Rewrite Systems

14. Conclusion

Tentative Course Outline

1. Introduction
2. Termination
3. Church-Rosser
4. Orthogonality
5. Diagrams
6. Completion
7. Saturation
8. Modularity
9. Unification
10. Induction
11. Polynomials
12. Boolean Rings
13. Extensions
14. Open Problems

Sources

- Terese, § 7.6
- Jouannaud, §§ 3.2, 8.5
- Comon, handout
- Open Problems, handout

Append & Reverse

□@z → z (A0)
(x:y)@z → x:(y@z) (A1)

$□^r$ → □ (R0)
$(x:y)^r$ → y^r@(x:□) (R1)

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Theorem

z@□ = z

(for all z)

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Proof
(By Induction)

z@□ = z

- z=□ □@□ = □ (A0)
- z=(x:y) (x:y)@□ = x:(y@□) (A1)
  = x:y (IH)

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Disproof (By Non-Standard Model)

- **Second nil:** □
- **Also:** □@z = z
- **Then** □@□ = □ ≠ □

Initial Algebra

- Congruence classes of terms
- Quotient of Herbrand Universe
- True in initial algebra iff provable by structural induction
- Isomorphic to normal-form algebra

Specifications

- The initial algebra ... captures the essential properties of a specification....
- It contains exactly what the specification requires and nothing more.
- It is the “best” model for a given specification....
- It contains no superfluous terms (no *junk*) and does not make two terms indistinguishable which were intended to be distinct (no *confusion*).

General Method

Find an axiomatization A
of the minimal Herbrand model I
such that a conjecture C
is consistent with A and hypothesis H
iff C is an inductive consequence
of the hypothesis H.

Proof (By Consistency)

□@z = z (A0)
(x:y)@z = x:(y@z) (A1)
z@□ = z (IH)

Were there an inconsistency
like x:y = □ between 2 terms
it would have an equational proof

Normal-Form Method

- H is given as a (ground) confluent and terminating system R
- A = { s≠t | s,t distinct R-normal forms}
- Inconsistent iff 2 normal forms are equated

Proof
(By Normal Forms)

$\square @ z \rightarrow z$ (A0)

$(x{:}y) @ z \rightarrow x{:}(y @ z)$ (A1)

$z @ \square = z$ (IH)

Were there an inconsistency
like x:y = □ between 2 nfs
it would have an equational proof

Saturation Method

- H is given as a (ground) confluent and terminating system R
- A = { s≠t | s>t, s irreducible}
- (Linear) Completion used to saturate R and C
- Inconsistent iff a left-side has an R-irreducible ground instance

Proof
(By Completion)

$\square @ z \rightarrow z$ (A0)

$(x{:}y) @ z \rightarrow x{:}(y @ z)$ (A1)

$z @ \square \rightarrow z$ (IH)

Were there an inconsistency
like x:y = □ between 2 nfs
it would have a rewrite proof

An Open Problem

Problem #13

Originator: Jean-Jacques Lévy

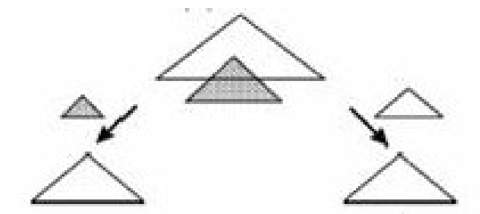
Date: April 1991

Summary:
Give decidable criteria for left-linear rewriting systems to be Church-Rosser.

