

Konig's Lemma

- A TREE IS FINITE (HAS FINITELY MANY EDGES)

IF AND ONLY IF

- ALL NODES HAVE FINITE DEGREE

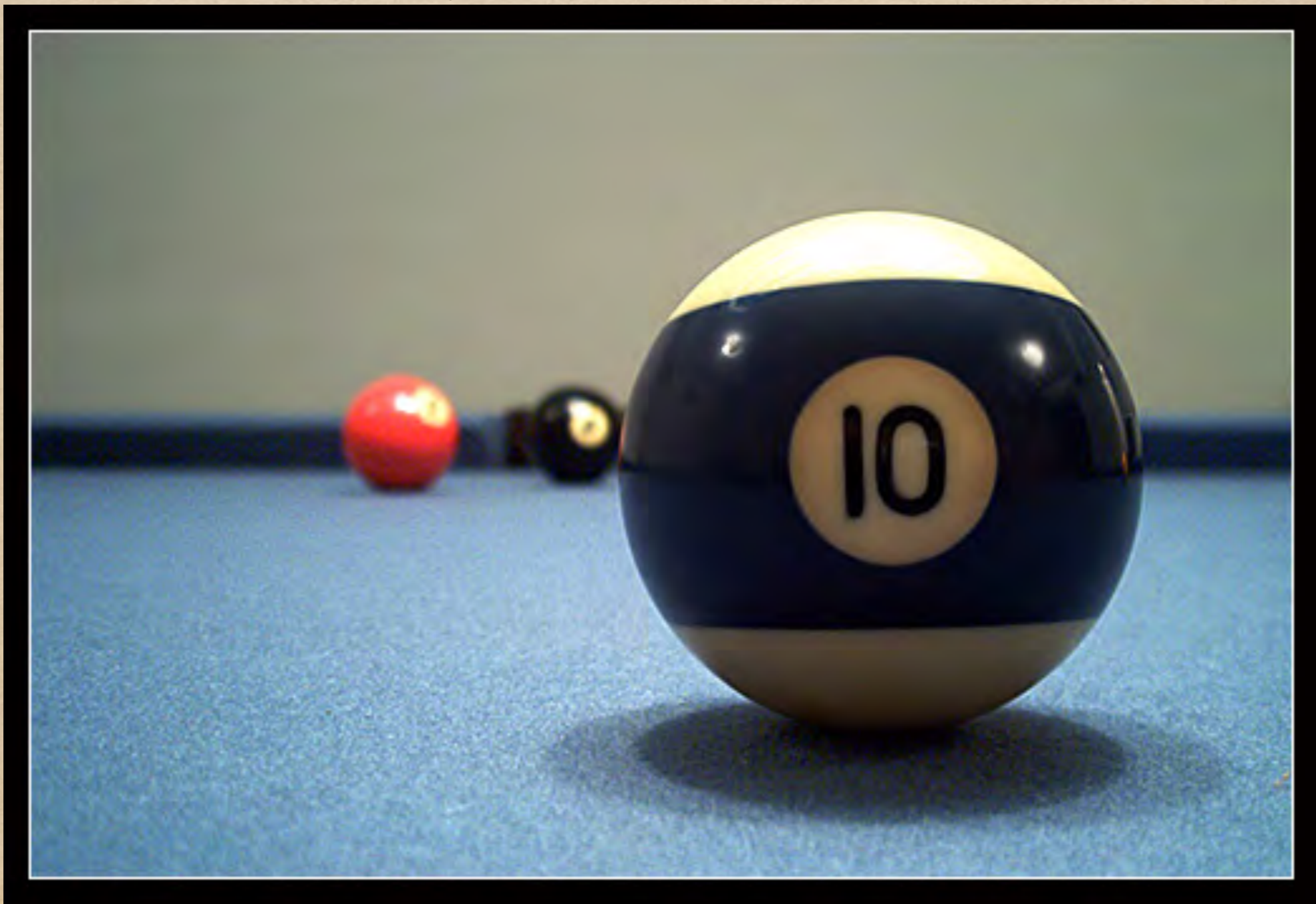
AND

- ALL BRANCHES (SIMPLE PATHS) HAVE

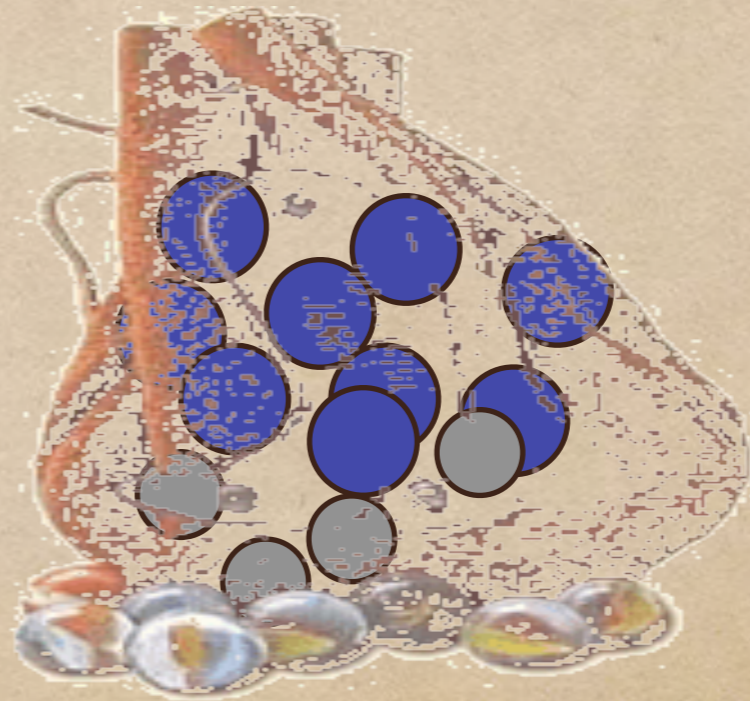
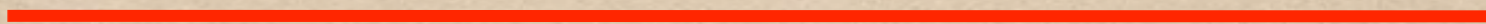
Billiards



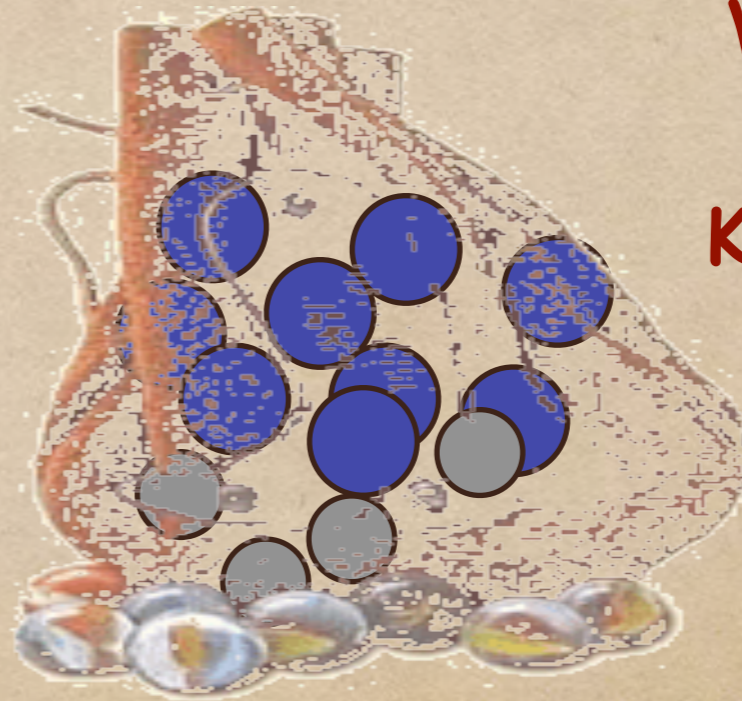
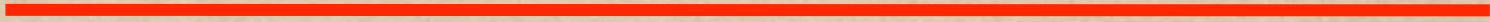
Smullyan's Billiards



