1. Show that \( P \) is closed under polynomial-time Cook reductions.

2. A \( k \)-CNF formula is \text{NAE-satisfiable} if it can be satisfied in such a way that each clause has at least one true literal and at least one false literal (NAE stands for Not-All-Equal). For example, the clause \((x_1 \lor x_2 \lor \overline{x_3})\) is NAE-satisfied by \(x_1 = T, x_2 = T, x_3 = T\), and not by \(x_1 = T, x_2 = T, x_3 = F\). Let \( \text{NAE-}k\text{SAT} \) be the language of all \( k \)-CNF NAE-satisfiable formulas.
   
   (a) Show that \( \text{NAE-}3\text{SAT} \) is \( \text{NP} \)-complete.
   
   Suggestion: Show that \( 3\text{SAT} \leq_p \text{NAE-}4\text{SAT} \leq_p \text{NAE-}3\text{SAT} \).
   
   (b) Is \( \text{EVEN-NAE-}3\text{SAT} = \{ \phi \mid \phi \text{ is a } 3\text{-CNF formula with an even number of NAE-satisfying assignments} \} \) \( \text{NP} \)-hard?

3. Recall that a graph is \( k \)-colorable if its vertices can be colored using up to \( k \) different colors in such a way that any two adjacent vertices have different colors. For any \( k \in \mathbb{N} \) define the language \( k\text{-Col} = \{ G \mid G \text{ is } k\text{-colorable} \} \).
   
   (a) Show that a graph is 2-colorable if and only if it has no cycle of odd length, and deduce that \( 2\text{-Col} \) is in \( \text{P} \).
   
   (b) Prove that \( 3\text{-Col} \) is \( \text{NP} \)-complete.
   
   Hint: Reduce from \( \text{NAE-}3\text{SAT} \). Given a formula generate a graph as follows: associate a vertex to each literal. Connect all these vertices to a vertex \( w \) and connect each variable vertex to its negation. Then, add a triangle for each clause and connect its vertices to the corresponding literals.
   
   (c) Deduce that the following languages are \( \text{NP} \)-complete.
   
   i. \( 2009\text{-Col} \).
   
   ii. \( \text{Coloring} = \{ (G,k) \mid G \text{ is } k\text{-colorable} \} \).
   
   iii. \( \text{CliqueCover} = \{ (G,k) \mid \text{the vertices of } G \text{ can be partitioned into } k \text{ sets, so that each set induces a clique} \} \).

4. A polynomial-time reduction \( f \) from a language \( L \in \text{NP} \) to a language \( L' \in \text{NP} \) is \textit{parsimonious} if the number of witnesses of \( x \) is equal to the number of witnesses of \( f(x) \). Show a polynomial-time parsimonious reduction from \( \text{SAT} \) to \( 3\text{SAT} \) (where by a witness for \( \text{SAT}/3\text{SAT} \) we mean a satisfying assignment).