Indian Calendrical Calculations*

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Summary. We analyse various Indian calendars. We discuss the Indian day count, a generic solar calendar that generalizes various calendars including the mean Indian solar calendar, the true and astronomical Indian solar calendars, a generic lunisolar calendar that generalizes the Indian version, and the true and astronomical Indian lunisolar calendars. We also discuss aspects of the traditional Indian calculation of the time of sunrise and the determination of lunisolar holidays.

> The months of the Hindus are lunar, their years are solar;
> therefore their new year's day must in each solar year
> fall by so much earlier as the lunar year is shorter than the solar. . . .
> If this precession makes up one complete month,
> they act in the same way as the Jews,
> who make the year a leap year of thirteen months . . . ,
> and in a similar way to the heathen Arabs.
>
> – Alberuni's *India*

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1 Introduction

The world's many calendars are of three primary types: diurnal, solar, and lunar; see the third edition of our *Calendrical Calculations* [3] (henceforth *CC*). All three are represented among the many calendars of the Indian sub-continent.

- A *diurnal* calendar is a day count, either a simple count, like the Julian day number used by astronomers, or a complex, mixed-radix count, like the Mayan long count (see Section 10.1 of *CC*). The classical Indian day count (*ahargaṇa*) is used for calendrical purposes.
- *Solar* calendars have a year length that corresponds to the solar year. All modern solar calendars add leap days at regular intervals to adjust the mean length of the calendar year to better approximate the true solar year. The solar (*saura*) calendar is more popular in northern India; a similar one is in use in Nepal.[3]
- A *lunar* (*cāndra*) calendar has as its primary component a mensural unit that corresponds to the lunar synodic month. It can be purely lunar, as in the twelve-month Islamic calendar year (see Chapter 6 of *CC*), or it can incorporate occasional leap months, as in the Hebrew lunisolar calendar (see Chapter 7 of *CC*). Several forms of the lunisolar calendar are in use in India today; the Tibetan *Phugpa* or *Phuklak* calendar is somewhat similar (see Chapter 19 of *CC*).

In general, a date is determined by the occurrence of a cyclical event (equinox, lunar conjunction, and so on) before some "critical" time of day, not necessarily during the same day. For a "mean" (*madhyama*) calendar, the event occurs at fixed intervals; for a "true" (*spaṣṭa*) calendar, the (approximate or precise) time of each actual occurrence of the event must be calculated. Various astronomical values were used by the Indian astronomers Āryabhaṭa (circa 300 C.E.), Brahmagupta (circa 630 C.E.), the author of *Sūrya-Siddhānta* (circa 1000 C.E.), and others.

We systematically apply the formulæ for cyclic events in Chapter 1 of *CC* to derive formulæ for generic mean single-cycle and dual-cycle calendars; see Chapter 12 of *CC* for more details. Solar calendars are based on the motion of the sun alone, so fit a single cycle pattern; lunisolar calendars, on the other hand, take both the solar and lunar cycles into account, so require double-cycle formulæ. We apply these generic algorithms to the old Indian solar and lunisolar calendars, which are based on mean values (see Chapter 9 of *CC*). We also use the code in *CC* to compare the values obtained by the much more complicated true Indian calendars (Chapter 18 of *CC*) with their modern astronomical counterparts. Unless noted otherwise, we centre our astronomical calculations on the year 1000 C.E.

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[3]We have been unable to ascertain the precise rules of the Nepalese solar calendar.

Table 1. Some eras, given as the offset from the Gregorian year.

Era	Current Year	Elapsed Year
Kali Yuga	+3102	+3101
Nepalese		+877
Kollam	+823	
Vikrama	+58	+57
Śaka	−77	−78
Bengal		−593
Caitanya		−1486

Table 2. Indian month names.

	Vedic	Zodiacal Sign			Month	
1	Madhu	Meṣa	(Aries)	मेष	Vaiśākha	वैशाख
2	Mādhava	Vṛṣabha	(Taurus)	वृषभ	Jyeṣṭha	ज्येष्ठ
3	Śukra	Mithuna	(Gemini)	मिथुन	Āṣāḍha	आषाढ
4	Śuchi	Karka	(Cancer)	कर्क	Śrāvaṇa	श्रावण
5	Nabhas	Siṃha	(Leo)	सिंह	Bhādrapada	भाद्रपद
6	Nabhasya	Kanyā	(Virgo)	कन्या	Āśvina	आश्विन
7	Issa	Tulā	(Libra)	तुला	Kārtika	कार्तिक
8	Ūrja	Vṛścika	(Scorpio)	वृश्चिक	Mārgaśīrṣa	मार्गशीर्ष
9	Sahas	Dhanus	(Sagittarius)	धनुस्	Pauṣa	पौष
10	Sahasya	Makara	(Capricorn)	मकर	Māgha	माघ
11	Tapas	Kumbha	(Aquarius)	कुम्भ	Phālguna	फाल्गुन
12	Tapasya	Mīna	(Pisces)	मीन	Caitra	चैत्र

We will ignore many trivial differences between alternative calendars, such as eras (year count). Some Indian calendars count "elapsed" years, beginning with year 0; others use "current," 1-based years. The offsets of some common eras from the Gregorian year are summarized in Table 1. Indian month names are given in Table 2. Tamil names are different. There are also regional differences as to which is the first month of the year. Finally, calendars are local, in the sense that they depend on local times of sunrise, and the like.

The next, brief, section describes the Indian day count. Section 3 presents a generic solar calendar and shows how the mean Indian solar calendar fits the pattern. It is followed by a section that compares the later, true calendar with modern astronomical calculations. Similarly, Section 5 presents a generic lunisolar calendar and its application to the Indian version, and is followed by a section on the true and astronomical versions. Section 7 discusses aspects of the traditional calculation of the time of sunrise. Finally, Section 8 outlines the difficulty of computing the day of observance of holidays based on the lunisolar calendar.

Table 3. Day count correlations.

Date (Julian)		J.A.D.	Ahargaṇa	R.D.
2 January 4713 B.C.E.	(noon)	1	−588,464.5	−1,721,423.5
18 February 3102 B.C.E.	(midnight)	588,465.5	0	−1,132,959
3 January 1 C.E.	(midnight)	1,721,425.5	1,132,960	1

Following the style of *CC*, the algorithms in this paper are presented as mathematical function definitions in standard mathematical format. All calendar functions were automatically typeset directly from the Common Lisp functions listed in the appendix.

2 Diurnal Calendars

In most cases, a calendar date is a triple $\langle y, m, d \rangle$, where year y can be any positive or negative integer, and month m and day d are positive integers, possibly designated "leap." A day count is convenient as an intermediate device for conversions between calendars. One can count days from the first day of a calendar, normally $\langle 1, 1, 1 \rangle$, called the *epoch*. So $\langle 1, 1, 1 \rangle$, $\langle 1, 1, 2 \rangle$, and so on correspond to (elapsed) day count 0, 1, and so on. Days prior to the epoch have negative day counts and nonpositive year numbers. Day counts can be 0-based or 1-based; that is, the epoch may be assigned 0 or 1. In *CC*, we use the *Rata Die* (R.D.) count, day 1 of which is 1 January 1 (Gregorian).

The *ahargaṇa* ("heap of days") is a 0-based day count of the Kali Yuga (K.Y.) era, used in Indian calendrical calculations. The K.Y. epoch, day 0 of the *ahargaṇa* count, is Friday, 18 February 3102 B.C.E. (Julian). Its correlations with R.D. and with the midday-to-midday Julian day number (J.A.D., popular among astronomers) are given in Table 3. An earlier count, with much larger numbers, was used by Āryabhaṭa. We use the onset of the Kali Yuga, R.D. −1,143,959, for our **hindu-epoch**.

3 Mean Solar Calendars

The modern Indian solar calendar is based on a close approximation to the *true* times of the sun's entrance into the signs of the sidereal zodiac. The Hindu names of the zodiac are given in Table 2. Traditional calendarists employ medieval epicyclic theory (see Section 18.1 of *CC*); others use a modern ephemeris. Before about 1100 C.E., however, Hindu calendars were usually based on *average* times. The basic structure of the calendar is similar for both the mean (*madhyama*) and true (*spaṣṭa*) calendars. The Gregorian, Julian, and Coptic calendars are other examples of mean solar calendars; the Persian

and French Revolutionary are two examples of true solar calendars. In this and the following section, we examine these two solar calendar schemes.

The Indian mean solar calendar, though only of historical interest, has a uniform and mathematically pleasing structure. (Connections between leap-year structures and other mathematical tasks are explored in [4].) Using the astronomical constants of the $\bar{A}rya$-$Siddh\bar{a}nta$ yields 149 leap years of 366 days, which are distributed evenly in a cycle of 576 years. Similarly, 30-day and 31-day months alternate in a perfectly evenhanded manner.

3.1 Single–Cycle Calendars

The mean solar calendar is an instance of a general single-cycle calendar scheme. Consider a calendar with mean year length of Y days, where Y is a positive real number. If Y is not a whole number, then there will be common years of length $\lfloor Y \rfloor$ and leap years of length $\lceil Y \rceil$, with a leap-year frequency of $Y \bmod 1$.