Multiset (Bag) Ordering



Multiset (Bag) Ordering



Well-founded
by
König's Lemma

HARDER A(m,n)

```
t := 1
s[t] := m
loop
  c := c + 1
  m := s[t]
  t := t - 1
  if m = 0
  then
    n := n + 1
  elseif n = 0
  then
    t := t + 1
    s[t] := m - 1
    n := 1
  else
    t := t + 2
    s[t-1] := m - 1
    s[t] := m
    n := n - 1
until t = 0
```

BAG OF PAIRS

$(S[I], \infty) \quad I < T$

$(S[T], N)$

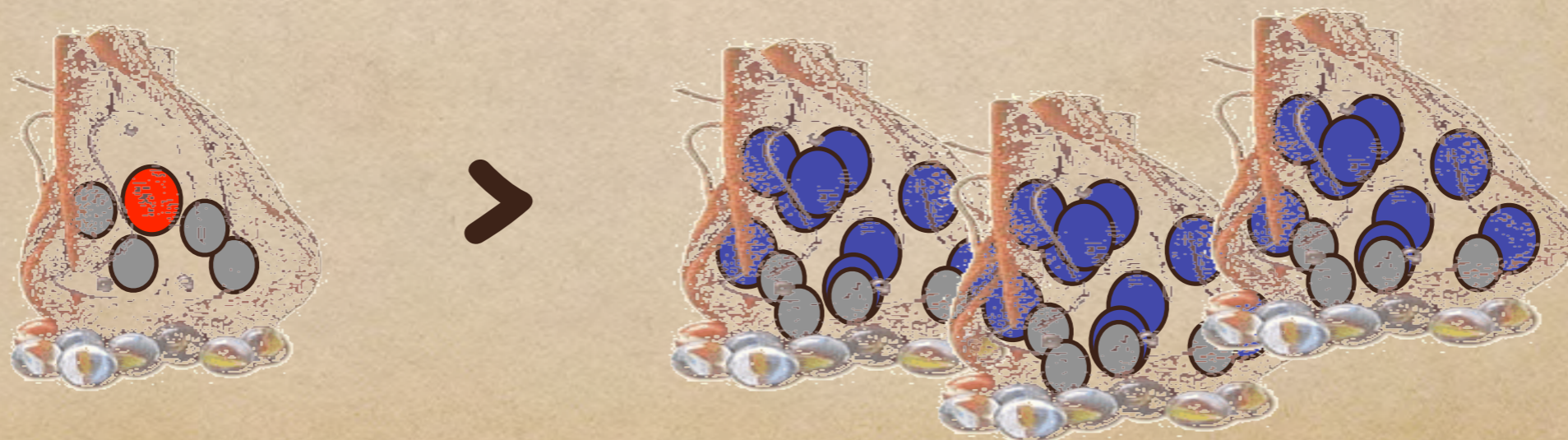


Nested Matryoshka Dolls

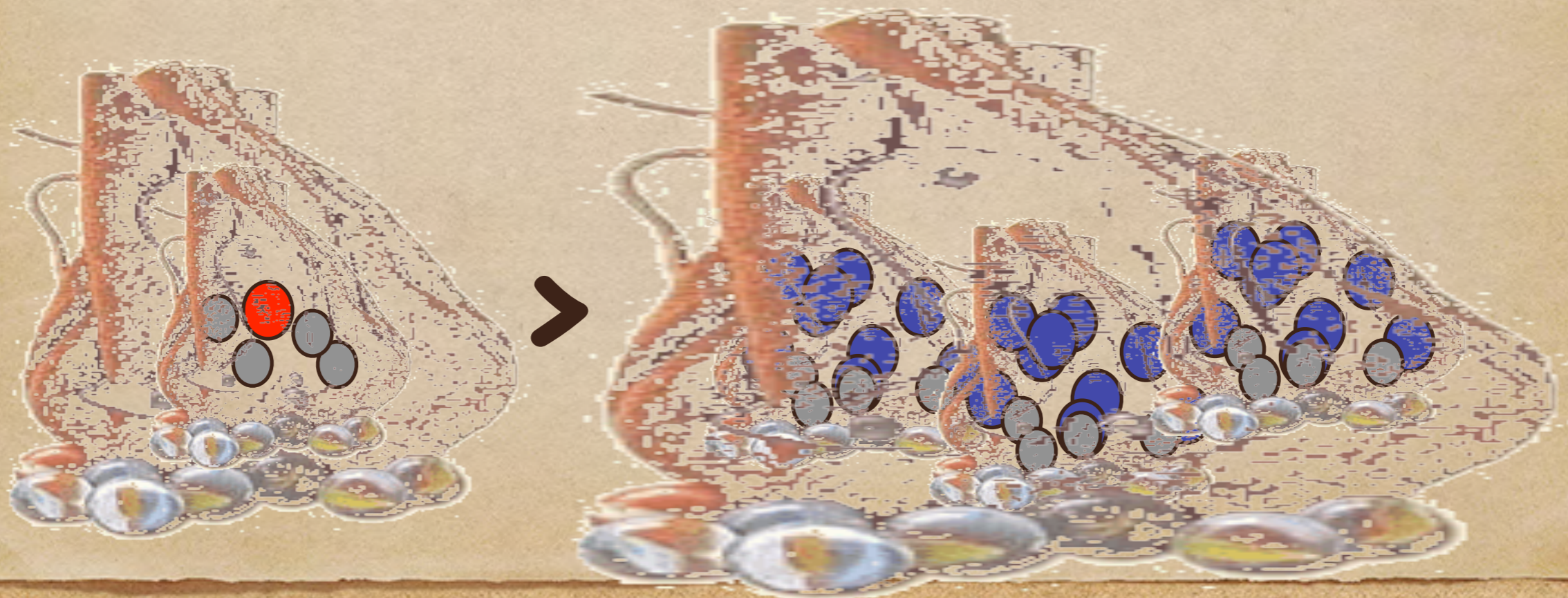


Nested Bags

Nested Ordering



Nested Ordering



Hydra





Hercules' Second Labor

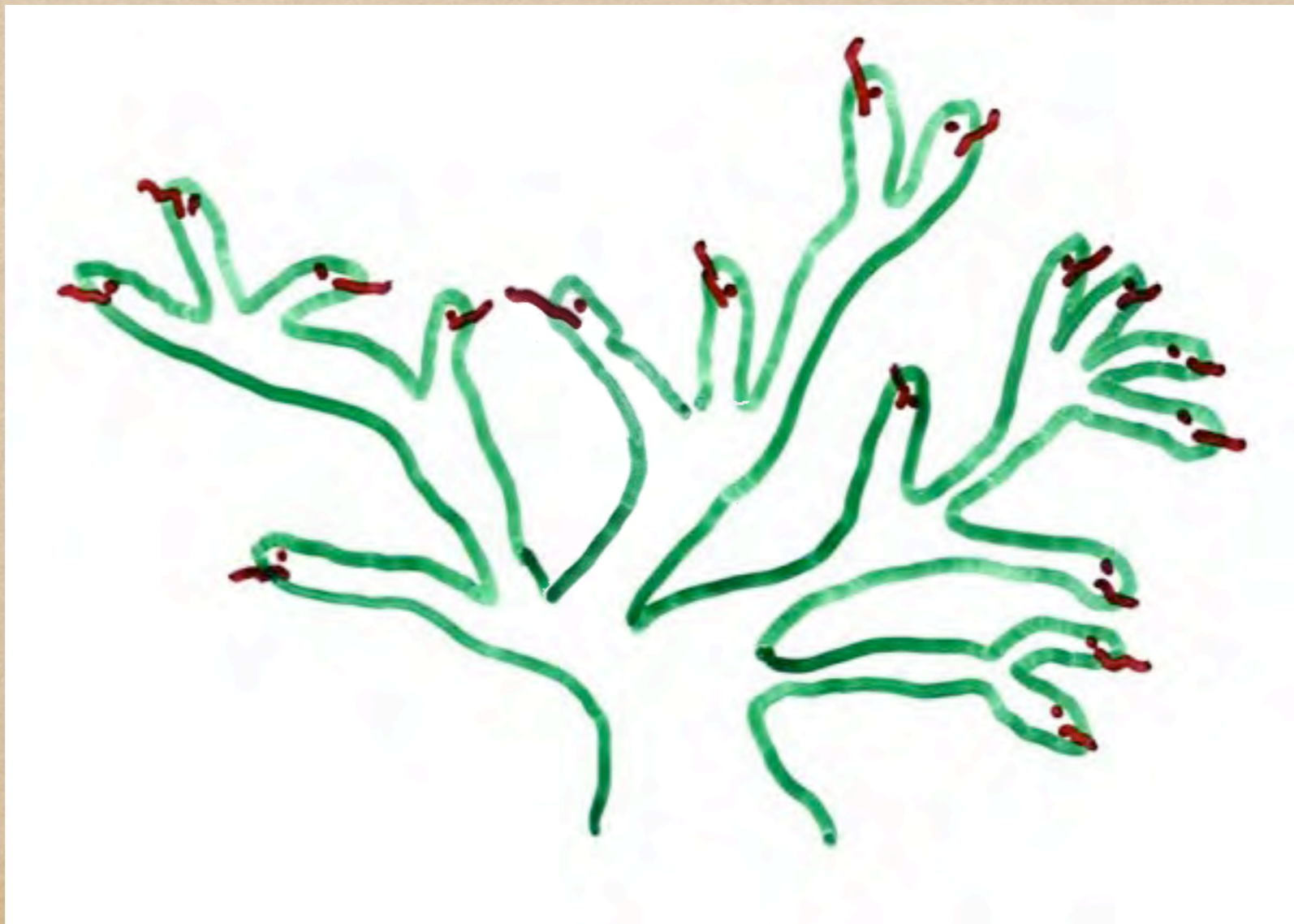


Each time Hercules bashed one of Hydra's heads, Iolaus held a torch to the headless neck.

After destroying eight mortal heads, Hercules chopped off the ninth, immortal head, which he buried at the side of the road from Lerna to Elaeus, and covered with a heavy rock.



