict clauses	Assumptions	Temporary conflict clauses

Temporary Partial Resolution



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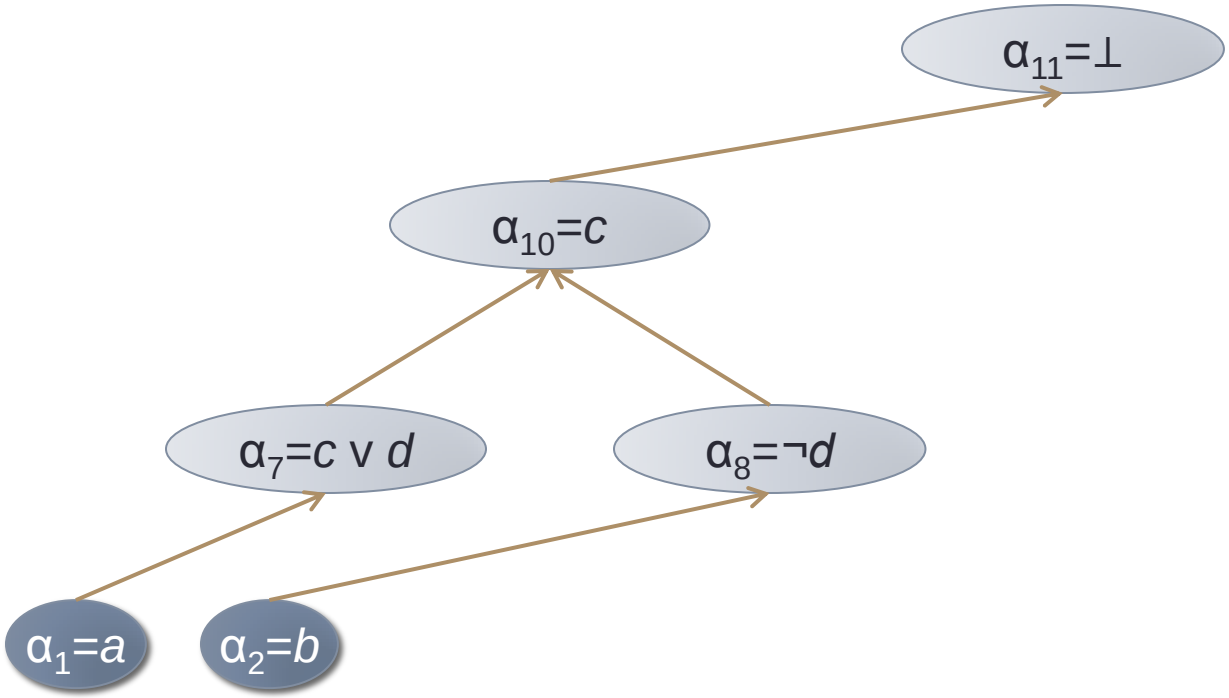

Legend:

Assumptions

Temporary conflict clauses

Assumptions

	.. as decision	.. as unit clauses
Compatible with Inc. SAT	+	-
Conflict clauses are pervasive	+	-
Simplification	-	+
Preprocessing	-	+

Previous Approaches

Algorithm	Instances	Assumption as units	SatELite	Assumption -dep. Clauses
Clause Sharing	Multiple	Yes	No	Discard
Minisat-Alg	One	No	No	Keep All
Assumption prop.	Multiple	Yes	Full	T2P
Incremental SatELite	One	No	Incremental	Keep All

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Previous Approaches

Algorithm	Instances	Assumption as units	SatELite	Assumption -dep. Clauses
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For every incremental call i:

- Create a new solver S_i
- Add ϕ_i to S_i
- Add A_i as temporary unit clauses to S_i
- Add pervasive conflict clauses from S_{i-1} to S_i
- Solve S_i

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Assu				2P
Incr Sa				p All

Create a solver S
For every incremental call i:
Add Δ_i to S
Add A_i as assumptions (decisions) to S
Solve S

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Previous Approaches

Algorithm	Instances	Assumption as units	SatELite	Assumption -dep. Clauses

For every incremental call i:

- In S_{i-1} convert learnt temporary clauses to pervasive
- Create a new solver S_i
- Add ϕ_i to S_i
- Copy all learnt clauses from S_{i-1} to S_i
- Add A_i as unit temporary clauses to S_i
- Run preprocessor over S_i
- Solve S_i

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Previous Approaches

Algorithm	Instances	Assumption as units	SatELite	Assumption -dep. Clauses
Clause				Discard
Mini				Keep All
Assu p				T2P
Incremental SatELite	One	No	Incremental	Keep All

