1. Let \( f, g : \mathbb{N} \to \mathbb{N} \) be two functions. Recall that \( f = O(g) \) if there exists a \( c > 0 \) such that \( f(n) \leq c \cdot g(n) \) for every sufficiently large \( n \). We say that \( f = \Omega(g) \) if \( g = O(f) \) and that \( f = \Theta(g) \) if \( f = O(g) \) and \( g = O(f) \). Also, we say that \( f = o(g) \) if for any \( \varepsilon > 0 \), \( f(n) \leq \varepsilon \cdot g(n) \) for every sufficiently large \( n \). Finally, we say that \( f = \omega(g) \) if \( g = o(f) \).

Prove or disprove:

(a) \((5n)! = O(n!^5)\).

(b) If \( f(n) = O(n) \) then \( 10^f(n) = O(2^n) \).

(c) \( \log(n!) = \Theta(n \log n) \).

(d) Every two functions \( f, g \) satisfy \( f = O(g) \) or \( g = O(f) \).

(e) There exists a function \( f \) such that \( f(n) = O(n^{1+\varepsilon}) \) for any \( \varepsilon > 0 \) but \( f(n) = \omega(n) \).

2. For two languages \( L_1, L_2 \) define \( L_1 \Delta L_2 = (L_1 \setminus L_2) \cup (L_2 \setminus L_1) \). We say that a class \( C \) is closed under \( \Delta \) if \( L_1, L_2 \in C \) implies \( L_1 \Delta L_2 \in C \). For each class decide if it is closed under \( \Delta \) (or show that it is equivalent to an open question): \( P, NP, NP \cap \text{coNP} \).

3. Prove that each of the following problems can be solved by a polynomial time algorithm:

(a) Input: A graph \( G \) and a positive integer \( k \).
   Question: Does \( G \) contain a vertex of degree at least \( \log_2 |V(G)| \) or a clique of size \( k \)?
   (\( V(G) \) denotes the vertex set of \( G \)).

(b) Input: A list of \( n \) positive integer numbers \( A_1, \ldots, A_n \) and a number \( T \). All the numbers are given in unary representation (i.e., a number \( k \) is represented as \( 1^k \)).
   Question: Does exist a subset \( S \subseteq \{1, 2, \ldots, n\} \) such that \( \sum_{i \in S} A_i = T \)?

(c) Input: A 3CNF formula \( \phi \) in which each clause contains exactly 3 distinct literals and each variable occurs exactly 3 times.
   Question: Is \( \phi \) satisfiable?
   Hint: Use the fact that any regular bipartite graph has a perfect matching.\(^1\)

4. Let \( A \subseteq \{0, 1\}^* \) be a language which satisfies \( |A \cap \{0, 1\}^n| = n^3 \) for all \( n \geq 10 \). Prove that \( A \in \text{NP} \) implies \( A \in \text{coNP} \).

\(^1\)A regular graph is a graph where each vertex has the same number of neighbors. A matching in a graph is a set of edges without common vertices. A perfect matching is a matching which matches all vertices of the graph.