To convert between R.D. dates and single-cycle dates, we apply formulæ (1.73) and (1.77) from Section 1.12 of CC. Suppose that a year is divided into months of length as close to equal as possible. For a standard 12-month year, the average length of a month would be $M = Y/12$. Some months, then, should be $\lfloor M \rfloor$ days long, and the rest $\lceil M \rceil$ days. Alternatively, a year may include a 13th short month, in which case M is the mean length of the first 12 months. (In any case, we may assume that $Y \geq M \geq 1$.)

A day is declared "New Year" if the solar event occurs *before* some critical moment. In other words, if t_n is the critical moment of day n, then n is New Year if and only if the event occurs during the interval $[t_n - 1, t_n)$. The beginnings of new months may be handled similarly or may be determined by simpler schemes, depending on the calendar; we discuss this below.

Suppose that the sun was at the critical longitude at the critical time t_{-1} of day -1, the day before the epoch, so that day -1 just missed being New Year. Finally, assume that a leap day, when there is one, is added at year's end. The number n of elapsed days from the calendar's epoch $\langle 0, 0, 0 \rangle$ until a date $\langle y, m, d \rangle$ (all three components are for now 0-based) is simply

$$\lfloor yY \rfloor + \lfloor mM \rfloor + d \,, \tag{1}$$

with inverse

$$\begin{aligned}
y &= \lceil (n+1)/Y \rceil - 1 \,, \\
n' &= n - \lfloor yY \rfloor \,, \\
m &= \lceil (n'+1)/M \rceil - 1 \,, \\
d &= n' - \lfloor mM \rfloor \,.
\end{aligned} \tag{2}$$

If the rule is that the event may occur up to *and including* the critical moment, then n is New Year if and only if the event occurs during the interval $(t_n - 1, t_n]$. Accordingly, we need to change some ceilings and floors in the

above formulæ. Supposing that the event transpired exactly at that critical moment t_0 of the epoch, the elapsed-day calculation becomes:

$$\lceil yY \rceil + \lceil mM \rceil + d \ . \tag{3}$$

The inverse function, assuming $Y \geq M \geq 1$, converting a day count n into a 0-based date, $\langle y, m, d \rangle$, is:

$$
\begin{aligned}
y &= \lfloor n/Y \rfloor \ , \\
n' &= \lfloor n \bmod Y \rfloor \ , \\
m &= \lfloor n'/M \rfloor \ , \\
d &= \lfloor n' \bmod M \rfloor \ .
\end{aligned}
\tag{4}
$$

The above four formulæ (1–4) assume that months are determined in the same way as are years, from a specified average value M, and, therefore, follow the same pattern every year (except for the leap day in the final month of leap years).[4] There is an alternative version of mean solar calendars in which month lengths can vary by one day, and are determined by the mean position of the sun each month. In this case, we combine the calculation of the number of days in the elapsed years and those of the elapsed months. Assuming a 12-month year with $M = Y/12$, we have:

$$\lfloor yY + mM \rfloor + d \tag{5}$$

or

$$\lceil yY + mM \rceil + d \ , \tag{6}$$

depending on whether the "before" or "not after" version is required. The inverses for these two variable-month versions are:

$$
\begin{aligned}
y &= \lceil (n+1)/Y \rceil - 1 \ , \\
m' &= \lceil (n+1)/M \rceil - 1 \ , \\
m &= m' \bmod 12 \ , \\
d &= n - \lfloor m'M \rfloor \ ,
\end{aligned}
\tag{7}
$$

and

$$
\begin{aligned}
y &= \lfloor n/Y \rfloor \ , \\
m &= \lfloor n/M \rfloor \bmod 12 \ , \\
d &= \lfloor n \bmod M \rfloor \ ,
\end{aligned}
\tag{8}
$$

respectively.

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[4]In *CC*, formulæ are given for the hybrid case where years are determined by the "not after" convention, but months by a "before" rule. There are also various cosmetic differences between the formulæ given here and in *CC*.

3.2 Generic Single–Cycle Calendars

The critical event for a calendar sometimes occurs exactly at the calendar's epoch (K.Y., in the Indian case). However, often an additional complication is introduced, wherein the relevant critical event occurred some fraction of a day before the critical time for the epoch. Furthermore, the cyclical month pattern may have its own starting point. Accordingly, for the fixed-month calendar, we are given the following constants:

1. the calendar epoch, **single-cycle-epoch**, an integer;
2. the offset of the first critical event, **delta-year**, a number in the range $[0, 1)$;
3. the average year length, **average-year-length**, of at least 1 day;
4. the average month length, **average-month-length**, at least 1 day long, but no longer than an average year; and
5. the offset for the first month, **delta-month**, also in the range $[0, 1)$.

In the "before" version of the rules, the critical yearly event for the epochal year occurred **delta-year** days after the earliest possible moment, which is $1 -$ **delta-year** days before the critical time. Similarly, the critical monthly event for the first month of the calendar occurred **delta-month** days after its earliest possible time.

To convert a single-cycle 1-based date to an R.D. date, we add to the epoch the days before the *year*, the days before *month* in *year*, and the days before *day* in *month*, taking the initial offsets into account:

$$\textbf{fixed-from-single-cycle}\left(\boxed{year \mid month \mid day} \right) \quad \overset{\text{def}}{=} \qquad (9)$$

$$\textbf{single-cycle-epoch}$$
$$+ \ \lfloor (year - 1) \times \textbf{average-year-length} + \textbf{delta-year} \rfloor$$
$$+ \ \lfloor (month - 1) \times \textbf{average-month-length} + \textbf{delta-month} \rfloor + day - 1$$

In the other direction, we compute the single-cycle date from an R.D. date by determining the year from the start of the mean year using (1.68) from *CC*, the month from (1.68) applied to the month parameters, and the day by calculating the remainder:

$$\textbf{single-cycle-from-fixed}\,(date) \quad \overset{\text{def}}{=} \qquad (10)$$

$$\boxed{year \mid month \mid day}$$

where

$$days \quad = \quad date - \textbf{single-cycle-epoch}$$

$$year \quad = \quad \left\lceil \frac{days + 1 - \textbf{delta-year}}{\textbf{average-year-length}} \right\rceil$$

$$n \quad = \quad days$$
$$- \; \lfloor \, \textbf{delta-year} + (year - 1) \times \textbf{average-year-length} \, \rfloor$$

$$month \quad = \quad \left\lceil \frac{n + 1 - \textbf{delta-month}}{\textbf{average-month-length}} \right\rceil$$

$$day \quad = \quad n + 1$$
$$- \lfloor \, \textbf{delta-month}$$
$$+ \, (month - 1) \times \textbf{average-month-length} \, \rfloor$$

The Coptic calendar, with **average-year-length** of $365\frac{1}{4}$ days, is also a single-cycle calendar, but we need to use an artificial **average-month-length** of 30 to accommodate its twelve 30-month days, which are followed by an extra "month" of epagomenē lasting 5–6 days. Also, **single-cycle-epoch** = R.D. $103,605$, **delta-year** $= \frac{1}{4}$, and **delta-month** $= 0$. Compare Table 1.4 and Chapter 4 of *CC*.

The Julian (old style) calendar, on the other hand, even though it has the same year length as the Coptic, does not fit our scheme, because of its irregular month lengths; see Chapter 3 of *CC*.

It should be stressed that for these functions to operate correctly for rational parameters, precise arithmetic is incumbent. Otherwise, four years, say, of average length $365\frac{1}{4}$ might not add up to an integral number of days, wreaking havoc on functions using floors, ceilings, and modular arithmetic.

For the alternate version, where the critical event may occur at the critical time, **delta-year** is the fraction of the day *before* the critical moment of the epoch at which the event occurred. The same is true for **delta-month**. So, we have, instead,

$$\textbf{alt-fixed-from-single-cycle} \left(\; \boxed{year \; | \; month \; | \; day} \; \right) \quad \overset{\text{def}}{=} \qquad (11)$$

$$\textbf{single-cycle-epoch}$$
$$+ \; \lceil (year - 1) \times \textbf{average-year-length} - \textbf{delta-year} \rceil$$
$$+ \; \lceil (month - 1) \times \textbf{average-month-length} - \textbf{delta-month} \rceil + day - 1$$

$$\textbf{alt-single-cycle-from-fixed} \, (date) \quad \overset{\text{def}}{=} \qquad (12)$$

$$\boxed{year \; | \; month \; | \; day}$$

where

$$days \quad = \quad date - \textbf{single-cycle-epoch} + \textbf{delta-year}$$

$$year \quad = \quad \left\lfloor \frac{days}{\textbf{average-year-length}} \right\rfloor + 1$$

$$n \quad = \quad \lfloor \textit{days} \mod \textbf{average-year-length} \rfloor + \textbf{delta-month}$$

$$\textit{month} \quad = \quad \left\lfloor \frac{n}{\textbf{average-month-length}} \right\rfloor + 1$$

$$\textit{day} \quad = \quad \lfloor n \mod \textbf{average-month-length} \rfloor + 1$$

This version, too, works for the Coptic calendar, but with $\textbf{delta-year} = \frac{1}{2}$.