Create a new solver S
For every incremental call i:
Add Δ_i to S
Add A_i as assumptions to S
Run preprocessor over S (Incremental)
Solve S

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Our New Approach

Algorithm	Instances	Assumption as units	SatELite	Assumption -dep. Clauses
Clause Sharing	Multiple	Yes	No	Discard
Minisat-Alg	One	No	No	Keep All
Assumption prop.	Multiple	Yes	Full	T2P
Incremental SatELite	One	No	Incremental	Keep All
UI-SAT	One	Yes	Incremental	Incremental T2P

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Clause Sharing	Multiple	Yes	No	Discard
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Our New Approach

Algorithm	Instances	Assumption as units	SatELite	Assumption -dep. Clauses
Clause S				rd
Minisa				All
Assum pro				
Increm SatE				All
UI-SAT	One	Yes	Incremental	Incremental T2P

Create a new solver S
For every incremental call i:
Undo assumptions in S (Inc. T2P)
Add A_i as temporary unit clauses to S
Add Δ_i clauses to S
Run preprocessor over S (Incremental)
Solve S

Incremental SatELite (NRS'12)

Freeze Assumptions // For current call
Preprocess
Solve // might run in-processing
UnFreeze Assumptions

Incremental SatELite (NRS'12)

```
Remove subsumptions.
For each eliminated variable in elimination order:
    if (*) // decide heuristically
        Re-Eliminate
    else
        Re-Introduce
Freeze Assumptions // For current call
Preprocess
Solve // might run in-processing
UnFreeze Assumptions
```

Incremental SatELite (NRS'12)

Core of Incremental Preprocessing

Remove subsumptions.
For each eliminated variable in elimination order:
  if (*) // decide heuristically
    Re-Eliminate
  else
    Re-Introduce
Freeze Assumptions // For current call
Preprocess
Solve // might run in-processing
UnFreeze Assumptions

Incremental SatELite (SAT'12)

Assumptions as Unit Clauses

Remove subsumptions.
For each eliminated variable in elimination order:
    if (*) // decide heuristically
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Freeze Assumptions // For current call
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Solve // might run in-processing
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        Re-Introduce

Preprocess
Solve // might run in-processing

Incremental SatELite (SAT'12)

Assumptions as Unit Clauses

Undo Previous Assumptions
Add assumptions as temporary unit clauses
Remove subsumptions.
For each eliminated variable in elimination order:
    if (*) // decide heuristically
        Re-Eliminate
    else
        Re-Introduce

Preprocess
Solve // might run in-processing

Data Structures

- We need to keep relevant data to undo assumptions.
- SubsumedClauses
  - Per clause – set of clauses subsumed by it

Subsumption (Example)

- Two clauses: $c_1=(a \vee b)$ $c_2=(a \vee b \vee c)$
- Normal subsumption: Delete c_2
- Our Subsumption: if c_1 marked as temporary:
  - c_1 subsumes c_2 → Add(c_2) to SubsumedClauses[c_1]
  - Delete c_2

Data Structures

- We need to keep relevant data to undo assumptions.
- SubsumedClauses
  - Per clause – set of clauses subsumed by it

- Resol - Partial Resolution for temporary clauses
  - If at least one of the parent clauses is temporary, then a new vertex is added to the resolution graph :
    - Conflict Analyze
    - Variable Elimination
    - Self Subsumption

Self-Subsumption (Example)

Two clauses: $c_1=(a \vee b)$ $c_2=(a \vee \neg b \vee c)$

- Normal self-subsumption:
  - Create a new clause $c_3= Resol(c_1, c_2) = (a \vee c)$
  - Delete c_2
  - (Normally implemented just as removal of $\neg b$ from c_2)
- In our case (in addition to normal):
  - c_3 subsumes c_2 → Add c_2 to $SubsumedClauses[c_3]$

