1. Show that the problem $\text{Gap - 3SAT}[\frac{11}{16} + \varepsilon, 3/4]$ is NP − Hard. Note that we consider here the non-exact version, that is the input formula might contain clauses of size smaller than 3.

2. Consider the following variation of $\text{CSG}_E$, called projection game. For every edge $uv$ and $\alpha \in \Sigma$, if $u$ is assigned to $\alpha$ there is exactly one assignment to $v$ such that the edge $uv$ is satisfied and if $v$ is assigned to $\alpha$ there is exactly one assignment to $u$ such that the edge $uv$ is satisfied. Denote by $\text{Gap - kPG}[a,b]$ the problem of deciding whether given an instance of a projection game, is it possible to satisfy at least $b$-fraction of the edges or every assignment satisfies at most $a$-fraction of the edges.

Prove or disprove, assuming $P \neq NP$: For every $\varepsilon > 0$, $\text{Gap - 7PG}[\varepsilon, 1 - \varepsilon]$ is NP − Hard.

3. Suppose that you are given a randomized algorithm $A$ that decides $L$ in the following way:
   
   - If $x \in L$ then $A$ accepts $x$ with probability at least 0.2
   - If $x \not\in L$ then $A$ accepts $x$ with probability at most 0.1

   where the probability is over the random coins of $A$. Provide an algorithm $B$ that given $x$, if $x$ is in $L$ then $B$ accepts with probability at least 0.9 and otherwise rejects with probability at least 0.9.

4. In this problem we deal with the conditional expectations method, which is used to transform randomized algorithms to deterministic ones. Let $G$ be a simple graph on $n$ vertices $x_1, x_2, \ldots, x_n$ and $m$ edges.

   (a) Reprove that $G$ contains a cut of size $m/2$. In other words, one can color the vertices in two colors, red and blue, such that at least half of the edges are between red and blue vertices.

   (b) For a fixed coloring for the vertices $x_1, \ldots, x_k$ let $S_k$ be the expected number of edges that connect a red vertex and a blue vertex, where each vertex from $\{x_{k+1}, \ldots, x_n\}$ is red with probability 1/2 and blue with probability 1/2 independently. Show that $S_0 \geq m/2$ and that there is a choice of a color for $x_{k+1}$ such that $S_{k+1} \geq S_k$.

   (c) Show that for a given coloring for the vertices $x_1, \ldots, x_k$, $S_k$ can be calculated in polynomial running time.

   (d) Use the previous items to show a 2-approximation deterministic polynomial time algorithm for the max cut problem.

   (e) Use a similar method to provide a deterministic 16/15-approximation algorithm for the Max-E4SAT problem.