Now for the variable-month version, as before, we have the epoch of the calendar **single-cycle-epoch**, the average year length **average-year-length**, and the initial offset **delta-year**. However, instead of the fixed-month structure given by **average-month-length** and **delta-month**, we simply specify the (integral) number of months in the year, **months-per-year**. To convert between R.D. dates and dates on this single-cycle mean calendar, we again apply formulæ (1.65) and (1.68) from Section 1.12 of *CC*, but with minor variations.

In this case, to convert a single-cycle date to an R.D. date, we add the days before the mean *month* in *year*, and the days before *day* in *month*:

$$\textbf{var-fixed-from-single-cycle} \left(\; \boxed{\textit{year} \; | \; \textit{month} \; | \; \textit{day}} \; \right) \quad \overset{\text{def}}{=} \tag{13}$$

$\qquad$ **single-cycle-epoch**

$\qquad + \lfloor (\textit{year} - 1) \times \textbf{average-year-length} + \textbf{delta-year}$

$\qquad \qquad + (\textit{month} - 1) \times \textit{mean-month-length} \rfloor$

$\qquad + \textit{day} - 1$

where

$$\textit{mean-month-length} = \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

In the other direction, we compute the single-cycle date from an R.D. date by determining the year from the start of the mean year using (1.68), the month from (1.68) applied to the month parameters, and the day by subtraction:

$$\textbf{var-single-cycle-from-fixed} \, (\textit{date}) \quad \overset{\text{def}}{=} \tag{14}$$

$\qquad \boxed{\textit{year} \; | \; \textit{month} \; | \; \textit{day}}$

where

$$\textit{days} \quad = \quad \textit{date} - \textbf{single-cycle-epoch}$$

$$\textit{offset} \quad = \quad \textit{days} + 1 - \textbf{delta-year}$$

$$\textit{year} \quad = \quad \left\lceil \frac{\textit{offset}}{\textbf{average-year-length}} \right\rceil$$

$$mean\text{-}month\text{-}length = \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

$$m' = \left\lceil \frac{\mathit{offset}}{mean\text{-}month\text{-}length} \right\rceil - 1$$

$$month = 1 + (m' \bmod \textbf{months-per-year})$$

$$\begin{aligned} day = \ & days + 1 \\ & - \lfloor \textbf{delta-year} + m' \times mean\text{-}month\text{-}length \rfloor \end{aligned}$$

3.3 Indian Mean Solar Calendar

Unlike other solar calendars, especially the universally used Gregorian, the Indian calendars are based on the sidereal (*nakshatra*) year. The old Hindu mean (*madhyama*) solar calendar is an example of the second version of our generic solar calendar, using an estimate of the length of the sidereal year and mean sunrise as the critical time.

However, we need the fourth version of the formulæ, with the determining event occurring before or at the critical time:

alt-var-fixed-from-single-cycle (15)
$$\left(\begin{array}{|c|c|c|} \hline \mathit{year} & \mathit{month} & \mathit{day} \\ \hline \end{array} \right) \quad \overset{\text{def}}{=}$$

single-cycle-epoch
$$+\big\lceil\ (year - 1) \times \textbf{average-year-length} - \textbf{delta-year}$$
$$+\ (month - 1) \times mean\text{-}month\text{-}length\ \big\rceil$$
$$+\ day - 1$$

where

$$mean\text{-}month\text{-}length = \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

and

alt-var-single-cycle-from-fixed (date) $\overset{\text{def}}{=}$ (16)

$$\begin{array}{|c|c|c|} \hline \mathit{year} & \mathit{month} & \mathit{day} \\ \hline \end{array}$$

where

$$days = date - \textbf{single-cycle-epoch} + \textbf{delta-year}$$

$$mean\text{-}month\text{-}length = \frac{\textbf{average-year-length}}{\textbf{months-per-year}}$$

$$year = \left\lfloor \frac{days}{\textbf{average-year-length}} \right\rfloor + 1$$

$$month = 1 + \left(\left\lfloor \frac{days}{\textit{mean-month-length}} \right\rfloor \bmod \textbf{months-per-year} \right)$$

$$day = \lfloor days \bmod \textit{mean-month-length} \rfloor + 1$$

Following the *First Ārya Siddhānta* regarding year length, the constants we would need are:

$$\textbf{single-cycle-epoch} \quad \overset{\text{def}}{=} \quad \textbf{hindu-epoch} \tag{17}$$

$$\textbf{average-year-length} \quad \overset{\text{def}}{=} \quad 365\frac{149}{576} \tag{18}$$

$$\textbf{delta-year} \quad \overset{\text{def}}{=} \quad \frac{1}{4} \tag{19}$$

$$\textbf{months-per-year} \quad \overset{\text{def}}{=} \quad 12 \tag{20}$$

The above algorithms give a 1-based year number. The necessary changes for versions of the Indian calendar that use elapsed years (including those in *CC*) are trivial.

4 True Solar Calendars

One may say that a solar calendar is astronomical if the start of its years is determined by the actual time of a solar event. The event is usually an equinox or solstice, so we presume that it is the moment at which the true solar longitude attains some given value, named **critical-longitude** below, and can assume that the true and mean times of the event differ by at most five days.

The astronomical Persian calendar (Chapter 14 of *CC*) uses a critical solar longitude of 0° for the vernal equinox and apparent noon in Tehran as its critical moment. The defunct French Revolutionary calendar (Chapter 16 of *CC*) used a critical solar longitude of 180° for the autumnal equinox and apparent midnight in Paris as its critical moment.[5]

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[5]A generic version of such calendars was mentioned in our paper [2] as Lisp macros.

4.1 Generic Solar Calendars

Fixed-month versions of the true calendar usually have idiosyncratic month lengths. This is true of both the Persian and Bahá'í calendars; see Chapters 14–15 of *CC*. So we restrict ourselves to the determination of New Year. First, we define a function to determine the true longitude at the critical time of any given day, where the critical time is determined by some function **critical-time**:

$$\textbf{true-longitude}\,(date) \quad \overset{\text{def}}{=} \tag{21}$$

$$\textbf{solar-longitude}\,(\textbf{critical-time}\,(date))$$

Since solar longitude increases at different paces during different seasons, we search for the first day the sun attains the **critical-longitude**, beginning five days prior to the mean time:

$$\textbf{solar-new-year-on-or-after}\,(date) \quad \overset{\text{def}}{=} \tag{22}$$

$$\underset{d \geq start}{\text{MIN}}$$

$$\left\{ \begin{aligned} &\textbf{critical-longitude} \leq \textbf{true-longitude}\,(d) \\ &\leq \textbf{critical-longitude} + 2 \end{aligned} \right\}$$

where

$$\lambda \quad = \quad \textbf{true-longitude}\,(date)$$

$$\begin{aligned} start \quad = \quad & date - 5 \\ & + \left\lfloor \textbf{average-year-length} \times \tfrac{1}{360} \right. \\ & \quad \left. \times ((\textbf{critical-longitude} - \lambda) \mod 360) \right\rfloor \end{aligned}$$

The initial estimate is based on the current solar longitude λ, with an average daily increase of $360°/Y$.

Solar New Year (*Sowramana Ugadi*) in a given Gregorian year is then computed as follows:

$$\textbf{hindu-solar-new-year}\,(g\text{-}year) \quad \overset{\text{def}}{=} \tag{23}$$

$$\begin{aligned} &\textbf{solar-new-year-on-or-after} \\ &\quad \left(\textbf{fixed-from-gregorian}\,\left(\boxed{g\text{-}year \;\; \textbf{january} \;\; 1} \right) \right) \end{aligned}$$

which uses the R.D. to Gregorian conversion function **gregorian-from-fixed** (equation 2.23) of *CC*.

For a variable-month version of the true calendar, such as the Indian solar calendar and its relatives, the start of *each* month is also determined by the true solar longitude:

Table 4. Sidereal year values.

Source	Length	
First Ārya-Siddhānta	365.258681	$365^{\mathrm{d}}6^{\mathrm{h}}12^{\mathrm{m}}30^{\mathrm{s}}$
Brahma-Siddhānta	365.258438	$365^{\mathrm{d}}6^{\mathrm{h}}12^{\mathrm{m}}\ 9^{\mathrm{s}}$
Original Sūrya-Siddhānta	365.258750	$365^{\mathrm{d}}6^{\mathrm{h}}12^{\mathrm{m}}36^{\mathrm{s}}$
Present Sūrya-Siddhānta	365.258756	$365^{\mathrm{d}}6^{\mathrm{h}}12^{\mathrm{m}}36.56^{\mathrm{s}}$
Modern Value (for 1000 C.E.)	365.256362	$365^{\mathrm{d}}6^{\mathrm{h}}\ 9^{\mathrm{m}}\ 8.44^{\mathrm{s}}$

$$\textbf{solar-from-fixed}\,(date) \quad \overset{\mathrm{def}}{=} \quad \boxed{year}\,\boxed{m+1}\,\boxed{date-begin+1} \qquad (24)$$

where

$$\lambda \quad = \quad \textbf{true-longitude}\,(date)$$

$$m \quad = \quad \left\lfloor \frac{\lambda}{30°} \right\rfloor$$

$$year \quad = \quad \mathrm{round}\left(\frac{\textbf{critical-time}\,(date) - \textbf{solar-epoch}}{\textbf{average-year-length}} - \frac{\lambda}{360°} \right)$$

$$approx \quad = \quad date - 3 - (\lfloor \lambda \rfloor \mod 30°)$$

$$begin \quad = \quad \underset{i \geq approx}{\mathrm{MIN}} \left\{ m = \left\lfloor \frac{\textbf{true-longitude}\,(i)}{30°} \right\rfloor \right\}$$

This function can be inverted using the methods of Section 18.5 of *CC*.