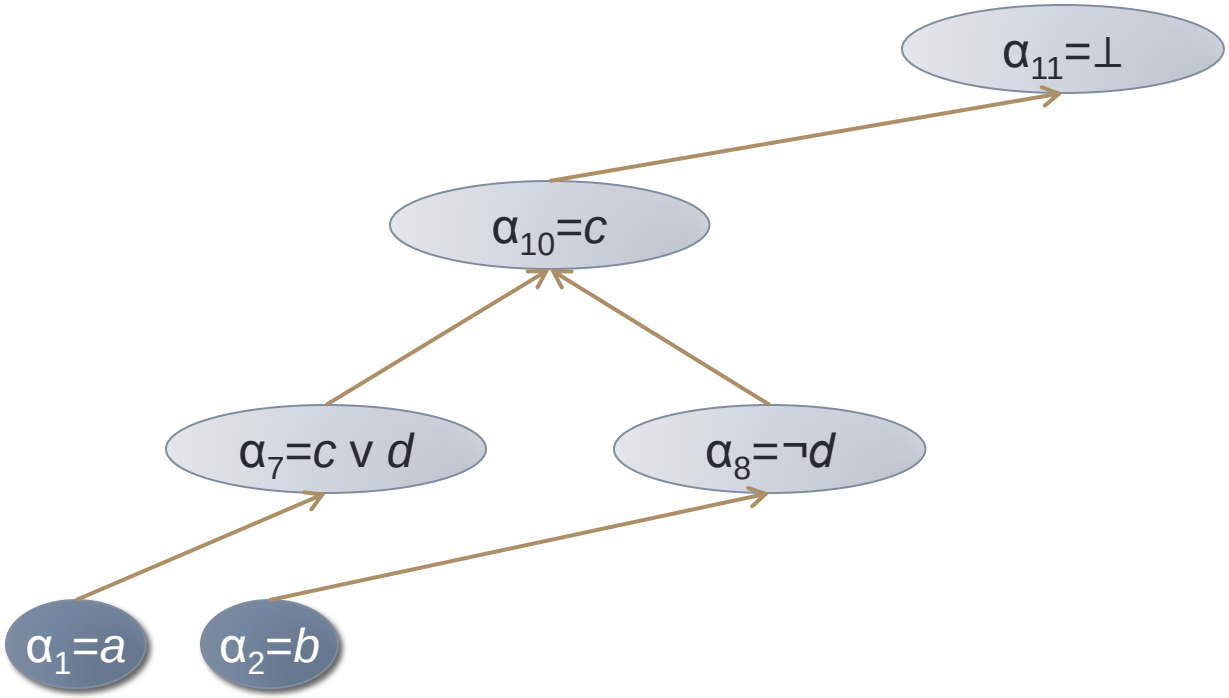
Undo Assumptions

Assumptions:

Iteration i : b, a

Iteration i+1: b

Undo Assumptions (Incremental T2P)

