Rewrite Systems

5. Confluence Properties

Normal Form

- Normal form -- cannot be rewritten
- An object's nf is any descendent nf
- Ambiguous -- has more than one nf
- Weakly normalizing -- at least one nf
- Immortal -- has a nonterminating path
- Strongly normalizing -- not immortal

Uniqueness

- Unique normal form

Notation

Binary relation

Inverse

Symmetric closure

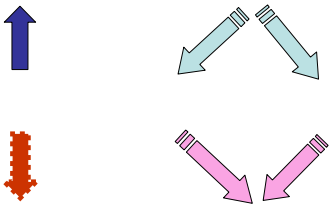
Transitive closure

Notation

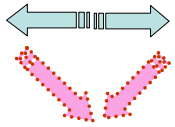
Given

Exists

Notation



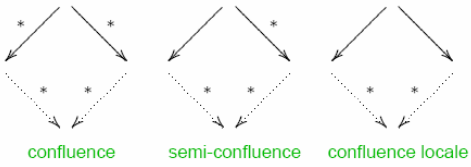
Church-Rosser



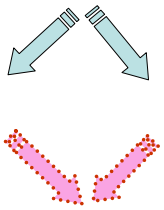
Decision Procedure

1. Finite
2. Sound
3. Complete
4. Terminating
5. Church-Rosser

Confluence Properties

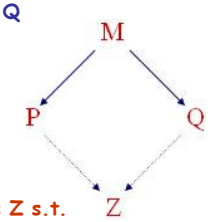


Confluence



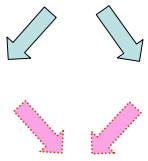
Confluence

for all M,P,Q

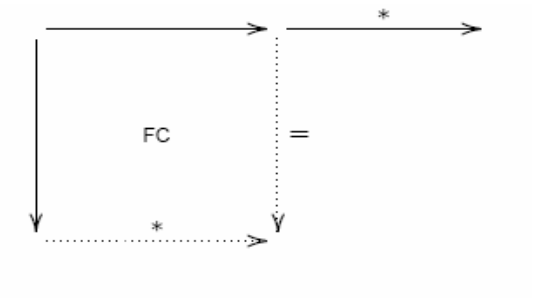


there exists Z s.t.