4.2 True Indian Solar Calendar

For the Indian solar calendar, we need to use the Indian sidereal longitude function (**hindu-solar-longitude** in *CC*) in place of **solar-longitude** (in the **true-longitude** function). The length of the sidereal year according to the *Sūrya-Siddhānta* is

$$\textbf{average-year-length} \;=\; 365\frac{279457}{1080000}\;.$$

(See Table 4.) The year begins when the sun returns to sidereal longitude $0°$. There are various critical times of day that are used to determine exactly which day is New Year.

- According to the Orissa rule (followed also in Punjab and Haryana), sunrise is used. In other words, the solar month is determined by the stellar position of the sun the following morning:

$$\textbf{orissa-critical}\,(date) \quad \overset{\mathrm{def}}{=} \quad \textbf{hindu-sunrise}\,(date+1) \qquad (25)$$

where **hindu-sunrise** is sunrise according to the Indian rule or practice [(18.33) in *CC*]. See Section 7 for details.

- According to the Tamil rule, sunset of the current day is used:

$$\textbf{tamil-critical}\,(date) \quad \overset{\text{def}}{=} \quad \textbf{hindu-sunset}\,(date) \tag{26}$$

where **hindu-sunset** is sunset according to the Indian rule or practice.

- According to the Malayali (Kerala) rule, 1:12 p.m. (seasonal time) on the current day is used:

$$\textbf{malayali-critical}\,(date) \quad \overset{\text{def}}{=} \tag{27}$$

$$\textbf{hindu-sunrise}\,(date)$$
$$+\,\tfrac{3}{5} \times (\textbf{hindu-sunset}\,(date) - \textbf{hindu-sunrise}\,(date))$$

Kerala also uses a different critical longitude.

- According to some calendars from Madras, apparent midnight at the end of the day is used:

$$\textbf{madras-critical}\,(date) \quad \overset{\text{def}}{=} \tag{28}$$

$$\textbf{hindu-sunset}\,(date)$$
$$+\,\tfrac{1}{2} \times (\textbf{hindu-sunrise}\,(date+1) - \textbf{hindu-sunset}\,(date))$$

- According to the Bengal rule (also in Assam and Tripura), midnight at the start of the civil day is usually used, unless the zodiac sign changes between 11:36 p.m. and 12:24 a.m. (temporal time), in which case various special rules apply, depending on the month and on the day of the week.

See [8, page 12] and [1, page 282]. The function **critical-time** should be set to one of these.

4.3 Indian Astronomical Solar Calendar

For an astronomical Indian solar calendar, we need to substitute an astronomical calculation of *sidereal* longitude for **solar-longitude** in **true-longitude**. We should also use astronomical geometric sunrise (and/or sunset) for **hindu-sunrise** (and **hindu-sunset**) in **critical-time**; see Section 7.

The difference between the equinoctal and sidereal longitude (the *ayanāmsha*) changes with time, as a direct consequence of precession of the equinoxes. It is uncertain what the zero point of Indian sidereal longitude is, but it is customary to say that the two measurements coincided circa 285 C.E., the so-called "Lahiri *ayanāmsha.*" Others (for example [10, Section 18]) suggest that the two measurements coincided around 560 C.E. Either way, the overestimate of the length of the mean sidereal year used by the siddhantas leads to a growing discrepancy in the calculation of solar longitude; see
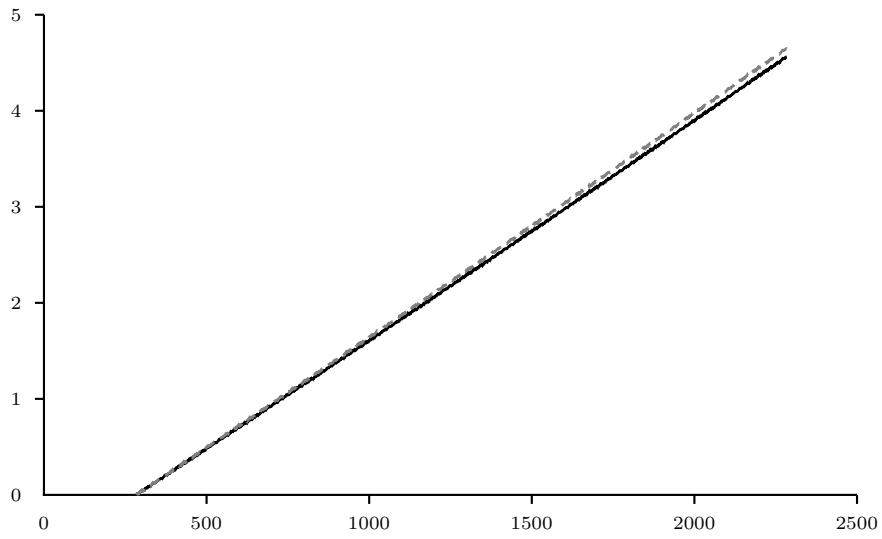
Fig. 1. Increasing difference, for 285–2285 C.E., between siddhāntic and astronomical sidereal longitudes in degrees (solid black) and days (dashed grey), assuming coincidence in 285.
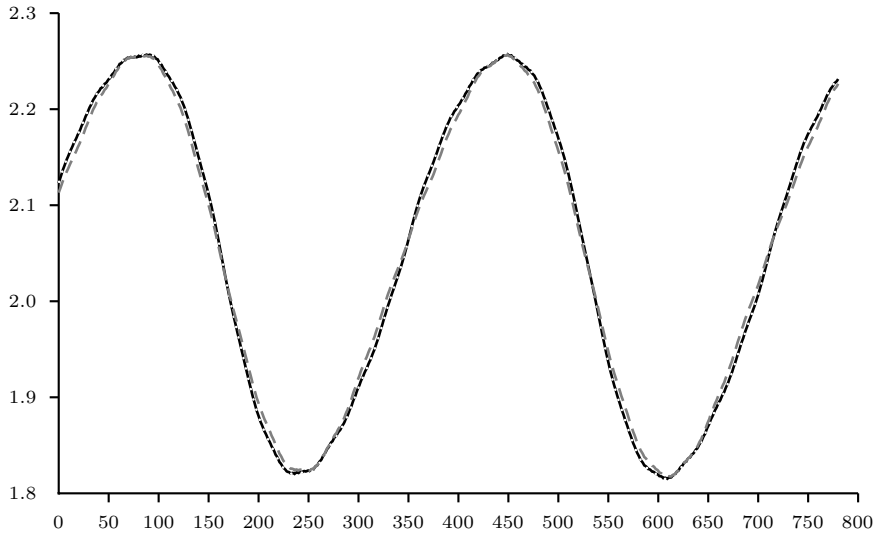
Fig. 2. Difference (1000–1001 C.E.) between siddhāntic and astronomical sidereal solar longitude (solid black line) in hours. Dotted white line (atop the black line) uses mathematical sine function, rather than an interpolated tabular sine; dashed grey line uses fixed epicycle; they are virtually indistinguishable.

Table 4. (The length of the sidereal year is increasing by about 10^{-4} seconds per year.)

The Indian vernal equinox, when the sun returns to the sidereal longitude $0°$, is called Mesha saṃkrānti. Solar New Year, the day of Mesha saṃkrānti, as computed by **hindu-solar-new-year** with traditional year lengths, is nowadays about four days later than that which would be obtained by astronomical calculation (assuming the Lahiri value).

To calculate the sidereal longitude, we use the algorithm for precession in [7, pages 136–137], as coded in (13.39) of CC, **precession**. The values given by this function need to be compared with its value when the *ayanāmsha* was 0, given, according to some authorities, by the following:

$$\textbf{sidereal-start} \quad \overset{\text{def}}{=} \tag{29}$$

$$\textbf{precession}$$
$$(\textbf{universal-from-local}$$
$$(\textbf{mesha-samkranti}\,(285 \text{ c.e.})\,, \textbf{hindu-locale})\,)$$

where **mesha-samkranti** (equation 18.51) of CC gives the local time of the (sidereal) equinox, Ujjain is our **hindu-locale**, and **universal-from-local** is one of the time conversion functions (equation 13.8) of CC. Then:

$$\textbf{sidereal-solar-longitude}\,(t) \quad \overset{\text{def}}{=} \tag{30}$$

$$(\,\textbf{solar-longitude}\,(t) \;-\; \textbf{precession}\,(t)$$
$$+\,\textbf{sidereal-start}\,) \mod 360$$

as in equation (13.40) of CC. That done, we can compare the astronomical calendar with the approximations used in the true Indian calendar.