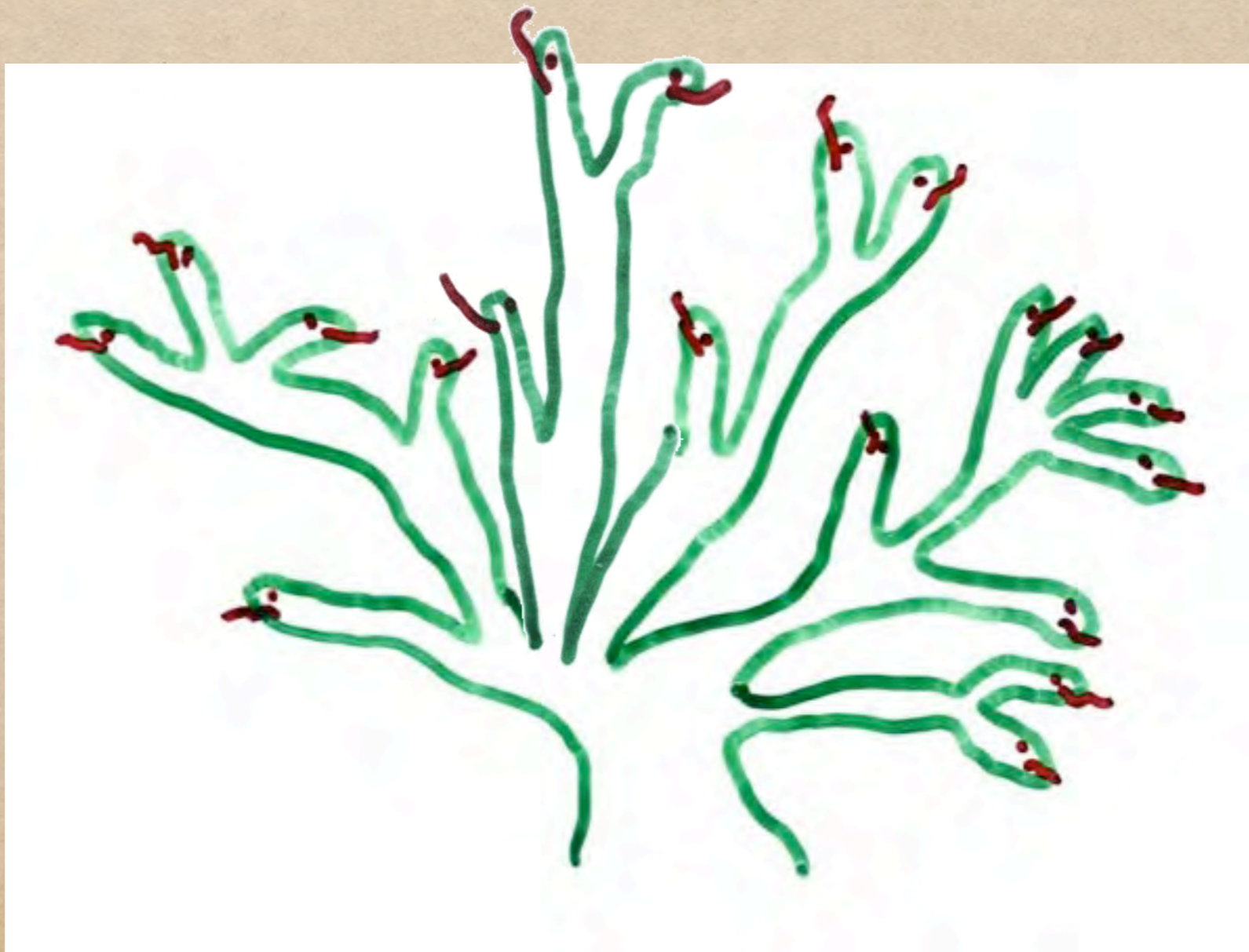
Hydra vs. Hercules



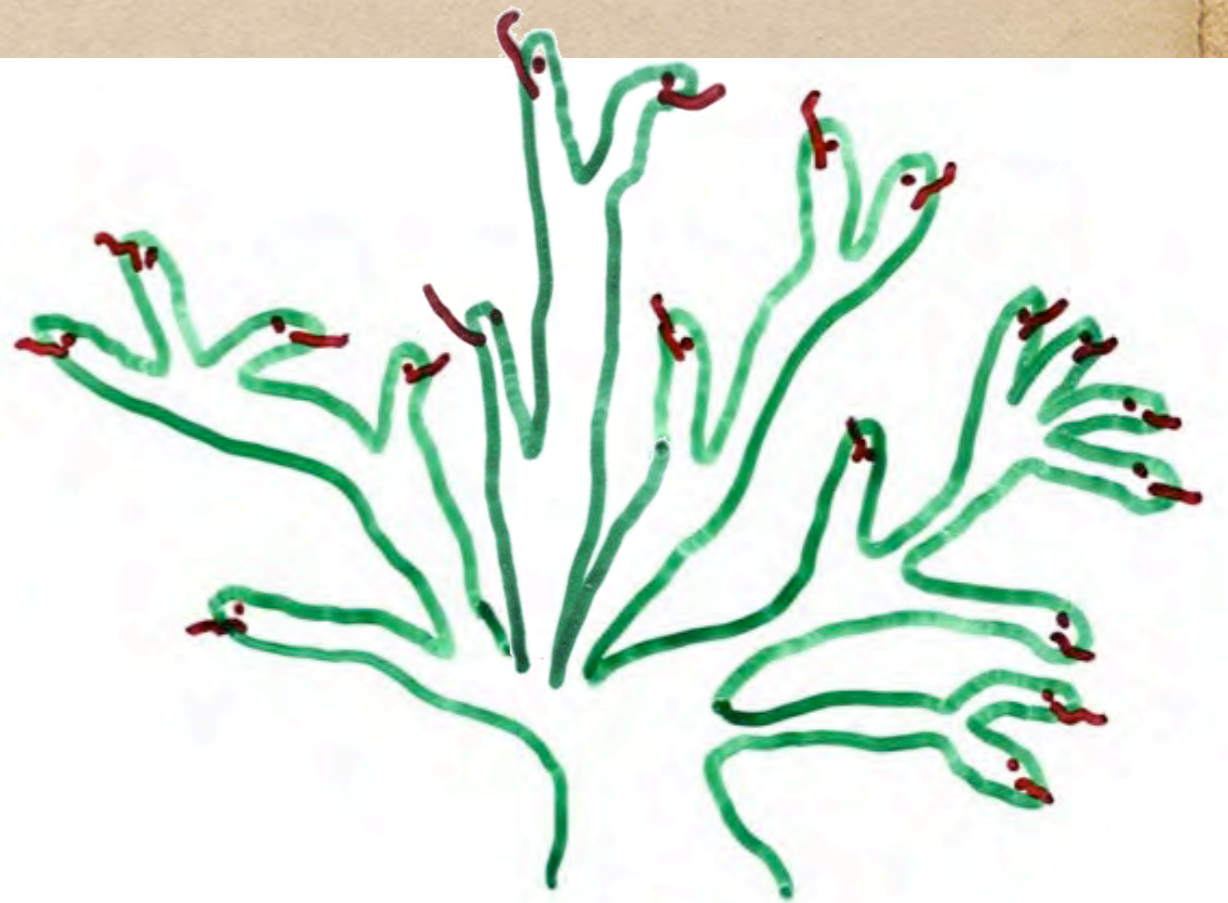
