Rewrite Systems

12. Strategies

Left Properties

- Non-overlapping = no critical pairs
- Overlaying = critical pairs at root only
- Left-linear = no repeat variables on left
- Orthogonal = non-overlapping & left-linear

Right Properties

- Right-linear = no repeats on right
- Non-duplicating = occurrences on right no more than on left
- Non-erasing = all left variables also on right
- Non-collapsing = no pure-variable right side

Thm. 9.3.10

An
orthogonal system
is normalizing
iff
it's outermost normalizing.

Proof Idea

- Inner step followed by outer can be mimicked by outer followed by zero or more inner [for orthogonal]
- Inner step cannot yield a normal form [for left-linear]

Orthogonal

- An inner step cannot create an outer redex

Cf. Thm. 4.8.7

A
locally-confluent
overlaying system
is terminating
iff
it's innermost normalizing.

Mortals & Immortals

- Immortal terms can be rewritten forever
- Mortals cannot be

Mortals & Immortals

1. Mortals remain mortal
2. Mortals beget mortals
3. Immortals
   can remain immortal

Constrictions

- All (proper) subterms of (applied) redex are mortal
- Non-terminating iff some constriction is non-terminating

Proof Idea

- Build non-terminating constriction
  - Consider non-terminating derivation
  - Normalize all mortal subterms
  - Normal forms are unique so don't interfere with higher redexes

Cf. Thm. 4.8.5

A
non-erasing
non-overlapping system
is terminating
iff
it's normalizing.

Subterming

- Rewriting terminates iff mixing rewriting with taking subterms is terminating
- Constricting terminates iff mixing constricting with taking subterms is terminating

Forward Derivation

- Start at the top and then rewrite only at *created* redexes
- Forward terminating if all forward derivations are finite

Top Rewrites

- A system is non-terminating iff it has a non-terminating derivation that starts with a top-rewrite (at the root)

A locally-confluent overlaying system is terminating iff it's innermost forward terminating.

A right-linear system is terminating iff it's forward terminating.

Disjoint Unions

- Share no function symbols or constants
- Disjoint unions of confluent systems are confluent
- Disjoint unions of terminating systems are not necessarily terminating
- Disjoint unions of normalizing systems are normalizing

Toyama's Example

$f(x,a,b) \rightarrow f(x,x,x)$

$g(x,y) \rightarrow x$
$g(x,y) \rightarrow y$

Disjoint Termination

1. Both non-duplicating
2. Both non-collapsing
3. Both overlaying and locally-confluent
4. Both left-linear and locally-confluent

Proof Idea

- Divide terms into layers