History

- Huet, 1980
- Toyama, 1988
- Gramlich, 1996
- van Oostrum, 1997
- Oyamaguchi & Ohta, 1997
- Okui, 1998
- Matsumoto, 1999

Critical Pair



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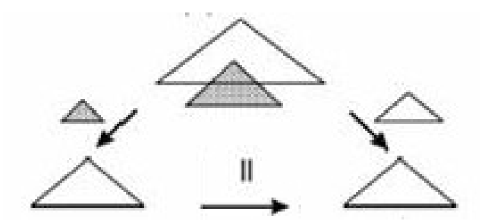
Huet



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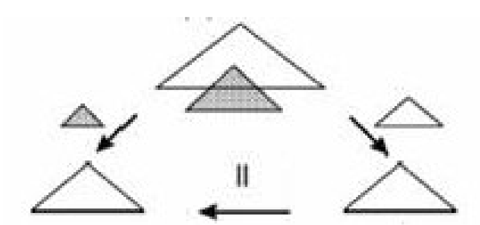
Open #13a



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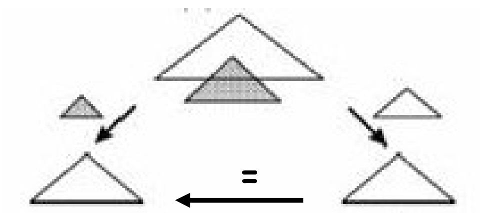
Open #13b



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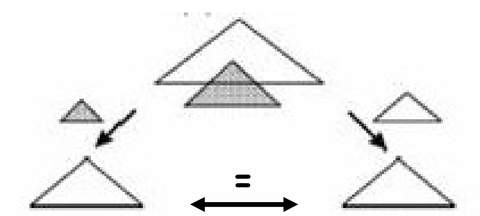
Open #13c



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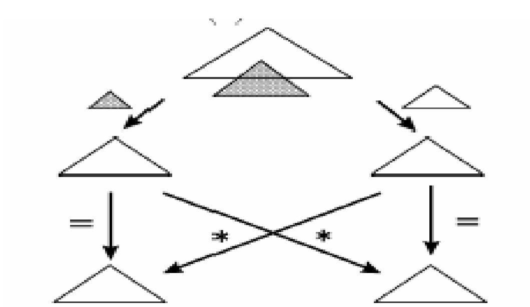
Huet #2 (Right-Linear)



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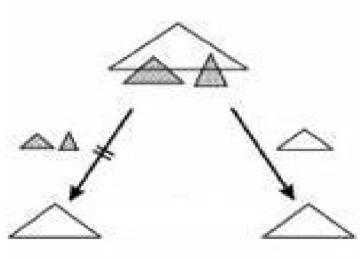
Parallel Critical Pair



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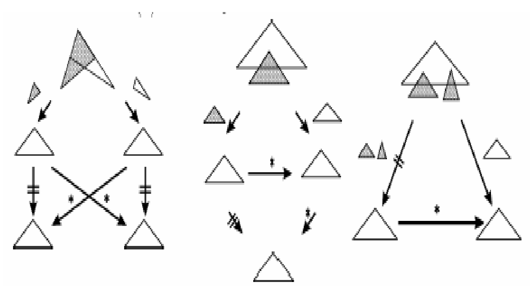
Gramlich



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(Complete) Development

- Given a set of non-overlapping redexes
- Apply <u>same</u> rules at (all) residual positions

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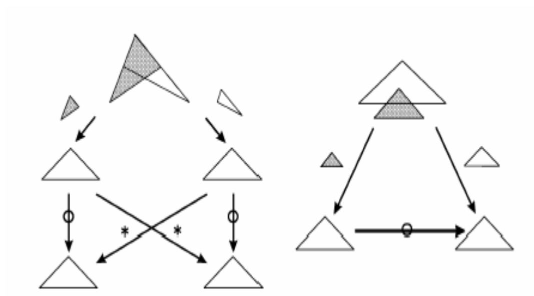
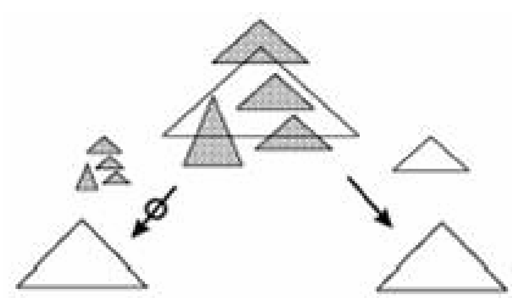
Huet-Toyama-Gramlich



Simultaneous Critical Pair



Okhi