The cumulative effect over the centuries of the difference in length of the sidereal year on the time of Mesha saṃkrānti, and on the sidereal longitude at that time, is shown in Figure 1. In 1000 c.e., it stood at about $1°37'$.

The average difference between the calculated sidereal longitude and the astronomical values was $2°3'$ during 1000–1002 c.e. In addition, Figure 2 shows a periodic discrepancy of up to $\pm12'$ between the siddhāntic estimate of solar longitude and the true values. The figure also suggests that neither the interpolated stepped sine function used in traditional astronomy nor the fluctuating epicycle of Indian theory (using the smallest [best] size, instead) make a noticeable difference for the sun. In other words, the tabular sine and arcsine functions (see Table 18.2 in CC) are precise enough for the purpose, while the theory of changing epicycle (see Figure 18.2 in CC) is unnecessary for the sun.

The difference in longitude for a given moment t, Ujjain local time, is calculated as:

hindu-solar-longitude(t)
$-$**sidereal-solar-longitude**(**universal-from-local**$(t, $**hindu-locale**$))$

5 Lunisolar Calendars

The lunisolar calendar type is represented by the Chinese (see Chapter 17 of *CC*), Hebrew (see Chapter 7 of *CC*), and Easter (see Chapter 8 of *CC*) calendars today, as well as those of some of the Indian and other Asian cultures (for example, the Tibetan *Phugpa* calendar; see Chapter 19 of *CC*), and was historically very popular. The basic idea is that months follow the lunar cycle, with leap months added every 2–3 years, so that the average year length matches the sun's apparent celestial revolution. Indian lunisolar calendars can be further subdivided into those whose months begin with each new moon (the *amānta* scheme) and those that go from full moon to full moon (*pūrṇimānta*).

The Hebrew and Easter calendars follow a fixed leap-year cycle, as did the old Hindu mean lunisolar calendar; the Chinese and modern Hindu calendars determine each month and year according to the true positions of the sun and moon. Unlike the Hebrew lunisolar calendar, with its 19-year cycle of 7 leap years, Indian intercalated months, in the mean scheme, do not follow a short cyclical pattern. Rather, in the *Ārya-Siddhānta* version, there are 66,389 leap years in every 180,000-year cycle. The Hebrew, Easter, and mean Indian leap-year rules all distribute leap years evenly over the length of the cycle.

Lunisolar calendars can also come in the same two flavours, fixed- and variable-month patterns. The Indian mean lunisolar calendar has variable months, like its solar sister; the Hebrew calendar has a more-or-less fixed scheme (see Chapter 7 of *CC* for details).

In the fixed-month scheme, one fixed month (usually the last) of the 13 months of a leap year is considered the leap month, so one can just number them consecutively. This is not true of the Indian calendar, in which any month can be leap, if it starts and ends within the same (sidereal) zodiacal sign.

Unlike other calendars, a day on the mean Indian calendar can be *omitted* any time in a lunar month, since the day number is determined by the phase of the mean moon. Here we concentrate on the leap year structure; see *CC* for other details.

5.1 A Generic Dual-Cycle Calendar

Let Y and M be the lengths of the mean solar year and lunar month in days, respectively, where $Y \geq M \geq 1$ are positive real numbers. If Y is not a multiple of M, then there will be common years of $\lfloor Y/M \rfloor$ months and leap years of length $\lceil Y/M \rceil$ months. Then a year has Y/M months on average, with a leap-year frequency of $(Y \bmod M)/M$.

The basic idea of the dual-cycle calendar is to first aggregate days into months and then months into years. Elapsed months are counted in the same way as years are on the single-cycle calendar, using an average length of M instead of Y. Then, years are built from multiple units of months, rather than days, again in a similar fashion to a single-cycle calendar.

For the Indian mean lunar calendar, according to the $\bar{A}rya$ $Siddh\bar{a}nta$, we would use the values:

$$M = 29\tfrac{2362563}{4452778} \,,$$
$$Y = 365\tfrac{149}{576} \,,$$

and sunrise as the critical time of day. The Hebrew calendar also follows a dual-cycle pattern, with:

$$M = 29\tfrac{13753}{25920} \,,$$
$$Y = \tfrac{285}{19}M \,,$$

and noon as critical moment, but exceptions can lead to a difference of up to three days.

To convert from a 0-based lunisolar date $\langle y, m, d \rangle$ to a day count, use

$$\lceil (\lfloor yY/M \rfloor + m)M \rceil + d \,. \tag{31}$$

In the other direction, we have:

$$
\begin{aligned}
m' &= \lfloor n/M \rfloor \,, \\
y &= \lceil (m'+1)M/Y \rceil - 1 \,, \\
m &= m' - \lfloor yY/M \rfloor \,, \\
d &= n - \lceil m'M \rceil \,.
\end{aligned}
\tag{32}
$$

When the leap month is not fixed and any month can be leap, as in the Indian calendar, we would use an extra, Boolean, component for dates, $\langle y, m, \ell, d \rangle$, and would need to determine which month is leap. On the Chinese calendar, the first lunar month in a leap year during which the sun does not change its zodiacal sign (counting from month 11 to month 11) is deemed leap. In the Indian scheme, the rule is similar: any month in which the sidereal sign does not change is leap.

As was the case for the solar calendars, there are variants corresponding to whether the critical events may also occur at the critical moments. See Section 12.2 of *CC*.

6 True Lunisolar Calendar

The general form of the determination of New Year on a lunisolar calendar is as follows:

1. Find the moment s when the sun reaches its critical longitude.
2. Find the moment p when the moon attains its critical phase before (or after) s.
3. Choose the day d before (or after) p satisfying additional criteria.

Some examples include:

- The Nicæan rule for Easter is the first Sunday after the first full moon on or after the day of the vernal equinox; see Chapter 8 of *CC*.

- The classical rule for the first month (*Nisan*) of the Hebrew year was that it starts on the eve of the first observable crescent moon no more than a fortnight before the vernal equinox; see Section 20.4 of *CC*.
- The eleventh month of the Chinese calendar almost always begins with the new moon on or before the day of the winter solstice (270°). The Chinese New Year is almost always the new moon on or after the day the sun reaches 300°; see Chapter 17 of *CC*.
- The Indian Lunar New Year is the (sunrise-to-sunrise) day of the last new moon before the sun reaches the edge of the constellation Aries (0° sidereal); see Chapter 18 of *CC*.

Using the functions provided in *CC*:

1. The moment s can be found with **solar-longitude-after** (equation 13.33).
2. Finding the moment p can be accomplished with **lunar-phase-at-or-before** (13.54) or **lunar-phase-at-or-after** (13.55).
3. Choosing the day is facilitated by **kday-on-or-after** and its siblings (Section 1.10).

For example, the winter-solstice-to-winter-solstice period is called a *suì* on the Chinese calendar. Hence, the start of the Chinese month in which a *suì* begins, that is, the month containing the winter solstice (almost always the eleventh month, but on occasion a leap eleventh month) is determined by:

$$\textbf{sui-month-start-on-or-after}\,(date) \quad \overset{\text{def}}{=} \tag{33}$$

$$\left\lfloor \textbf{standard-from-universal} \right.$$
$$\left. (moon, \textbf{chinese-location}\,(date)) \right\rfloor$$

where

$$
\begin{aligned}
sun \quad = \quad & \textbf{universal-from-standard} \\
& \quad (\, \lfloor \textbf{standard-from-universal} \\
& \qquad (\, \textbf{solar-longitude-after}\,(270°, date)\,, \\
& \qquad \quad \textbf{chinese-location}\,(date)\,)\, \rfloor\,, \\
& \qquad \textbf{chinese-location}\,(date)\,) \\
moon \quad = \quad & \textbf{lunar-phase-at-or-before}\,(0°, sun + 1)
\end{aligned}
$$

For the Indian calendars, the functions should use sidereal longitudes and can be traditional or astronomical, as desired. Using the astronomical code of *CC*, we can define:

$$\textbf{sidereal-solar-longitude-after}\,(\phi, t) \quad \overset{\text{def}}{=} \tag{34}$$

$$\underset{x\in[a,b]}{\overset{u-l<\varepsilon}{\mathrm{MIN}}}\left\{\begin{array}{l}((\textbf{sidereal-solar-longitude}\,(x)-\phi)\mod 360)\\ <180°\end{array}\right\}$$

where

$$\varepsilon = 10^{-5}$$

$$rate = \frac{\textbf{average-year-length}}{360°}$$

$$\tau = t + rate \times \big(\,(\,\phi-\textbf{sidereal-solar-longitude}\,(t)\,)\mod 360\,\big)$$

$$a = \max\{t,\tau-5\}$$

$$b = \tau+5$$

The function MIN performs a bisection search in $[a,b]$ with accuracy ε.

