Termination

Semantic Path Order

DNF3

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg\neg\neg x) \wedge (\neg\neg\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg\neg\neg x) \vee (\neg\neg\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (y \wedge x) \vee (z \wedge x)$

$\neg\neg(a \wedge (b \vee c))$

DNF4

- $\neg \neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z) \vee (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (x \wedge y) \vee (x \wedge z) \vee (x \wedge y) \vee (x \wedge z)$
- $x \vee x \Rightarrow x$

DNF5

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y) \wedge (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y) \vee (\neg x) \vee (\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $x \vee x \Rightarrow x$
- $x \wedge x \Rightarrow x$

DNF6

- $\neg \neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg \neg \neg x) \wedge (\neg \neg \neg y) \wedge (\neg \neg \neg x) \wedge (\neg \neg \neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg \neg \neg x) \vee (\neg \neg \neg y) \vee (\neg \neg \neg x) \vee (\neg \neg \neg y)$
- $x \vee x \Rightarrow x$
- $x \wedge x \Rightarrow x$

DNF6

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg\neg\neg x) \wedge (\neg\neg\neg y) \wedge (\neg\neg\neg x) \wedge (\neg\neg\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg\neg\neg x) \vee (\neg\neg\neg y) \vee (\neg\neg\neg x) \vee (\neg\neg\neg y)$
- $x \vee x \Rightarrow x$
- $x \wedge x \Rightarrow x$

$\neg\neg(x \vee y)$

Labeling

- ffx ⇨ fgfx
- ffx ⇨ fgfx
- fffx ⇨ ffgfx

Semantic Path Order

- Given a well-founded term order $\gtrsim$
- $s = f(s_1,\ldots,s_m) \quad t = g(t_1,\ldots,t_n)$
- $s > t$ if $s_i \gtrsim t$ for some i
- $s > t$ if $(s, s_1,\ldots,s_m) >_{lex} (t, t_1,\ldots,t_n)$
    - and $s > t_j$ for all j
- $s \approx t$ iff $(s, s_1,\ldots,s_m) \approx (t, t_1,\ldots,t_n)$

Semantic Path Order

- require $s \Rightarrow t \Rightarrow f(\ldots s \ldots) \geq f(\ldots t \ldots)$

Proof

- ~~Extend base order to a total w.f. order~~
- Consider minimal bad sequence
- Subterms are well-founded
- No use of $s_i \gtrsim t$ case
- So base order decreases and stabilizes