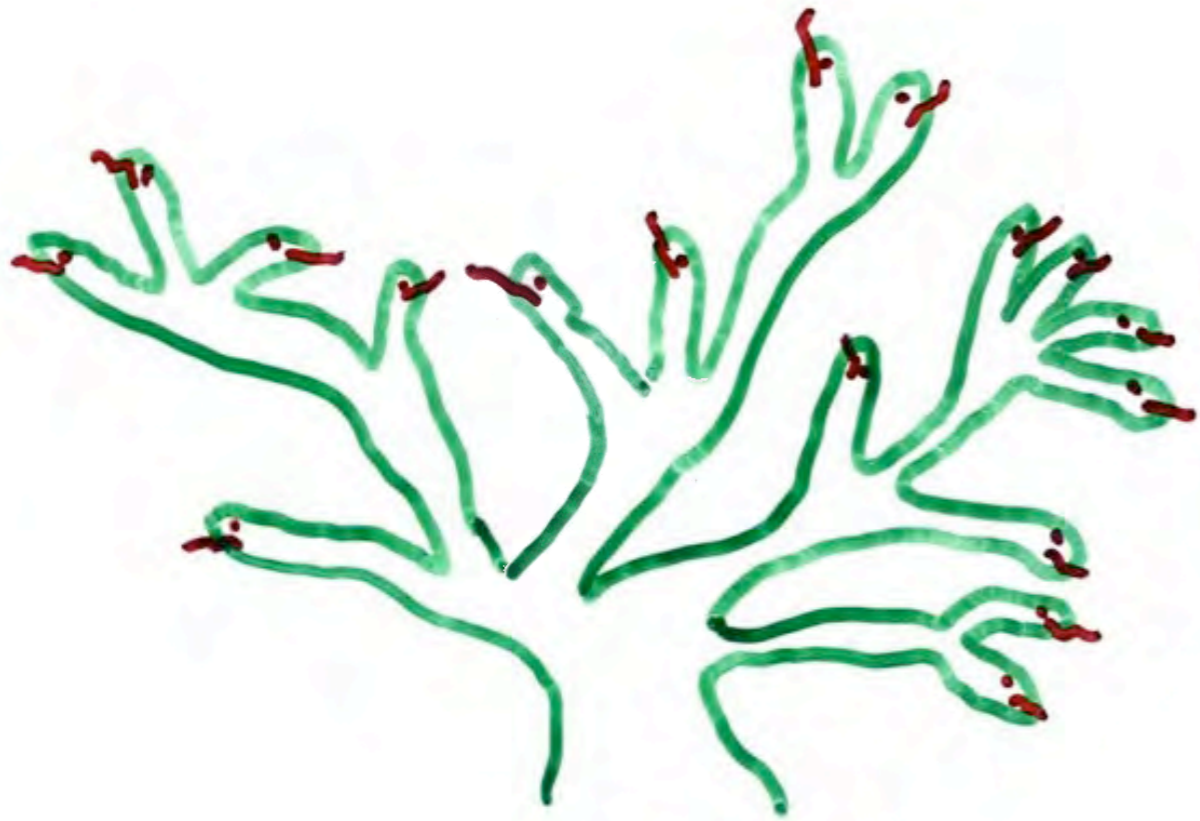
Hydra vs. Hercules



