1. Prove that the following problems are self reducible by a (direct) polynomial Cook reduction from the search version to the decision version of the same problem.

   (a) Clique = \{ (G, k) | G contains a clique of size k \}.¹

   (b) GraphIsomorphism = \{ (G_1, G_2) | G_1 and G_2 are isomorphic \}.²

2. For a number \( n \in \mathbb{N} \), denote by \( \operatorname{bin}(n) \) the binary representation of \( n \), e.g., \( \operatorname{bin}(13) = 1101 \).

   Let \( L \subseteq \{1\}^* \) be a unary language, and define \( \operatorname{bin}(L) = \{ \operatorname{bin}(n) | 1^n \in L \} \). Show that \( L \in \mathbb{P} \) if and only if \( \operatorname{bin}(L) \in \mathbb{E} \), where \( \mathbb{E} = \bigcup_{c \geq 1} \mathbb{DTIME}(2^{cn}) \).

3. Let \( \text{UpToOneSat} \) be the following language:

   \[ \text{UpToOneSat} = \{ \phi | \phi \text{ is a CNF formula that has at most one satisfying assignment} \} \]

   Prove that \( \text{UpToOneSat} \in \mathbb{NP} \) if and only if \( \mathbb{NP} = \mathbb{coNP} \).

4. We say that a non-deterministic machine is \textit{nice} if for every input \( x \in \{0, 1\}^* \) the following holds: every computation path returns either ‘accept’, ‘reject’ or ‘quit’. There is at least one non-quit path, and all non-quit paths have the same value. Let \( \mathbb{NICE} \) be the class of all languages that are accepted by some non-deterministic, polynomial time, nice machine. Prove that \( \mathbb{NICE} = \mathbb{NP} \cap \mathbb{coNP} \).

5. The class \( \mathbb{DP} \) is defined as the set of all languages \( L \) for which there are two languages \( L_1 \in \mathbb{NP} \) and \( L_2 \in \mathbb{coNP} \) such that \( L = L_1 \cap L_2 \). Let \( \text{SAT-UNSAT} \) be the language of all the pairs \( (\phi_1, \phi_2) \) such that \( \phi_1 \) and \( \phi_2 \) are CNF formulas, \( \phi_1 \) is satisfiable and \( \phi_2 \) is not. Show that \( \text{SAT-UNSAT} \) is \( \mathbb{DP} \)-complete, i.e., \( \text{SAT-UNSAT} \in \mathbb{DP} \) and every language in \( \mathbb{DP} \) is polynomial-time reducible to it.

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¹The decision version is “Given a pair \( (G, k) \) does \( G \) contain a clique of size \( k \)” and the search version is “Given a pair \( (G, k) \) find a clique of size \( k \) in \( G \) if exists, and reject otherwise”.

²Two graphs are \textit{isomorphic} if there is a way to label the vertices of one graph, such that the two graphs become identical.