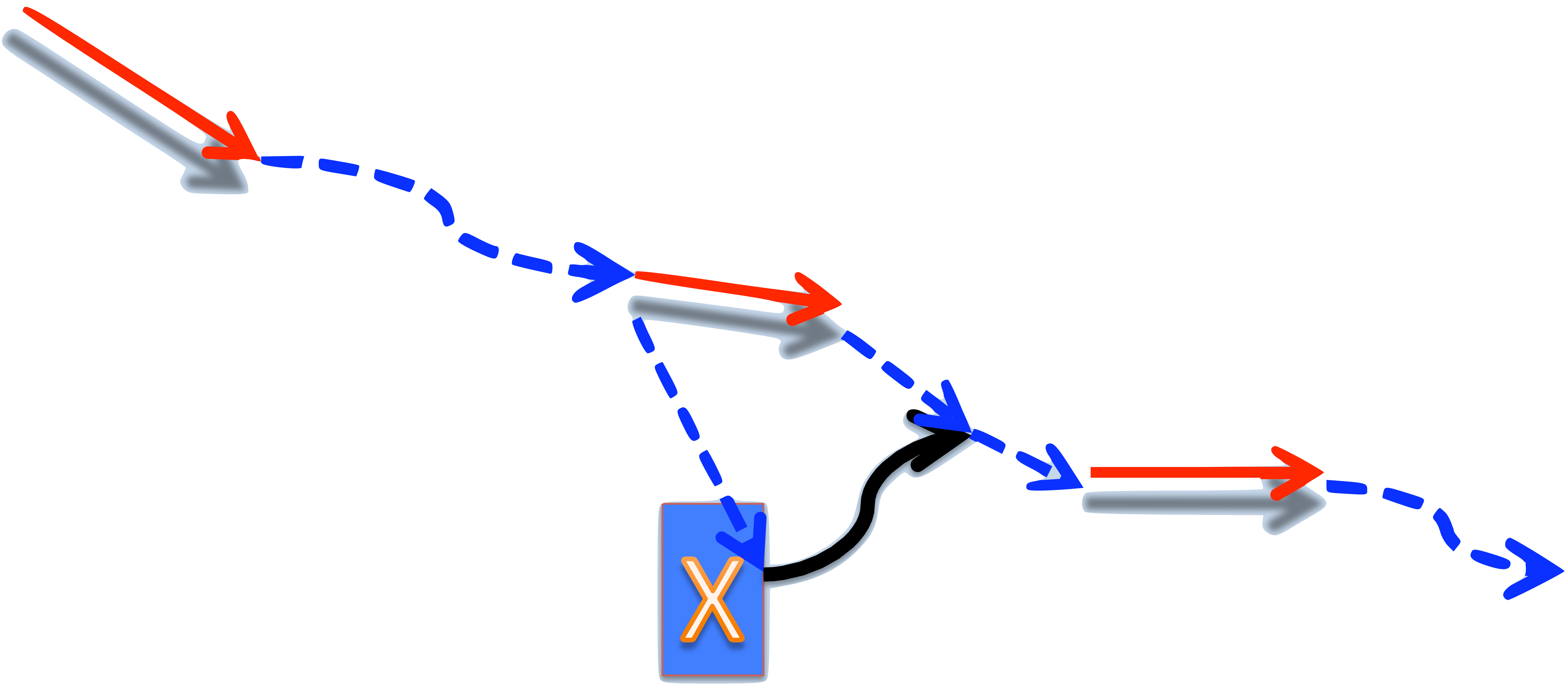
Jumping

- Let $P = R \cup B$
- If $s\ R \cup B\ t$
  - then $s\ R\ t$
  - or $s\ B\ v_1\ P\ v_2\ P \ldots P\ v_n\ P\ t$
- In short $RB \subseteq R \cup BP^*$
- Hence (induction) $RB^* \subseteq R \cup BP^*$