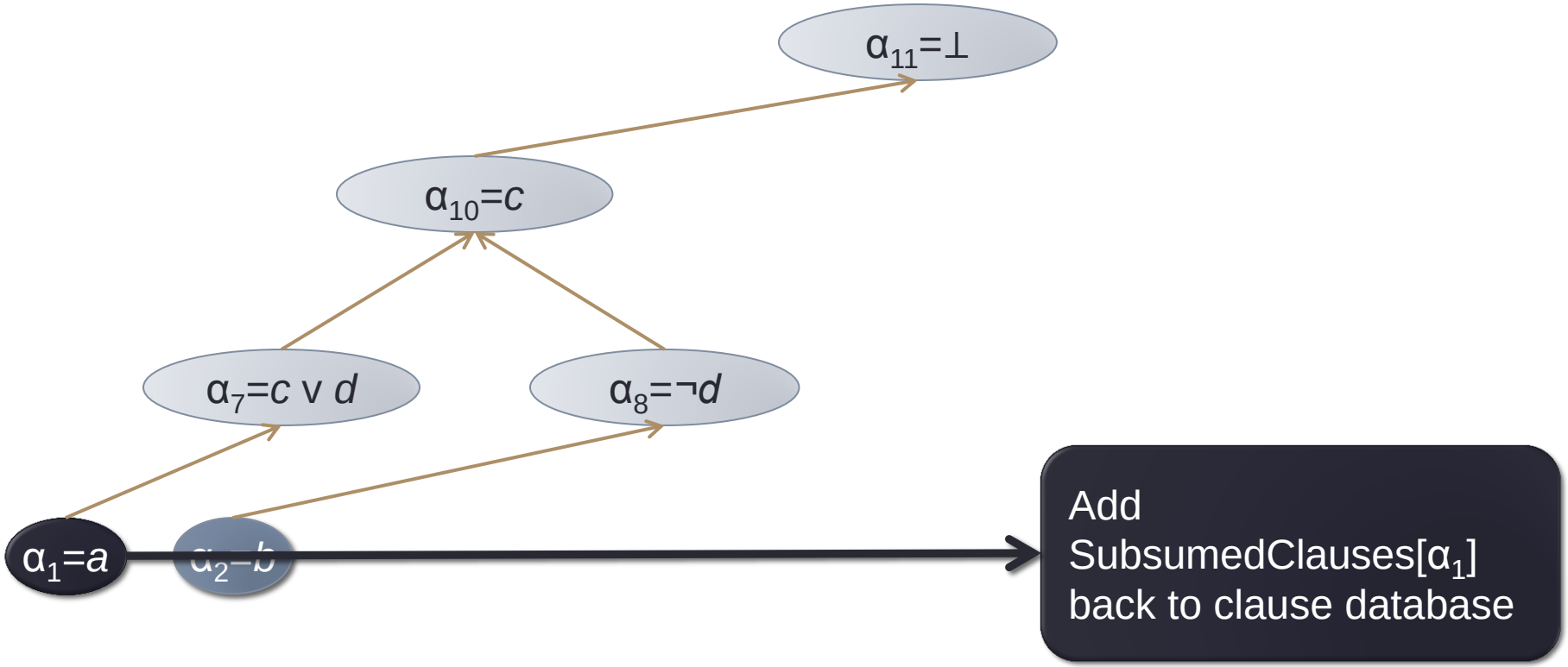


Legend:

Assumptions

Temporary conflict clauses

Undo Assumptions (Incremental T2P)



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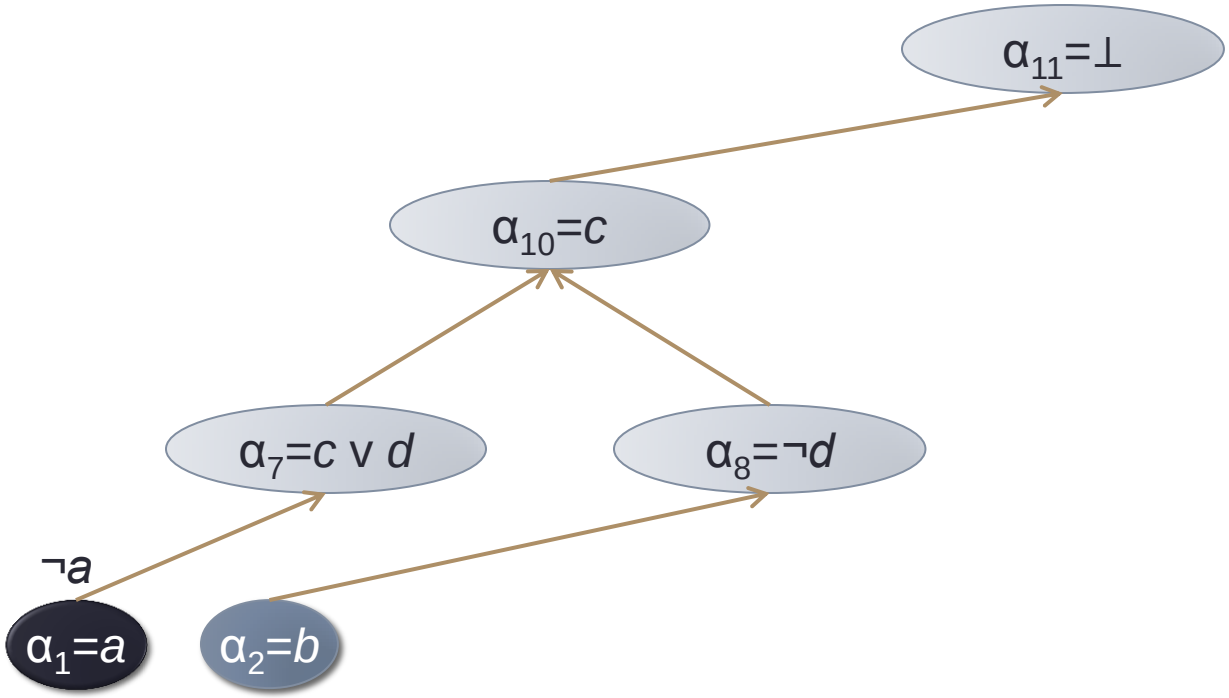