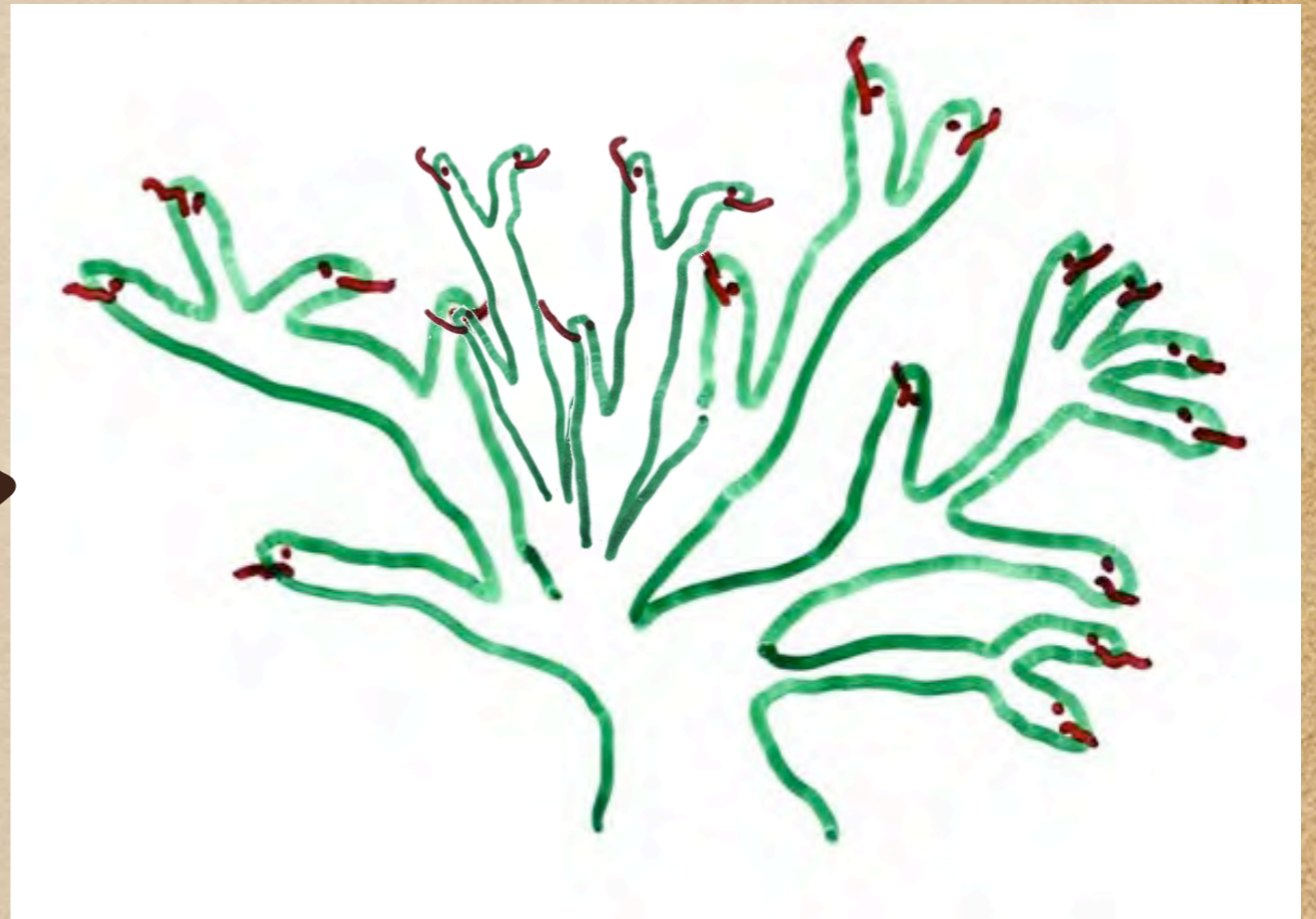
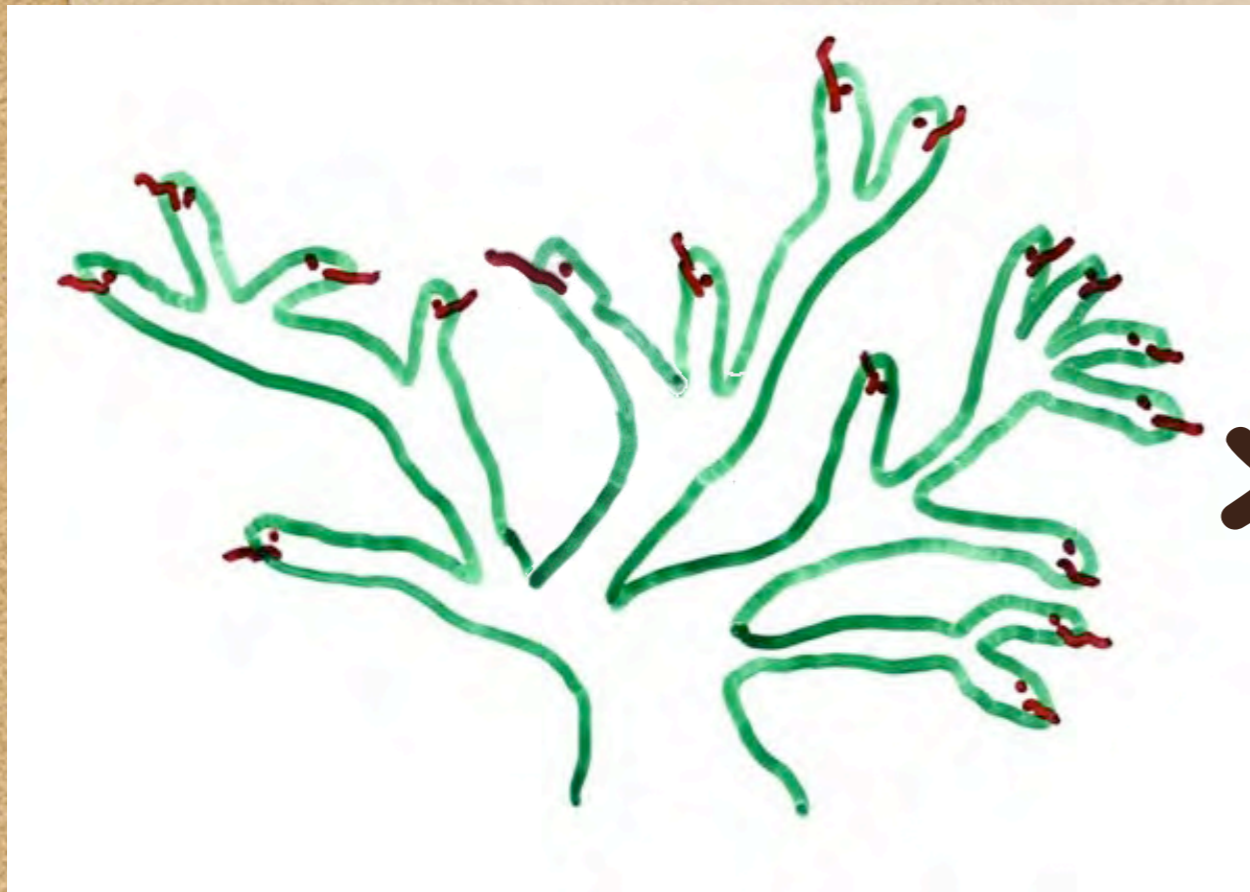
Hydra vs. Hercules