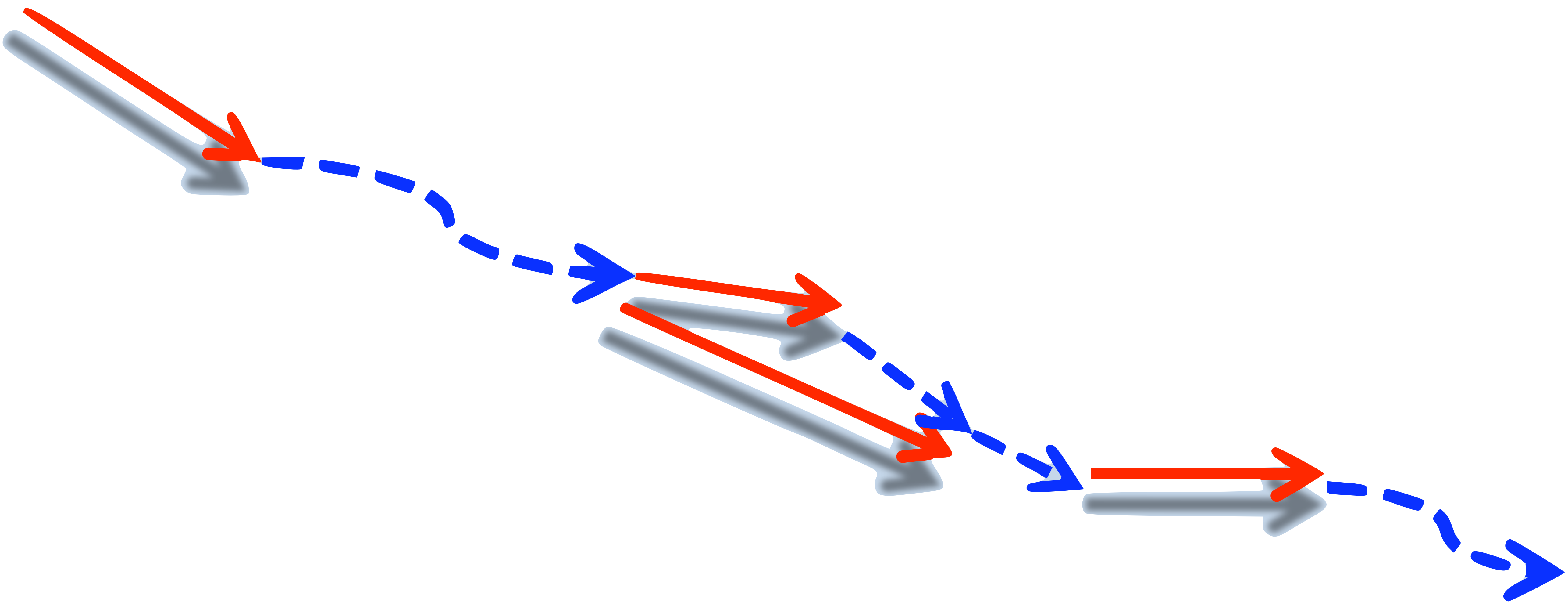
Constricting

- Let P = R∪B
- If there is an immortal purple chain
  s_1 P s_2 P s_3 P...
- then there is an immortal constricting chain
  s_1 BB...B t_1 R u_1 BB...B t_2 R...
  - R only when "necessary"
  - if t_i B v, then v is mortal

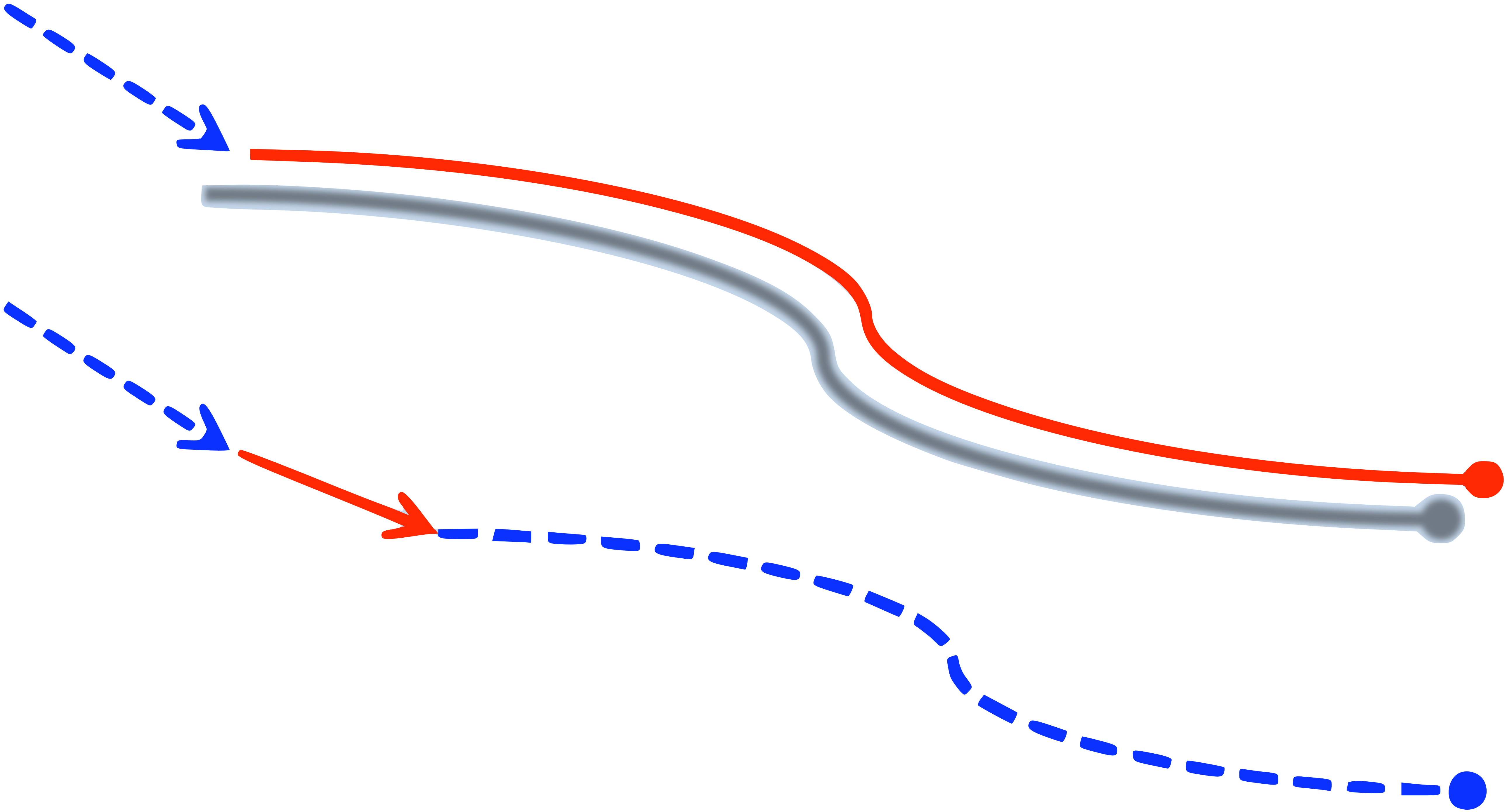
Constriction + Jumping



Constriction + Jumping



Constriction + Jumping



Constricted Jumping

- Constricted s_1 BB...B t_2 R t_3 BB...B t_4 R...
- Jumping RB* ⊆ R ∪ BP*
- Jumping RB* ⊆ R
- s_1 BB...B t_2 R t_4 R...