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Assumptions

Temporary conflict clauses

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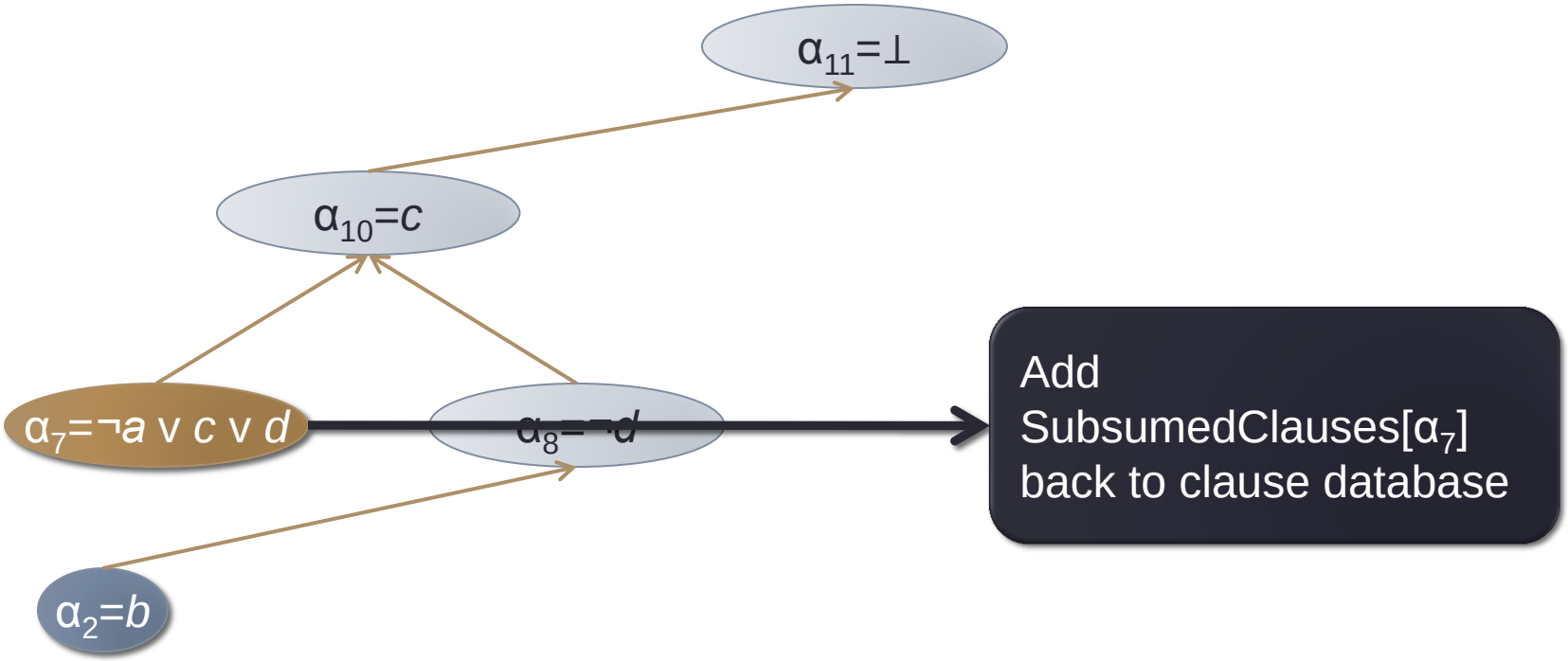


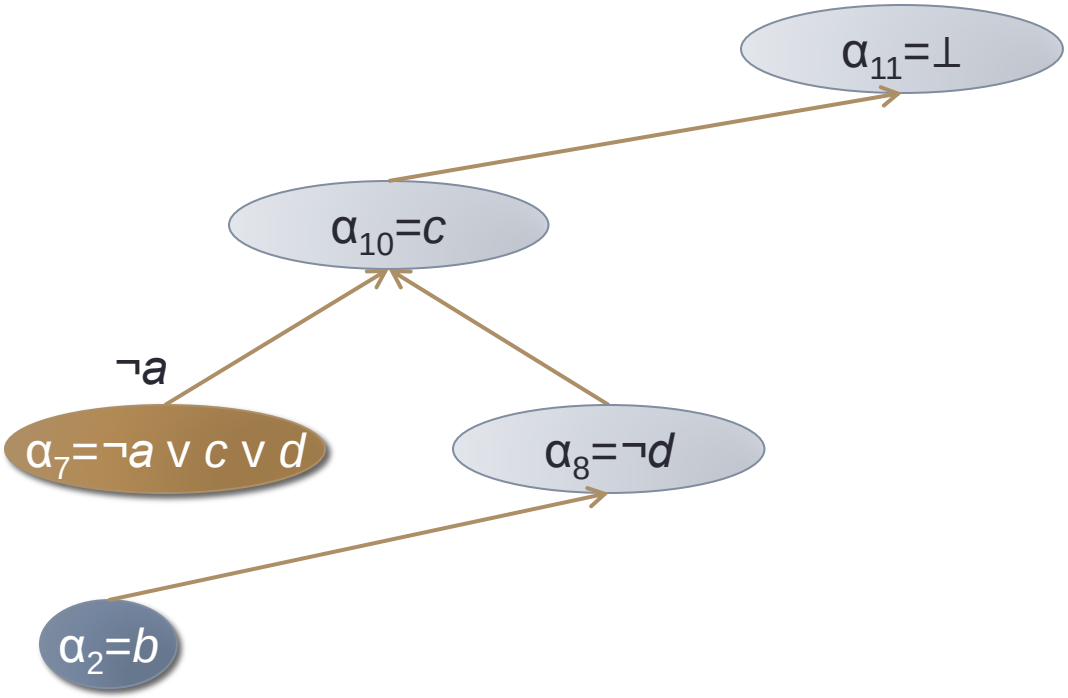
Legend:

Pervasive conflict clauses

Assumptions

Temporary conflict clauses

Undo Assumptions (Incremental T2P)



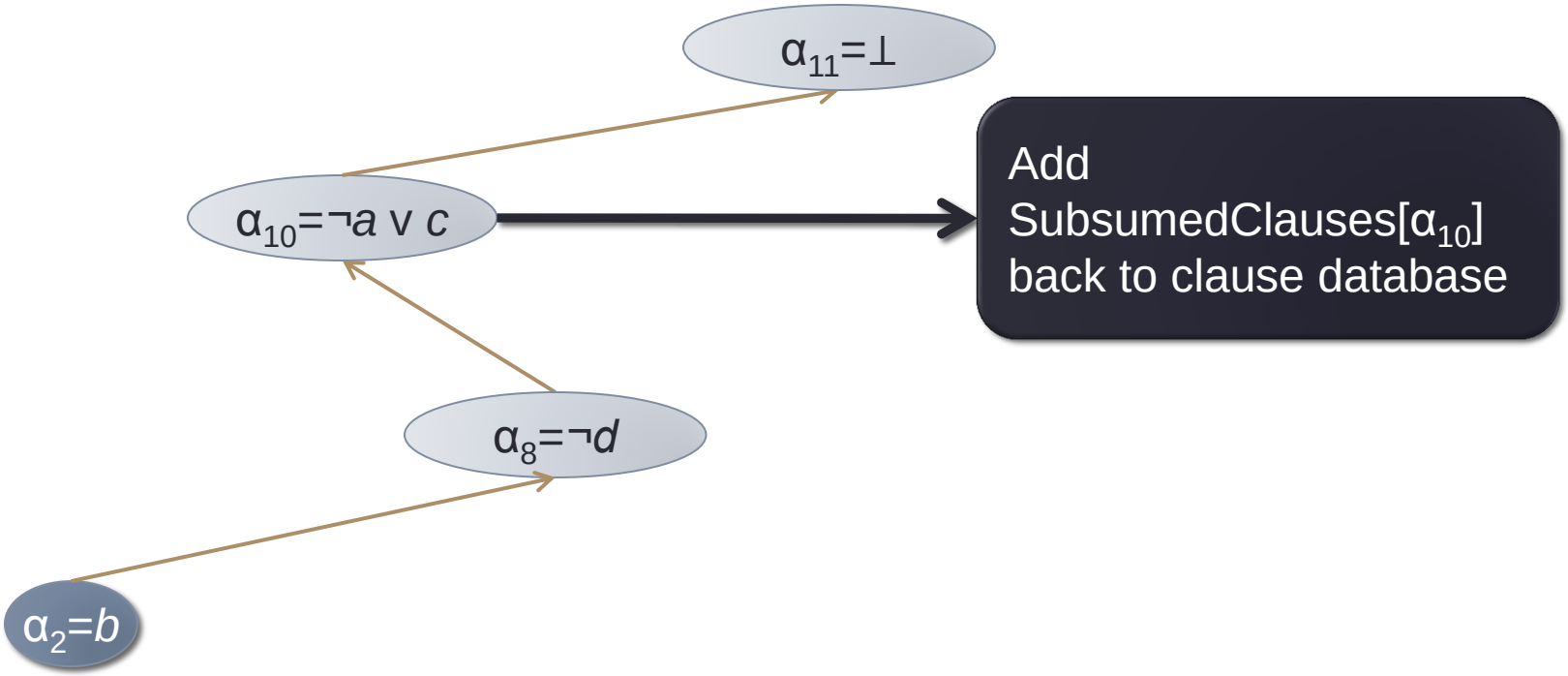
Legend:

Pervasive conflict clauses

Assumptions

Temporary conflict clauses

Undo Assumptions (Incremental T2P)