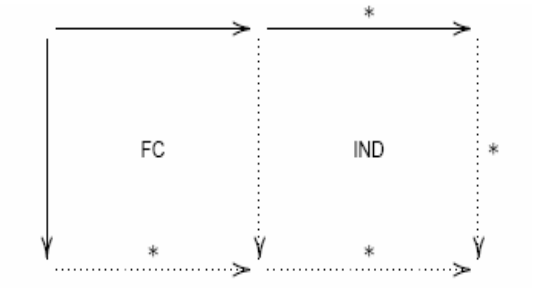
Strong Confluence



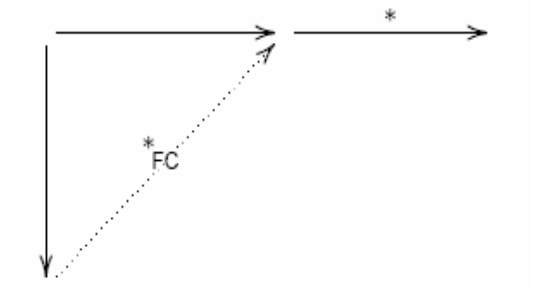
Strong Confluence



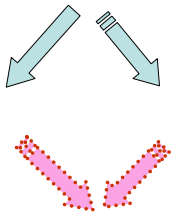
Strong Confluence



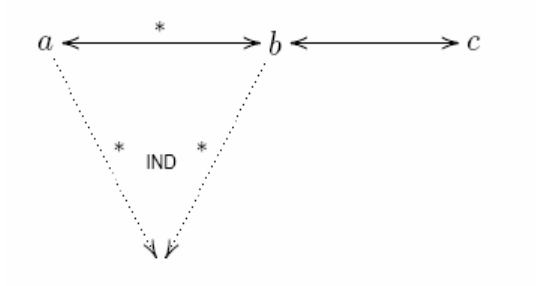
Strong Confluence

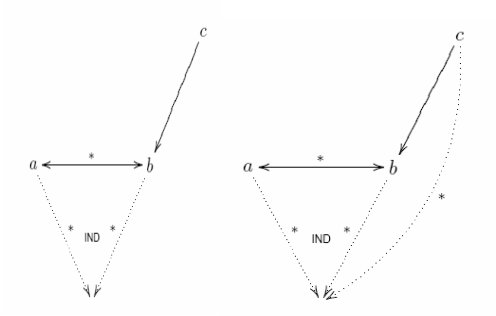


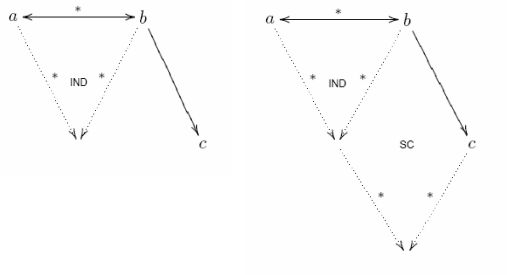
Semi-Confluence



1C => CR







Hindley-Rosen Lemma

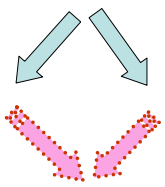
is SC

is SC

⊆

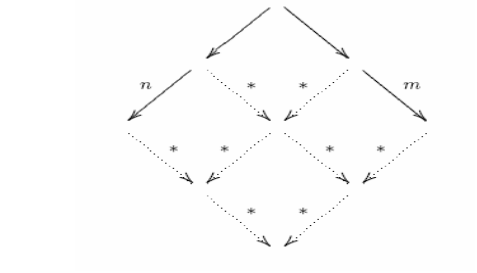
U is CR

Local Confluence

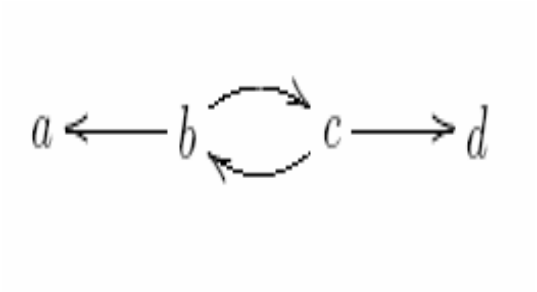


Question

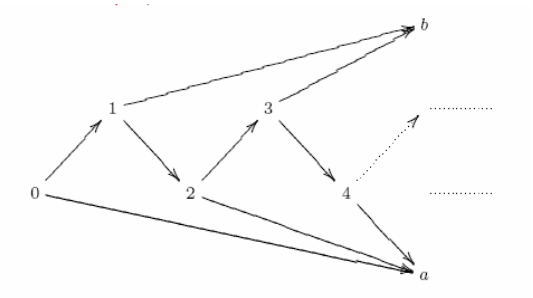
$LC \Rightarrow CR$



Counterexample [Curry]



Counterexample



Newmann's Lemma

LC + SN ⇒ CR

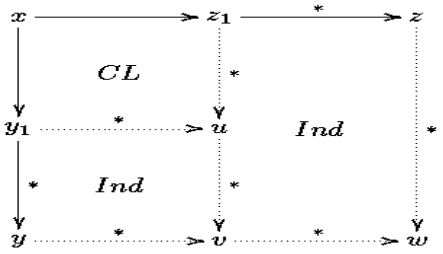
Well-founded Induction

$$\frac{\forall x.\ \{[\forall y.\ x > y \Rightarrow P(y)] \Rightarrow P(x)\}}{\forall x.\ P(x)}$$

Noetherian Induction

$$\frac{[\forall\, y.\ x \to y \Rightarrow P(y)] \Rightarrow P(x)}{\forall x.\ P(x)}$$

Proof



Decreasing Diagrams

5.11. THEOREM (van Oostrom 1994). *Let K be a well-founded set of indices, divided into two not-necessarily disjoint sets R and S. Suppose the relation $\leftarrow_i \circ \rightarrow_j$ is contained in the relation $\rightarrow_{I'}^* \circ \rightarrow_j^= \circ \rightarrow_{K'}^* \circ \leftarrow_{K'}^* \circ \leftarrow_i^= \circ \leftarrow_{J'}^*$, for all $i \in R$ and $j \in S$, where $I' = \{k \in S | k < i\}$, $J' = \{k \in R | k < j\}$, and $K' = I' \cup J'$. Then $\rightarrow_R$ and $\rightarrow_S$ commute.*