Jumping Union

- If B jumps over R
- then union well-founded iff both are
- s_1 BB...B t_1 RB* t_2 RB* t_2 RB* ...
  - s_1 BB...B t_1 R t_2 R t_3 R ...
- s_1 BB...B t_1 RB* t_2 RB* t_2 RBBBB...
  - s_1 BB...B t_1 R R R u_k BBBB...

Lifting

- For any immortal red chain
  s_1 R s_2 R s_3 R...
- there is also an immortal purple chain after taking an immediate blue turn
  s_1 B t_1 P t_2 P...
- Example: R is multiset; B is subset

Lifting Union

- If B jumps over R
- and B lifts to R
- then union well-founded iff B is
  - s_1 BB...B t_1 <u>R</u> t_2 <u>R</u> t_3 <u>R</u> ... XXX
  - s_1 BB...B t_1 <u>R</u> <u>R</u> <u>R</u> u_k BBBB...

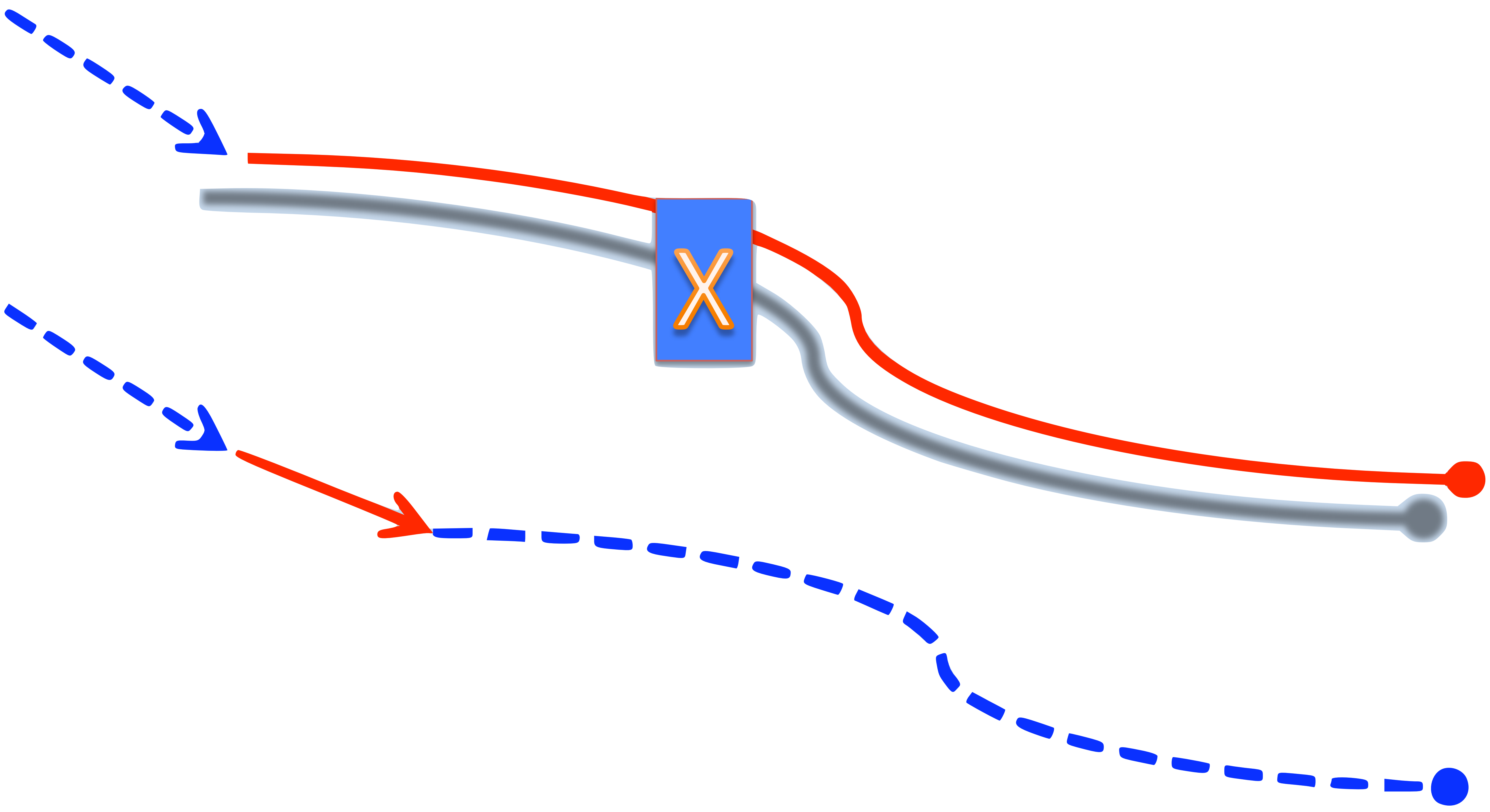
Nested Multisets

- subset jumps over multiset
- subset lifts to multiset
- well-founded since subset is

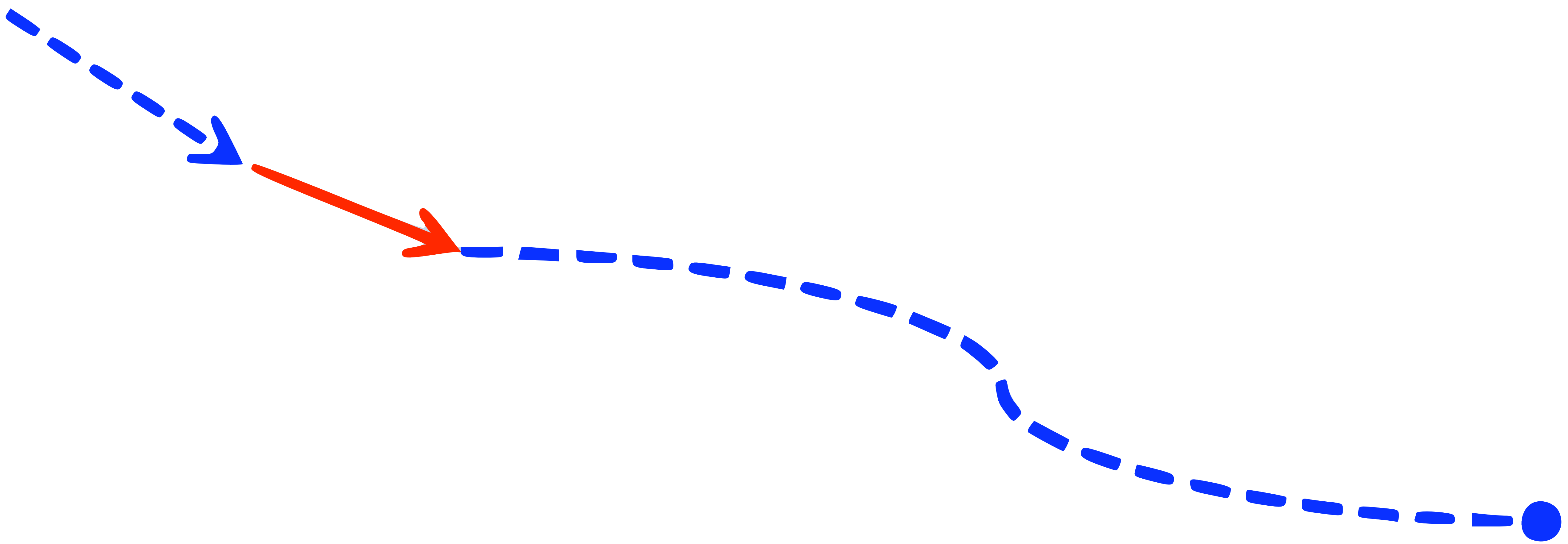
Escaping

- For any immortal red chain
  s_1 R s_2 R s_3 R...
- there is also an immortal purple chain after some blue turn
  s_1 R s_2 R ... R s_k B t_1 P t_2 P...

Jumping + Escaping



Jumping + Escaping



Escaping Union

- If B jumps over R
- and B escapes from R
- then union well-founded iff B is
  - s_1 BB...B t_1 R t_2 R t_3 R ... XXX
  - s_1 BB...B t_1 R R R u_k BBBB...