Exercise 1

\(k - NAE\) is a language of formulas such that:

- \(\phi\) is in \(k\)-CNF form
- \(\phi\) has a satisfying assignment such that in each clause at least one literal is assigned to False.

Prove that \(3 - NAE\) is \(\text{NPC}\) using the following steps:

1. Show that \(3 - \text{SAT} \leq \text{P} 4 - NAE\) using the following reduction. Given a set of variables \(x_1, \ldots, x_n\) add a new variable \(z\). Every clause of the form \(x_{i1} \lor x_{i2} \lor x_{i3}\) is transformed to \(x_{i1} \lor x_{i2} \lor x_{i3} \lor z\).

2. Show that \(4 - NAE \leq \text{P} 3 - NAE\) using the following reduction. Let \(i\)-th clause be \(x_1 \lor x_2 \lor x_3 \lor x_4\). Construct the following two clauses \(x_1 \lor x_2 \lor w_i\) and \(x_3 \lor x_4 \lor \overline{w_i}\), where \(w_i\) is a new variable that appears in these clauses only.

Exercise 2

1. We say that graphs \(G\) and \(H\) are isomorphic if nodes of \(G\) can be reordered to make it identical to \(H\). Define a language

\[\text{ISO} = \{ \langle G, H \rangle \mid G \text{ and } H \text{ are isomorphic} \}\]

Prove that \(\text{ISO} \in \text{NP}\).

2. A triangle in undirected graph is a 3-clique. Define a language

\[\text{TRIANGLE} = \{ G \mid G \text{ has a triangle} \}\]

Prove that \(\text{TRIANGLE} \in \text{P}\).

3. Let \(\text{IND}_{2012} = \{ G \mid G \text{ is undirected graph containing an independent set of size } 2012\}\). Prove that \(\text{IND}_{2012} \in \text{L}\).

Exercise 3

Given languages \(A\) and \(B\) over \(\Sigma_A\) and \(\Sigma_B\) respectively we say that \(f : \Sigma_A^* \rightarrow \Sigma_B^*\) is poly-time Karp' reduction from \(A\) to \(B\) (denoted \(A \leq_{\text{P}} B\)) if \(f\) is poly-time computable and one of the following holds:

1. \(\forall x : x \in A \iff f(x) \in B\).
2. \(\forall x : x \in A \iff f(x) \notin B\).

Prove that \(\text{NP}\) is closed under Karp' reduction if and only if \(\text{NP} = \text{coNP}\).

Exercise 4

We say that a non-deterministic machine is nice if for every input \(x \in \{0, 1\}^*\) the following holds:

- every computation path returns either ‘accept’, ‘reject’ or ‘quit’;
- there is at least one non-quit path, and all non-quit paths have the same value.

Let \(\text{NICE}\) be the class of all languages that are accepted by some non-deterministic, polynomial time, nice machine. Prove that \(\text{NICE} = \text{NP} \cap \text{coNP}\).

GOOD LUCK