For the traditional Hindu calendar, we would use equation (18.50) in *CC*, **hindu-solar-longitude-at-or-after**, instead of **sidereal-solar-longitude-after** and use the following in place of **lunar-phase-at-or-before**:

$$\textbf{hindu-lunar-phase-at-or-before}\,(\phi,t)\;\overset{\mathrm{def}}{=}\qquad\qquad(35)$$

$$\underset{x\in[a,b]}{\overset{u-l<\varepsilon}{\mathrm{MIN}}}\left\{((\textbf{hindu-lunar-phase}\,(x)-\phi)\mod 360)<180°\right\}$$

where

$$\varepsilon = 2^{-17}$$

$$\tau = t - \textbf{hindu-synodic-month}\times\tfrac{1}{360}$$
$$\qquad\times((\textbf{hindu-lunar-phase}\,(t)-\phi)\mod 360)$$

$$a = \tau-2$$

$$b = \min\{t,\tau+2\}$$

Then we can use the following to compute the start of Indian lunisolar month m:

$$\textbf{hindu-lunar-month-on-or-after}\,(m,\mathit{date})\;\overset{\mathrm{def}}{=}\qquad\qquad(36)$$

$$\left\{\begin{array}{ll}\mathit{date} & \textbf{if } moon\le\textbf{hindu-sunrise}\,(\mathit{date})\\ \mathit{date}+1 & \textbf{otherwise}\end{array}\right.$$

where

$$\lambda \quad = \quad (m-1) \times 30°$$

$$sun \quad = \quad \textbf{hindu-solar-longitude-after}\,(\lambda, date)$$

$$moon \quad = \quad \textbf{hindu-lunar-phase-at-or-before}\,(0°, sun)$$

$$date \quad = \quad \lfloor moon \rfloor$$

The time of the *tithis* (lunar "days," corresponding to 30ths of the lunar phase cycle) differs an average of less than 13 minutes between the traditional and astronomical calculations (again in 1000 C.E.). See Figure 3.

Figure 4 shows the same (nil) impact of sine and epicycle (using the biggest epicycle) on the calculation of lunar sidereal longitude as we found in the solar case. Moreover, the sixteenth century correction (*bīja*) of Gaṇṇeśa Daivajna for the length of the anomalistic months (from 488,203 revolutions of the apogee in a *yuga* to 488,199) also is of no consequence (in the sixteenth century as well as in the eleventh). The difference between the calculated longitude and astronomical values was $1°56' \pm 3°25'$.

In the true version of the Indian lunisolar calendar, months, called *kshaya*, may also be expunged, when two zodiacal sign transitions occur in one lunar month. Thus, even a 12-month year can have a leap month (as was the case in 1963–1964), and a leap year can even have two (as in 1982–1983). See our *Calendrical Tabulations* [6]. The code above does not check whether month m is expunged.

There are several competing conventions as to the placement and naming of leap months and excision of suppressed months; see [8, page 26].

7 Sunrise

Generally, Indian calendarists advocate the use of geometric sunrise for calendrical determinations:[6]

$$\textbf{hindu-sunrise}\,(date) \quad \overset{\text{def}}{=} \quad \textbf{dawn}\,(date, \textbf{hindu-locale}, 0°) \qquad (37)$$

Lahiri, however, suggests a depression angle of $47'$ (including $31'$ for refraction); astronomers typically use $50'$.

As is well known, the original siddhāntic calculation for sunrise uses a simple approximation for the equation of time. Figure 5 compares the two versions. Using an accurate equation of time, but otherwise following the siddhāntic method for sunrise, gives close agreement with geometric sunrise. See Figure 6.

---

[6]Pal Singh Purewal [personal communication, April 29, 2002]: "Most Indian almanac editors give and advocate the use of the centre of the solar disk for sunrise without refraction."
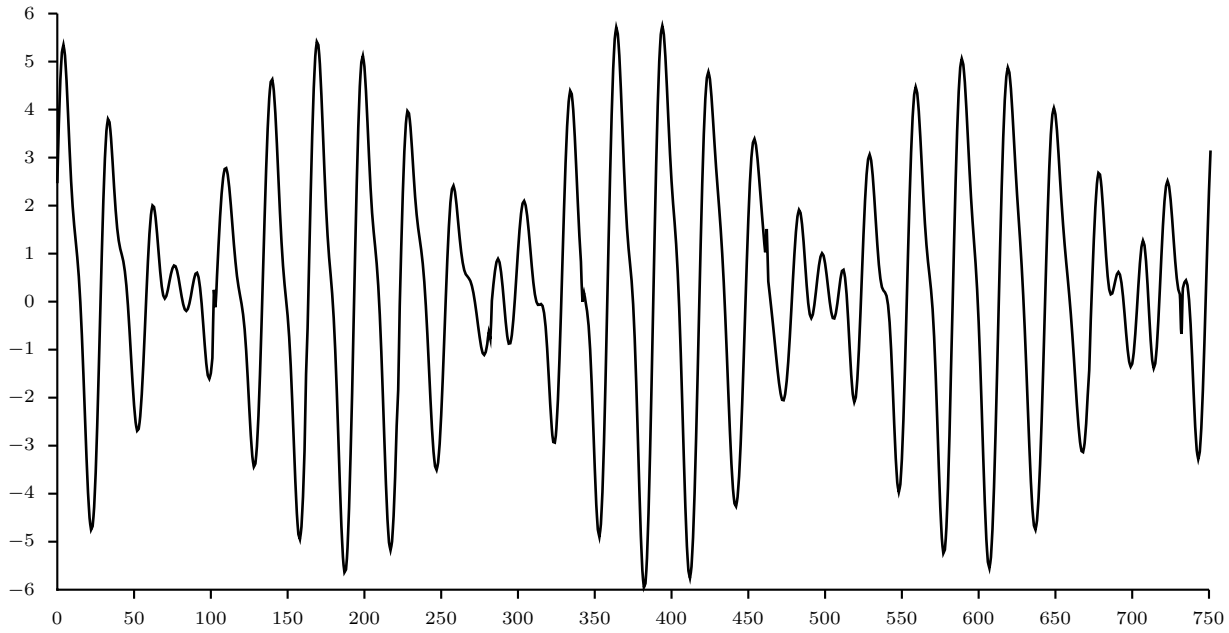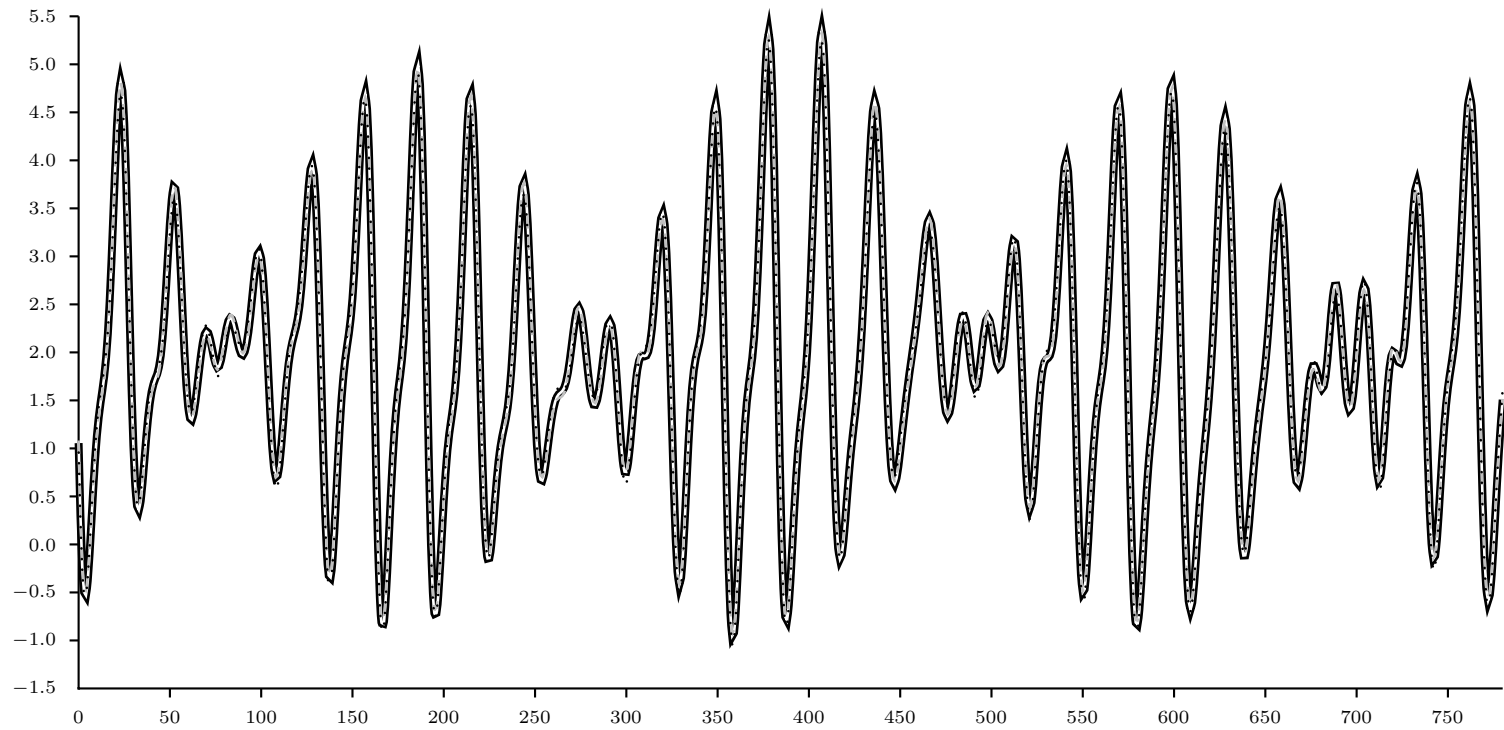
Nachum Dershowitz and Edward M. Reingold

Fig. 3. Tithi time differences, 1000–1001 C.E. in hours.

