Rewrite Systems

6. Critical Pairs

Properties

- Termination
  - Normalization
    - Has at least one normal form
- Confluence
  - Unique normalization
    - Has at most one normal form
- Order of evaluation does not matter!

Equational Reasoning

- Reflexivity

  $x=x$
- Commutativity

  $x=y \Rightarrow y=x$
- Transitivity

  $x=y \ \& \ y=z \Rightarrow x=z$
- Functional Reflexivity

  $x=x' \ \& \ y=y' \Rightarrow f(x,y)=f(x',y')$

Equals for Equals

- Step (left/right does not matter)

$$\frac{u = v \quad s[u] = t}{s[v] = t}$$

- Proof (sequence of steps)

$$s_0 = s_1 = s_1 = \ldots = s_n$$

Counterexample

$$f(x,x) \rightarrow a$$
$$f(x,g(x)) \rightarrow b$$
$$c \rightarrow g(c)$$

Provers

- CiME
- daTac
- E Prover
- EQP
- LP
- **ReDuX**
- **SATURATE**
- **SETHEO**
- **SPIKE**
- **Watson**