1. Let $A$ be a minimization problem. The decision problem Gap-$A[\alpha, \beta]$ is, given an input $x$, decide whether there exists a solution for $A$ of size at most $\alpha$ or every solution is of size at least $\beta$ (the other instances are not allowed). Recall that an algorithm is a $c$-approximation for $A$ if it finds a solution of size at most $c$ times the optimal one ($c > 1$). Prove that if there exists a polynomial-time $c$-approximation algorithm for $A$ for some $c < \frac{\beta}{\alpha}$ then Gap-$A[\alpha, \beta] \in P$.

2. Suppose that a CNF formula has less than $n^k$ clauses, each with at least $2k \log_2 n$ distinct variables. Use the probabilistic method to show that it has a satisfying assignment.

3. Represent the following conditions as graph constraints problems. You may use the two variants we saw in class.
   
   (a) The graph is bipartite.
   
   (b) A $4 - NAE$ formula is satisfiable.
   
   (c) The graph can be partitioned to 3 sets such that at least 0.95-fraction of the edges connect vertices from different sets.

4. (a) Prove that Gap - Max - $k$SAT$[1 - 2^{-k} + \epsilon, 1]$ is $NP - Hard$ for every $k \geq 3$ and $\epsilon > 0$.
   
   (b) Show that Gap - Max - $Ek$SAT$[1 - 2^{-k} - \epsilon, 1] \in P$ for every $k$ and $\epsilon > 0$.

5. Prove that for every $\epsilon > 0$ it is $NP - Hard$ to approximate the minimal VC within a factor of $17/16 - \epsilon$.

6. We say that a graph is special if the size of the largest independent set equals the chromatic number. Prove that it is $NP - Hard$ to decide if a graph is special.