Fig. 4. Difference (1000–1001 C.E.) between siddhāntic and astronomical sidereal lunar longitude (thick solid black line). Thinner white line (largely atop the black line) uses the mathematical sine function, rather than an interpolated tabular sine; gray dashed line (largely overlaying the white line) uses a fixed epicycle; the black dotted line (largely overlaying the white line) is sans *bīja* correction; they are virtually indistinguishable.
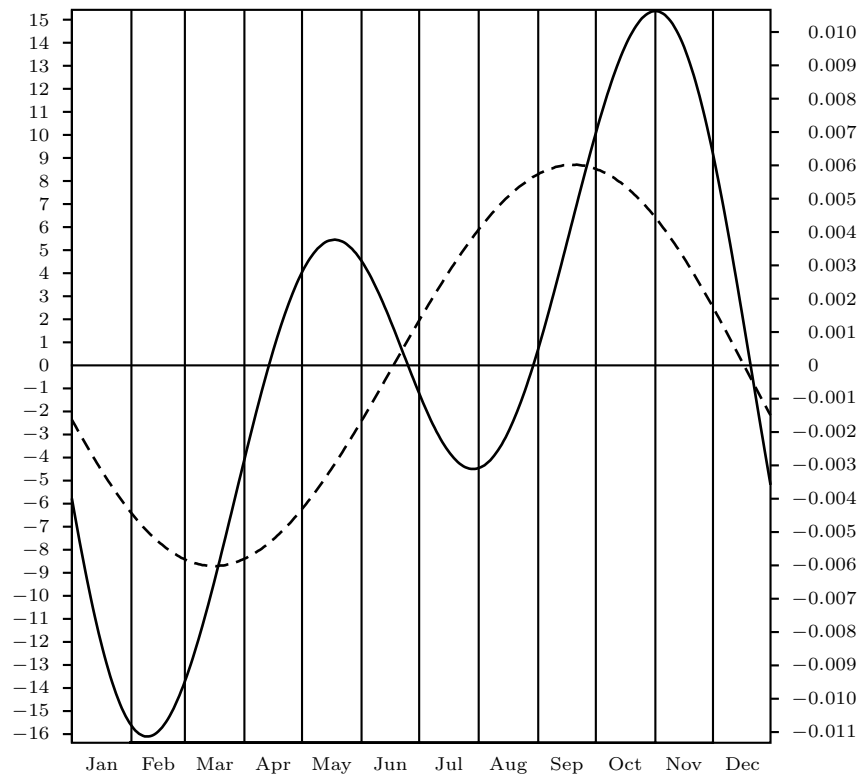
Fig. 5. The equation of time in 1000 C.E. The astronomical version is shown as a solid line; the Hindu version is shown as a dashed line. The left vertical axis is marked in minutes and the right vertical axis is marked in fractions of a day.

8 Holidays

Many of the holidays in India depend on the local lunisolar calendar. Table 5 lists some of the more popular holidays. (For a comprehensive list in English, see [9].) There is a very wide regional variance in timing and duration of holidays.

In general, holidays do not occur in leap months or on leap days. If a month is skipped, as happens intermittently (with gaps of 19 to 141 years between occurrences), then the "lost" holidays are moved to the next month, depending again on regional conventions. In many places, rather than skip a whole month, two half months are skipped and their holidays are moved backward or forward, depending on which lost half-month they are meant to occur in.

The precise day of observance of a lunisolar event usually depends on the time of day (sunrise, noon, midnight, etc.) at which the moon reaches a critical
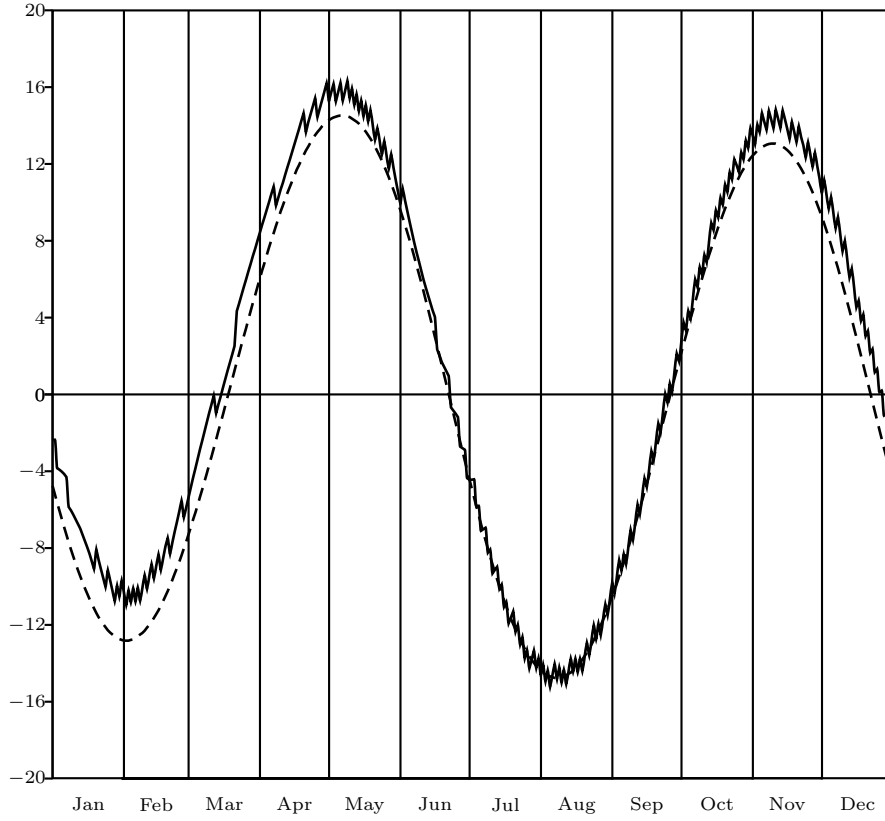
Fig. 6. Sunrise, Hindu and astronomical, 1000 C.E. Difference in sunrise times is shown as a solid line; the difference of just the equation of time calculation is shown as a dashed line. The vertical scale is in minutes.

Table 5. Some Hindu lunisolar holidays.

Holiday	Lunar Date(s)
New Year (Chandramana Ugadi)	Caitra 1
Birthday of Rāma	Caitra 9
Birthday of Krishna (Janmāshṭamī)	Śrāvaṇa 23
Ganēśa Chaturthī	Bhādrapada 3 or 4
Dashara (Nava Rathri), last 3 days	Āśvina 8–10
Diwali, last day	Kārtika 1
Birthday of Vishnu (Ekadashi)	Mārgaśīrṣa 11
Night of Śiva	Māgha 28 or 29
Holi	Phālguna 15

phase (*tithi*). According to [5], for example, Gaṇēśa Chaturthī is celebrated
on the day in which *tithi* (lunar day) 4 is current in whole or in part during
the midday period from 10:48 a.m. to 1:12 p.m. (temporal time). If that lunar
day is current during that time frame on two consecutive days, or if it misses
that time frame on both days, then it is celebrated on the former day.[7]

Some functions for holiday calculations are provided in Section 19.6 of *CC*.