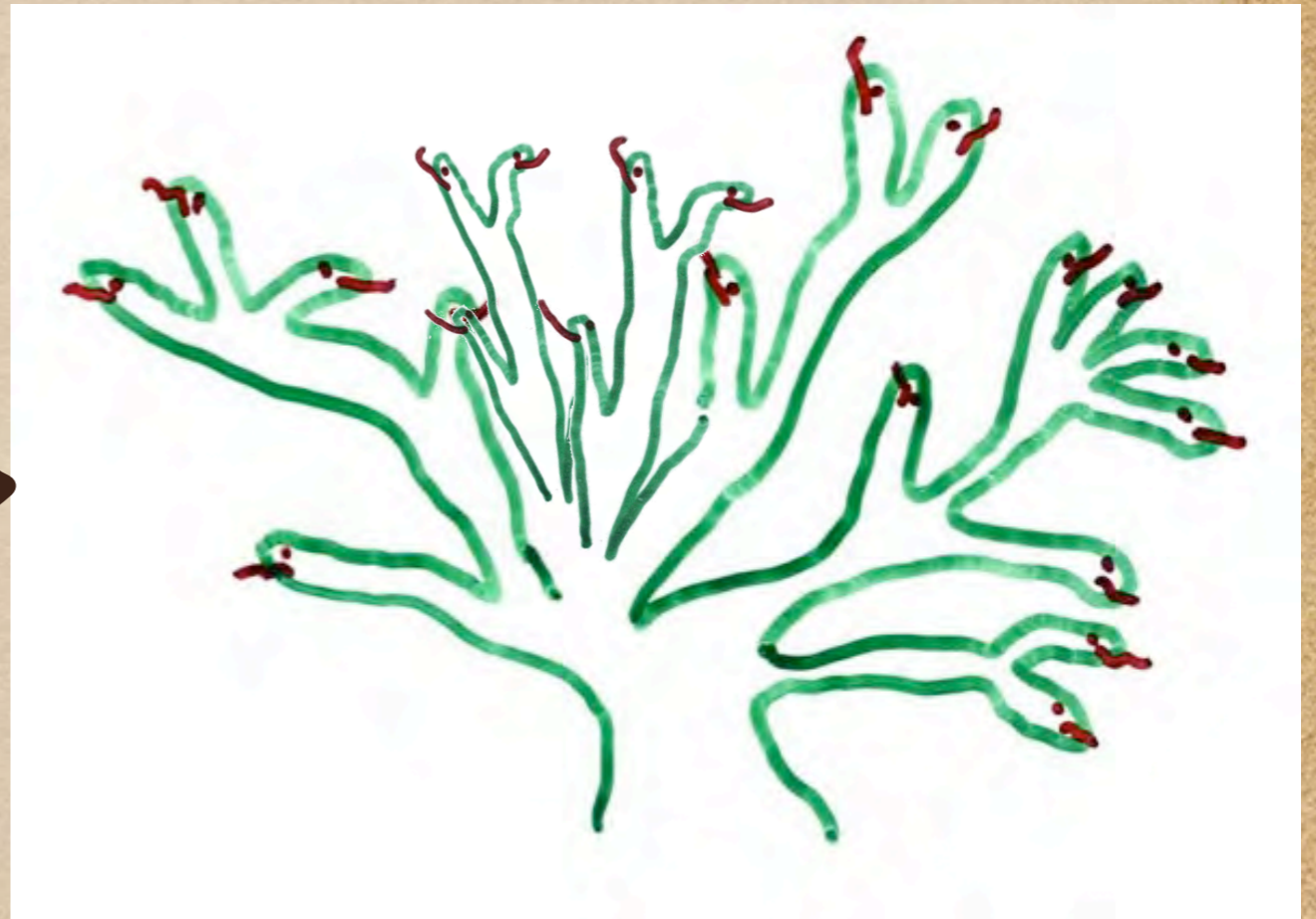
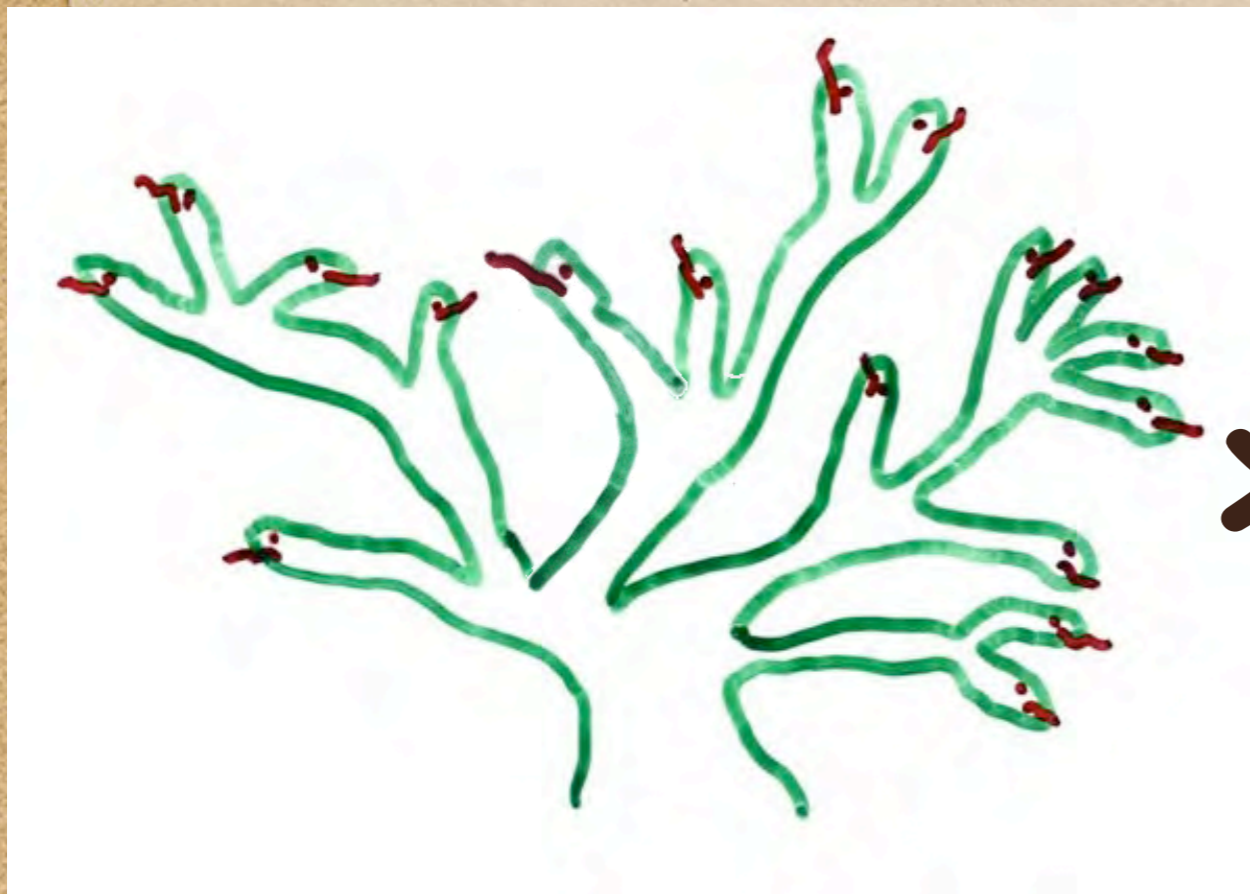
Hercules > Hydra