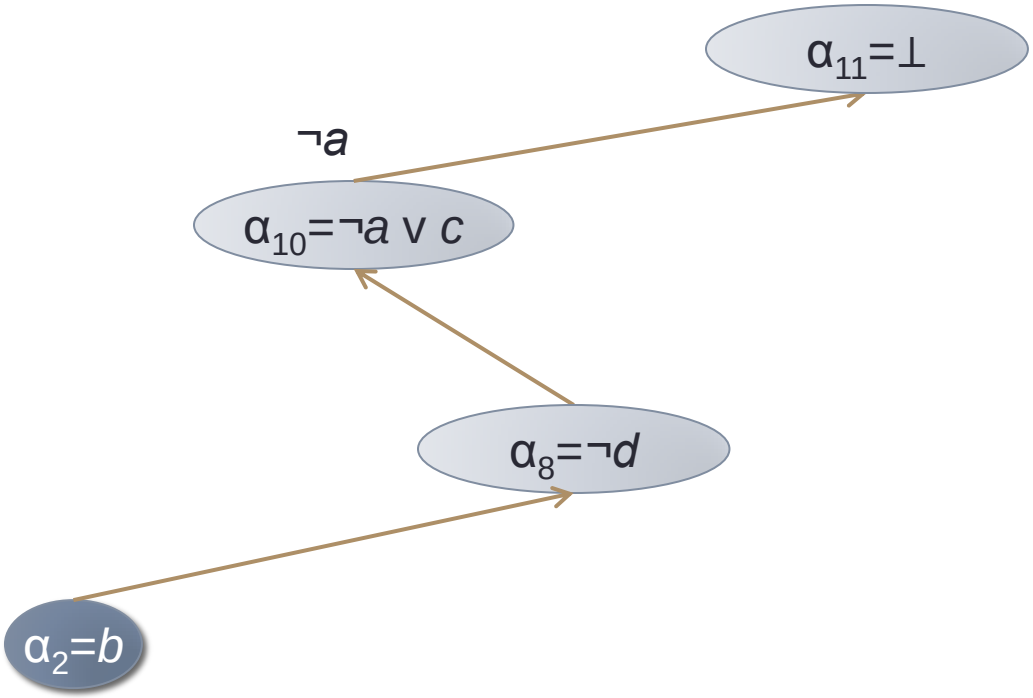


Legend:

Assumptions

Temporary conflict clauses

Undo Assumptions (Incremental T2P)



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Assumptions

Temporary conflict clauses

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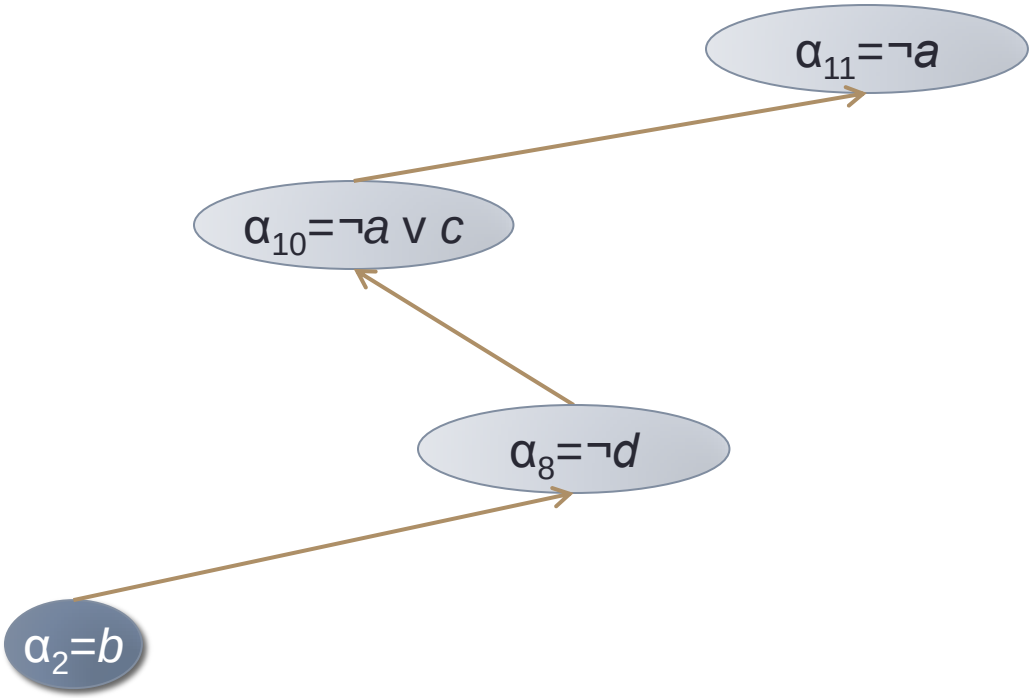


Legend:

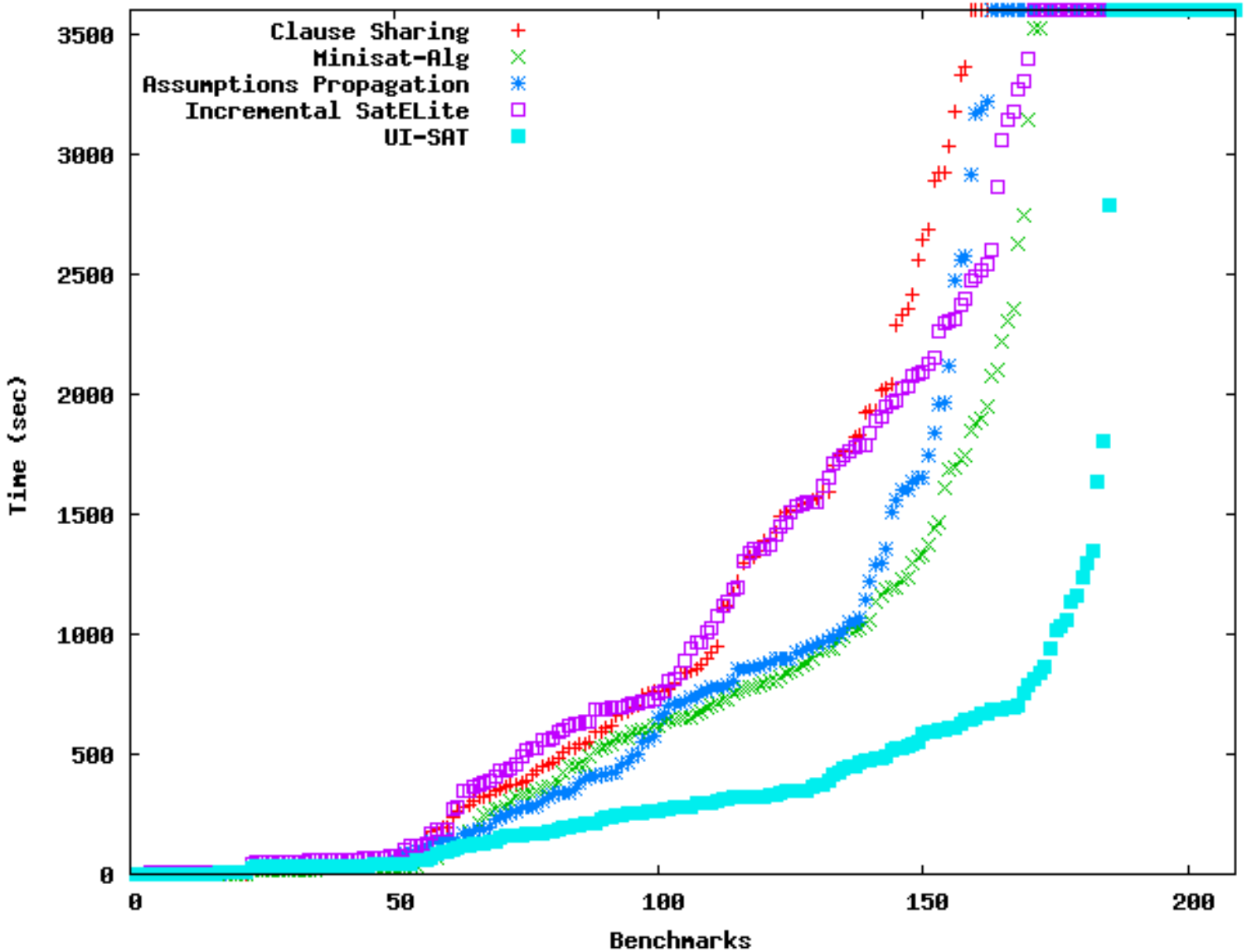
Assumptions

Temporary conflict clauses

Experimental Results

- Benchmark Set:
  - Instances generated by BMC (without look-ahead) under assumptions
    - Generated by an incremental model checker
      - May be invoked multiple times with different assumptions and properties
      - Essential to reduce the debug loop time for validation engineers
  - 3 satisfiable families – 128 instances
  - 4 unsatisfiable families – 81 instances
  - Algorithm Implementation in Intel's internal Fiver SAT Solver
  - Timeout: 3600sec
- Machines:
  - Intel® Xeon® 4Ghz 32Gb of memory

Experimental Results



Experimental Results

Method	Time-outs	Run-time
Clause Sharing	28	223,424
Minisat-Alg.	14	159,423
Assumption Prop.	24	182,530
Incremental SatELite	16	209,781
UI-SAT	**1**	**64,176**

Thank You!