Appendix – The Lisp Code

This appendix contains the Common Lisp source code for the algorithms
presented in the preceding sections. *CC* should be consulted for undefined
functions and macros.

```
1   (defun fixed-from-single-cycle (s-date)
2     ;; TYPE single-cycle-date -> fixed-date
3     (let* ((year (standard-year s-date))
4            (month (standard-month s-date))
5            (day (standard-day s-date)))
6       (+ single-cycle-epoch
7          (floor (+ (* (1- year) average-year-length)
8                    delta-year))
9          (floor (+ (* (1- month) average-month-length)
10                    delta-month))
11         day -1)))
```

```
1   (defun single-cycle-from-fixed (date)
2     ;; TYPE fixed-date -> single-cycle-date
3     (let* ((days (- date single-cycle-epoch))
4            (year (ceiling (- days -1 delta-year)
5                           average-year-length))
6            (n (- days (floor (+ delta-year
7                                 (* (1- year) average-year-length)))))
8            (month (ceiling (- n -1 delta-month) average-month-length))
9            (day (- n -1 (floor (+ delta-month
10                                  (* (1- month)
11                                     average-month-length))))))
12      (hindu-solar-date year month day)))
```

```
1   (defun alt-fixed-from-single-cycle (s-date)
2     ;; TYPE single-cycle-date -> fixed-date
3     (let* ((year (standard-year s-date))
4            (month (standard-month s-date))
5            (day (standard-day s-date)))
6       (+ single-cycle-epoch
7          (ceiling (- (* (1- year) average-year-length)
8                      delta-year))
```

---

[7]Precise details for the individual holidays are difficult to come by.

```
9           (ceiling (- (* (1- month) average-month-length)
10                         delta-month))
11          day -1)))
```

```
1   (defun alt-single-cycle-from-fixed (date)
2     ;; TYPE fixed-date -> single-cycle-date
3     (let* ((days (+ (- date single-cycle-epoch) delta-year))
4            (year (1+ (quotient days average-year-length)))
5            (n (+ (floor (mod days average-year-length)) delta-month))
6            (month (1+ (quotient n average-month-length)))
7            (day (1+ (floor (mod n average-month-length)))))
8       (hindu-solar-date year month day)))
```

```
1   (defun var-fixed-from-single-cycle (s-date)
2     ;; TYPE single-cycle-date -> fixed-date
3     (let* ((year (standard-year s-date))
4            (month (standard-month s-date))
5            (day (standard-day s-date))
6            (mean-month-length (/ average-year-length
7                                   months-per-year)))
8       (+ single-cycle-epoch
9          (floor (+ (* (1- year) average-year-length)
10                    delta-year
11                    (* (1- month) mean-month-length)))
12         day -1)))
```

```
1   (defun var-single-cycle-from-fixed (date)
2     ;; TYPE fixed-date -> single-cycle-date
3     (let* ((days (- date single-cycle-epoch))
4            (offset (- days -1 delta-year))
5            (year (ceiling offset average-year-length))
6            (mean-month-length (/ average-year-length
7                                   months-per-year))
8            (m-prime (1- (ceiling offset mean-month-length)))
9            (month (+ 1 (mod m-prime months-per-year)))
10           (day (- days -1
11                   (floor
12                    (+ delta-year
13                       (* m-prime mean-month-length))))))
14      (hindu-solar-date year month day)))
```

```
1   (defun alt-var-fixed-from-single-cycle (s-date)
2     ;; TYPE single-cycle-date -> fixed-date
3     (let* ((year (standard-year s-date))
4            (month (standard-month s-date))
5            (day (standard-day s-date))
6            (mean-month-length (/ average-year-length
7                                   months-per-year)))
8       (+ single-cycle-epoch
9          (ceiling (+ (* (1- year) average-year-length)
```

```
10                       (- delta-year)
11                       (* (1- month) mean-month-length)))
12           day -1)))

1   (defun alt-var-single-cycle-from-fixed (date)
2     ;; TYPE fixed-date -> single-cycle-date
3     (let* ((days (+ (- date single-cycle-epoch) delta-year))
4            (mean-month-length (/ average-year-length
5                                  months-per-year))
6            (year (1+ (quotient days average-year-length)))
7            (month (+ 1 (mod (quotient days mean-month-length)
8                             months-per-year)))
9            (day (1+ (floor (mod days mean-month-length)))))
10      (hindu-solar-date year month day)))

1   (defun true-longitude (date)
2     ;; TYPE moment -> longitude
3     (solar-longitude (critical-time date)))

1   (defun solar-new-year-on-or-after (date)
2     ;; TYPE fixed-date -> fixed-date
3     ;; Fixed date of solar new year on or after fixed date.
4     (let* ((lambda (true-longitude date))
5            (start
6             (+ date -5
7                (floor (* average-year-length 1/360
8                          (mod (- critical-longitude lambda) 360))))))
9       (next d start
10              (<= critical-longitude
11                  (true-longitude d)
12                  (+ critical-longitude 2)))))

1   (defun hindu-solar-new-year (g-year)
2     ;; TYPE gregorian-year -> fixed-date
3     ;; Fixed date of Hindu solar New Year in Gregorian year.
4     (solar-new-year-on-or-after
5       (fixed-from-gregorian
6         (gregorian-date g-year january 1))))

1   (defun solar-from-fixed (date)
2     ;; TYPE fixed-date -> solar-date
3     ;; Solar date equivalent to fixed date.
4     (let* ((lambda (true-longitude date))
5            (m (quotient lambda (deg 30)))
6            (year (round (- (/ (- (critical-time date) solar-epoch)
7                               average-year-length)
8                            (/ lambda (deg 360)))))
9            (approx ; 3 days before start of mean month.
10            (- date 3
11               (mod (floor lambda) (deg 30))))
12           (begin ; Search forward for beginning...
```

```
13            (next i approx ; ... of month.
14                  (= m (quotient (true-longitude i)
15                                 (deg 30))))))
16      (hindu-solar-date year (1+ m)  (- date begin -1))))

1  (defun orissa-critical (date)
2    ;; TYPE fixed-date -> moment
3    ;; Universal time of critical moment on or after date
4    ;; according to the Orissa rule
5    (hindu-sunrise (1+ date)))

1  (defun tamil-critical (date)
2    ;; TYPE fixed-date -> moment
3    ;; Universal time of critical moment on or after date
4    ;; according to the Tamil rule
5    (hindu-sunset date))

1  (defun malayali-critical (date)
2    ;; TYPE fixed-date -> moment
3    ;; Universal time of critical moment on or after date
4    ;; according to the Malayali rule
5    (+ (hindu-sunrise date)
6       (* 3/5 (- (hindu-sunset date) (hindu-sunrise date)))))

1  (defun madras-critical (date)
2    ;; TYPE fixed-date -> moment
3    ;; Universal time of critical moment on or after date
4    ;; according to the Madras rule
5    (+ (hindu-sunset date)
6       (* 1/2 (- (hindu-sunrise (1+ date)) (hindu-sunset date)))))

1  (defconstant sidereal-start
2    (precession (universal-from-local
3                  (mesha-samkranti (ce 285))
4                  hindu-locale)))

1  (defun sidereal-solar-longitude (tee)
2    ;; TYPE moment -> angle
3    ;; Sidereal solar longitude at moment tee
4    (mod (+ (solar-longitude tee)
5            (- (precession tee))
6            sidereal-start)
7         360))

1  (defun sui-month-start-on-or-after (date)
2    ;; TYPE fixed-date -> fixed-date
3    ;; Fixed date of start of Chinese month containing solstice
4    ;; occurring on or after date.
5    (let* ((sun (universal-from-standard
6                  (floor (standard-from-universal
7                           (solar-longitude-after (deg 270) date)
```

```
8                         (chinese-location date)))
9                    (chinese-location date)))
10           (moon (lunar-phase-at-or-before (deg 0) (1+ sun))))
11       (floor (standard-from-universal moon (chinese-location date)))))
```

```
1   (defun sidereal-solar-longitude-after (phi tee)
2     ;; TYPE (season moment) -> moment
3     ;; Moment UT of the first time at or after tee
4     ;; when the sidereal solar longitude will be phi degrees.
5     (let* ((varepsilon 1d-5) ; Accuracy of solar-longitude.
6            (rate ; Mean days for 1 degree change.
7             (/ average-year-length (deg 360)))
8            (tau ; Estimate (within 5 days).
9             (+ tee
10               (* rate
11                  (mod (- phi (sidereal-solar-longitude tee)) 360))))
12           (a (max tee (- tau 5))) ; At or after tee.
13           (b (+ tau 5)))
14       (binary-search ; Bisection search.
15        l a
16        u b
17        x (< (mod (- (sidereal-solar-longitude x) phi) 360)
18             (deg 180l0))
19        (< (- u l) varepsilon))))
```

```
1   (defun hindu-lunar-phase-at-or-before (phi tee)
2     ;; TYPE (phase moment) -> moment
3     ;; Moment UT of the last time at or before tee
4     ;; when the Hindu lunar-phase was phi degrees.
5     (let* ((varepsilon (expt 2 -17)) ; Accuracy.
6            (tau ; Estimate.
7             (- tee
8                (* hindu-synodic-month 1/360
9                   (mod (- (hindu-lunar-phase tee) phi) 360))))
10           (a (- tau 2))
11           (b (min tee (+ tau 2)))) ; At or before tee.
12       (binary-search ; Bisection search.
13        l a
14        u b
15        x (< (mod (- (hindu-lunar-phase x) phi) 360)
16             (deg 180l0))
17        (< (- u l) varepsilon))))
```

```
1   (defun hindu-lunar-month-on-or-after (m date)
2     ;; TYPE (hindu-lunar-month fixed-date) -> fixed-date
3     ;; Fixed date of first lunar moon on or after fixed date.
4     (let* ((lambda (* (1- m) (deg 30)))
5            (sun (hindu-solar-longitude-after lambda date))
6            (moon (hindu-lunar-phase-at-or-before (deg 0) sun))
7            (date (floor moon)))
```

```
8      (if (<= moon (hindu-sunrise date))
9          date (1+ date))))

1  (defun hindu-sunrise (date)
2    ;; TYPE fixed-date -> moment
3    ;; Geometrical sunrise at Hindu locale on date.
4    (dawn date hindu-locale (deg 0)))
```

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