On the Construction of Analytic Sequent Calculi for Sub-classical Logics

Ori Lahav and Yoni Zohar*

School of Computer Science, Tel Aviv University, Israel
orilahav@post.tau.ac.il yoni.zohar@cs.tau.ac.il

Proof theory reveals a wide mosaic of possibilities for sub-classical logics. These are logics that are strictly contained (as consequence relations) in classical logic. Thus, by choosing a subset of axioms and derivation rules that are derivable in (some proof system for) classical logic, one easily obtains a (proof system for a) sub-classical logic. Various important and useful non-classical logics can be formalized in this way, with the most prominent example being intuitionistic logic. In general, the resulting logics come at first with no semantics. They might be also unusable for computational purposes, since the new calculi might not be *analytic*: it is often the case that proofs of some formula φ must contain formulas that are not subformulas of φ. This is evident within the framework of Hilbert-style calculi, that are rarely analytic. But, even for Gentzen-type sequent calculi, where the initial proof system for classical logic **LK** is analytic, there is no guarantee that an arbitrary collection of classically derivable sequent rules constitutes an analytic sequent calculus.

We focus on a general family of relatively simple sequent calculi for propositional logics, called *pure* sequent calculi (originally studied in [2]), of which (the propositional fragment of) **LK** is the prototype example. Our contribution is twofold. First, we generalize the coherence condition from [3] to provide a decidable sufficient syntactic criterion for analyticity of a given pure sequent calculus. Here we employ a general concept of analyticity, based on a parametrized notion of a subformula, that shares the attractive features with the usual subformula property. This criterion is useful in many cases, e.g. for proving the analyticity of a sequent calculus for the logic of first-degree entailment [1], and of course, the analyticity of the propositional fragment of **LK**. Second, we show that calculi admitting this criterion can be utilized for constructing other analytic sequent calculi. Taking a basic calculus **B**, we present a method for obtaining other analytic-by-construction sub-calculi of **B**, by collecting derivable rules of **B** that have a certain "safe" form.

The proposed method is general enough to capture a wide variety of known sequent calculi for sub-classical logics. This includes:

- A large family of sequent calculi for propositional paraconsistent logics, originated from philosophical motivations, and obtained by replacing the usual left introduction rule of negation with weaker rules, each of which is derivable in **LK**.
- A sequent calculus for primal logic (without quotations) from [4], as well as some natural extensions of it. This calculus originated from practical computational motivations, aiming to allow efficient proof search. It is obtained by replacing the usual right introduction rule of implication with a weaker rule, and discarding the rule for introducing disjunction on the left hand-side.

Our approach is semantic: as in [5], we formulate and use a semantic property of sequent calculi that is equivalent to analyticity. The semantics, however, plays a role only in our arguments, while the actual use of the proposed methods includes only syntactic considerations.

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References

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