Hercules > Hydra



Hercules > Hydra

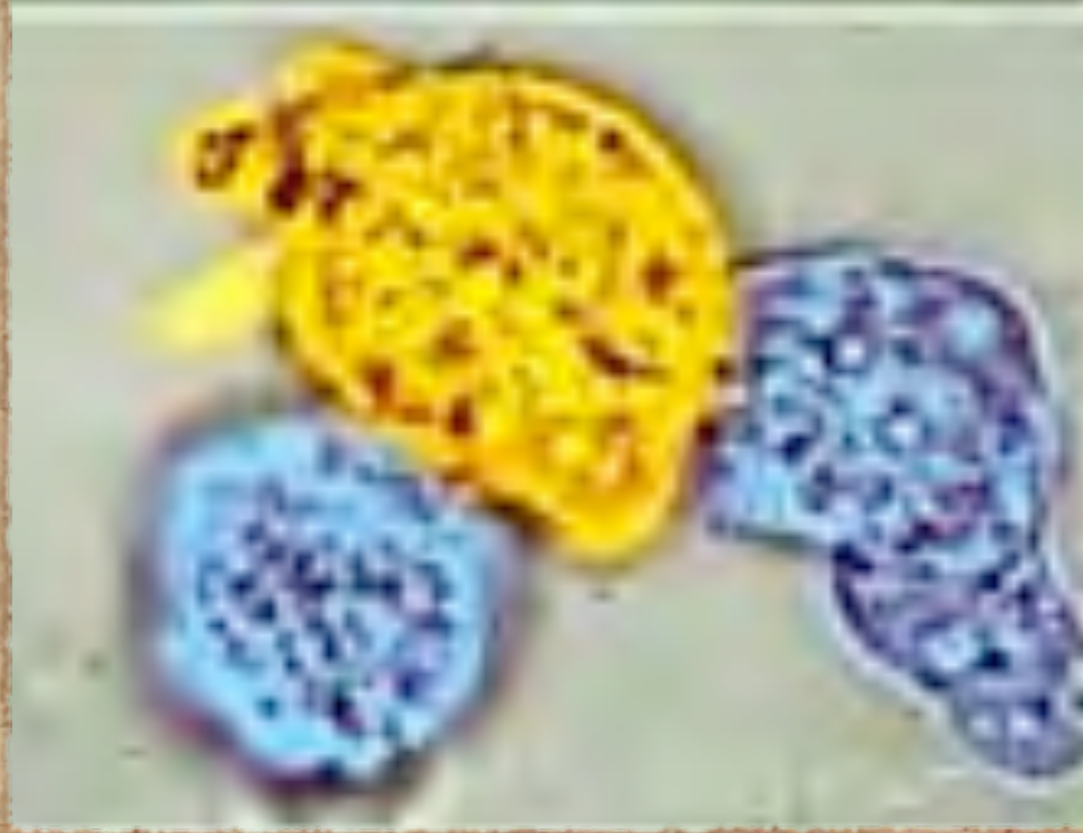
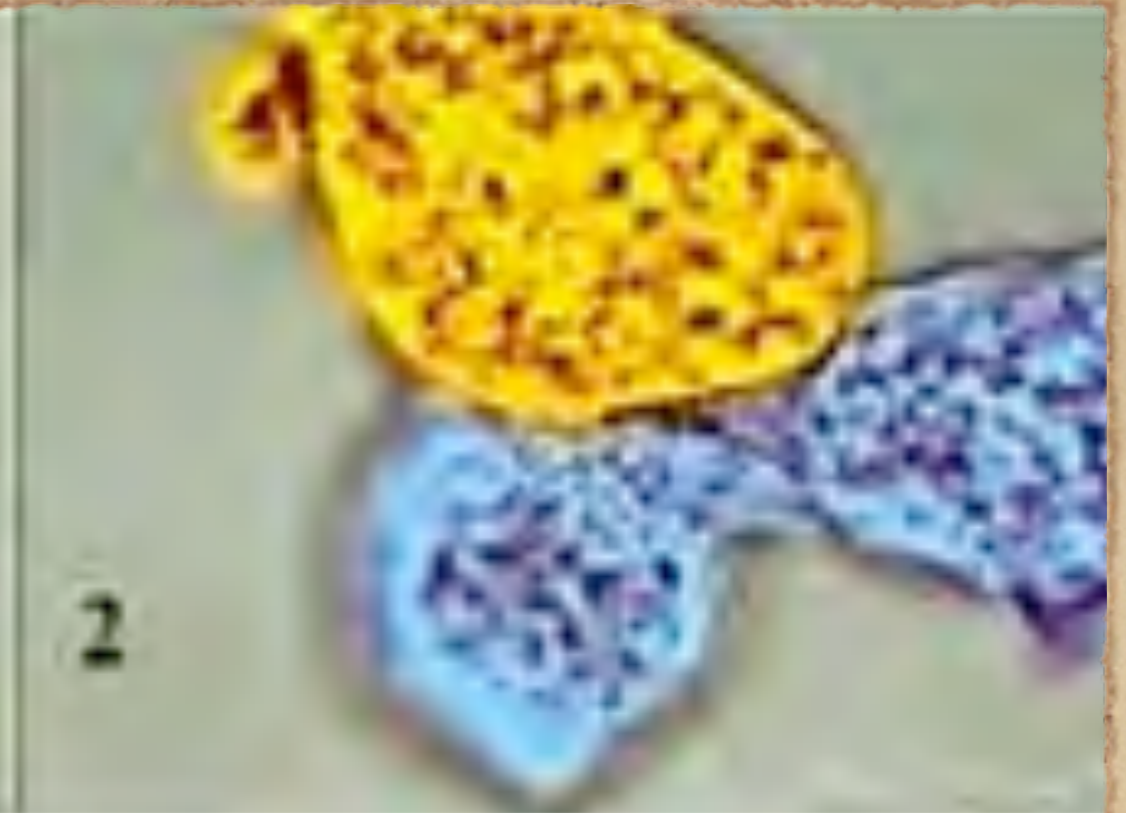


