Exercise 1

Given an undirected graph with \( n \) vertices’s recall that its diameter is defined by:

\[
\text{diam}(G) = \max_{u,v \in V(G)} \{d(u, u)\}.
\]

Is the problem of deciding weather the diameter of \( G \) is at most \( \frac{n}{4} \) in \( \text{NPC} \) or in \( \text{NL} \)? Prove your answer.

Exercise 2

Let \( L \) be a language of all graphs on \( n \) vertices’s with diameter \( \log(n) \). Prove that \( L \in \text{Space}(\log(n) \cdot \log(\log(n))) \).

Exercise 3

You proved in class that each undirected graph \( G = (V, E) \) possess a cut with at least \( \frac{|E|}{2} \) edges. Give a deterministic poly-time algorithm, giddy on the conditional expectation, which constructs such a cut for a given \( G \). Prove that your algorithm satisfies all the conditions.

Exercise 4

Define \( \text{ZPP} \) as the class of all languages decided by a probabilistic Turing machine running in expected polynomial time. That is, for a language \( L \in \text{ZPP} \) there is a probabilistic Turing machine \( M(x, y) \) with the following behavior:

\( M \) always accepts any input \( x \in L \) and always reject any input \( x \notin L \). Also, for every \( x \):

\[
E_y \left[ \text{the number of steps before} \ M(x, y) \text{ halts} \right] < |x|^c,
\]

for some fixed \( c > 0 \). Prove that \( \text{ZPP} = \text{RP} \cap \text{coRP} \).

GOOD LUCK