Remarks:

- Write both your name and your ID number very clearly on the top of the exercise. Write your exercises in pen, or in clearly visible pencil. Please write very clearly.
- Recall that 80% of the theoretical exercises must be submitted. The exercises can and must be worked on and submitted alone.
- Give correctness and complexity proofs for every algorithm you write.
- For every question where you are required to write pseudo-code, also explain your solution in words.

1. Which of the following statements are true and which are false? Only give an answer, you do not have to explain:
   (a) \( 34n^{10} = o(2^n) \).
   (b) \( 4 \log n = O(8 \log n) \).
   (c) \( n^3 + \log n = \Omega(n) \).
   (d) \( 5^n/50 = O(n) \).
   (e) \( n + 1 = O(2^n) \).
   (f) \( n + 1 = o(n) \).
   (g) \( n/\log n = \Theta(\log n) \).

2. (a) Implement, in pseudo-code, a ternary (base-3) counter: You are given an infinite array such that each cell can only hold the digits 0,1,2, and you want the counter to support the operation \( \text{increment()} \) that increases the value of the counter by 1.
   (b) Suppose you start from a counter initialized to 0 and you perform \( m \text{ increment()} \) operations. What is their total cost? What is the amortized time complexity of increment? Prove your answer in both the \text{bank} and the \text{potential} methods.

3. Let \( L \) be a singly-linked list that consists of \( n \) elements. Each element contains a pointer “next”. Someone possibly made a mistake, and the final element of the list, instead of having a “null” pointer, might be pointing to an element inside the list. What does the following pseudo-code do? What is its worst-case time complexity?

   \begin{verbatim}
   INPUT: x, which is a pointer to the first element of the list
   y ← x.next
   IF (y == NULL) THEN RETURN “true”
   \end{verbatim}
WHILE \((y \neq x)\)
\[ y \leftarrow y.next \]
IF \((y == NULL)\) THEN RETURN “true”
\[ y \leftarrow y.next \]
IF \((y == NULL)\) THEN RETURN “true”
\[ x \leftarrow x.next \]
END WHILE
RETURN “false”