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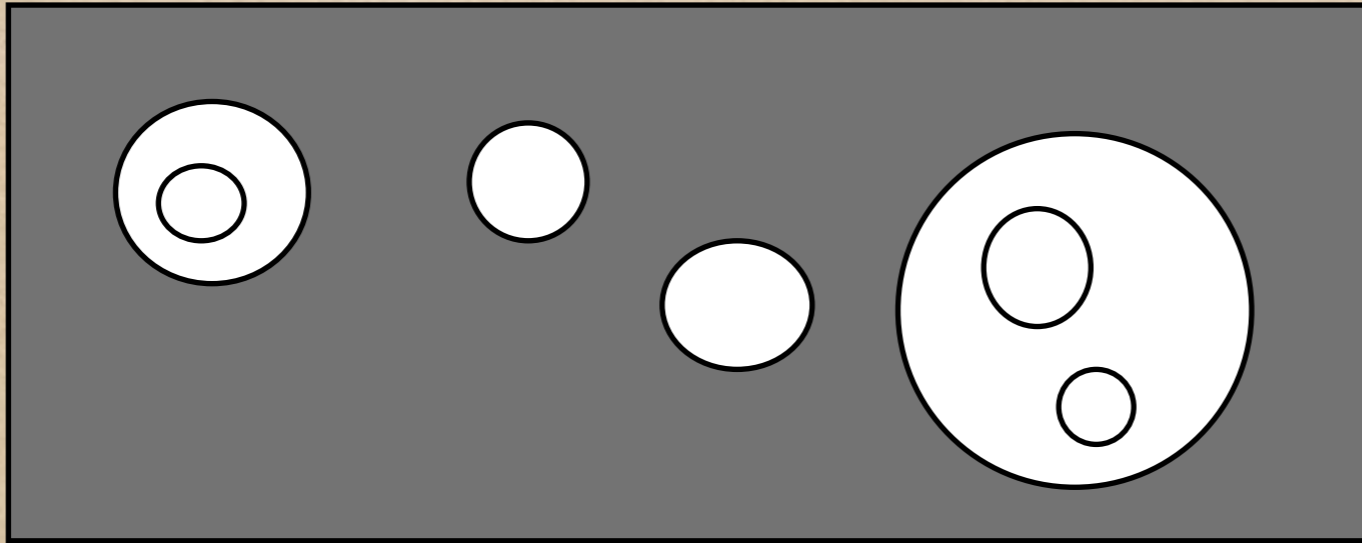
Hercules Defeats Hydra

- Cannot be proved in Peano Arithmetic
[Paris & Kirby]
- Requires induction up to ε_0
 - ◆ Natural numbers do not suffice
 - ◆ Sophisticated variants require more powerful systems [Friedman]

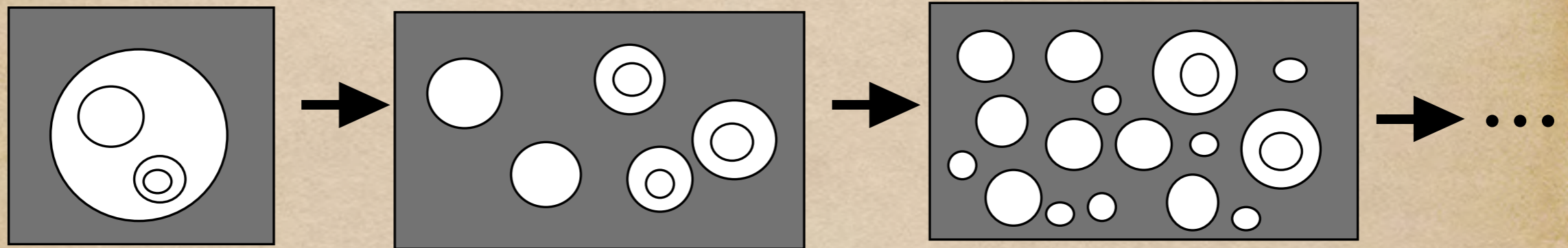


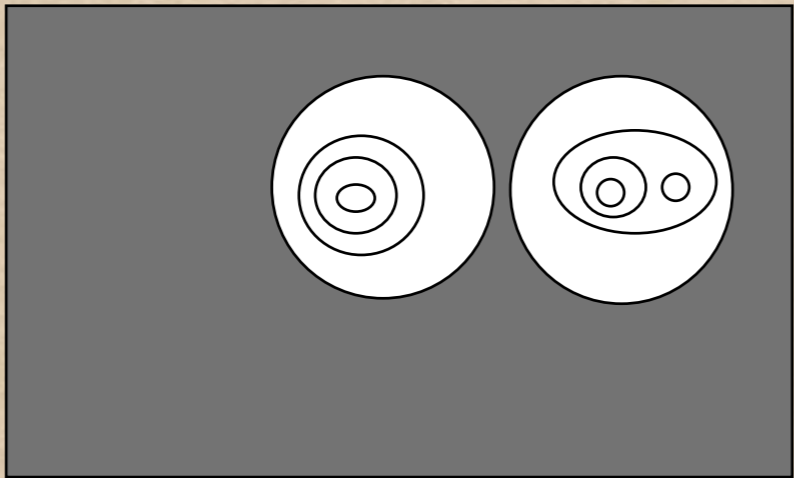


Amoebae

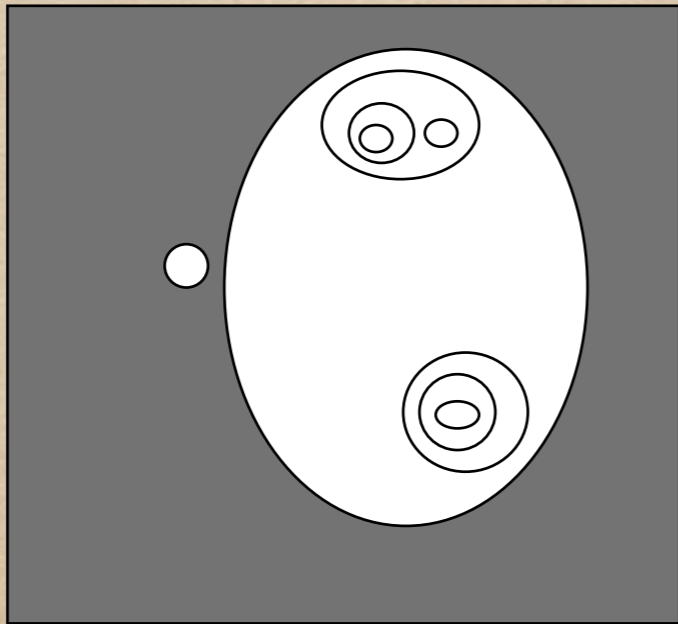
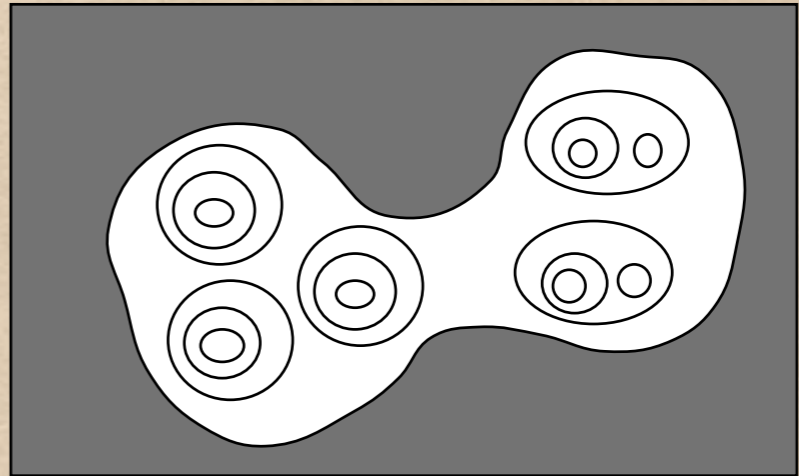


Amoebae





→
fusion



→
fusion

