

# **The Equivalence of Contests**

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# Some basic terms

- 2-player contest
  - $i, j$  are the players
  - $x_i, x_j$  are their efforts for winning
- Contest designer (e.g. government, company,...)
- Lottery contest success function:

$$P_i(x_i, x_j) = \begin{cases} x_i / (x_i + x_j) & \text{if } x_i + x_j \neq 0 \\ 1/2 & \text{if } x_i = x_j = 0 \end{cases}$$

# Some basic terms (cont'd)

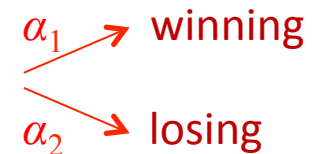
- Payoff function:

$$\pi_i(x_i, x_j) = \begin{cases} W + \alpha_1 x_i + \beta_1 x_j & \text{with prob. } P_i(x_i, x_j) \\ L + \alpha_2 x_i + \beta_2 x_j & \text{with prob. } 1 - P_i(x_i, x_j) \end{cases}$$

$\{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\}$  is the parameter space

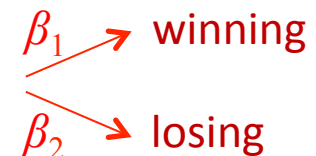
- $W > L \geq 0$  are the winning/losing prizes
- $\alpha_1, \alpha_2$  are the cost parameters

Reflect the effect of own effort  $x_i$  on payoff  $\pi_i$



- $\beta_1, \beta_2$  are the spillover parameters

Reflect the effect of rival's effort  $x_j$  on payoff  $\pi_i$



# Example – Tullock-type contest

- $\{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\} =$   
 $\{W, 0, -1, -1, 0, 0\}$

- Payoff function

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with prob. } P_i(x_i, x_j) \\ -x_i & \text{with prob. } 1 - P_i(x_i, x_j) \end{cases}$$

# Some basic terms (cont'd)

- Expected Payoff:

$$E(\pi_i(x_i, x_j)) = \frac{x_i}{x_i + x_j} (W + \alpha_1 x_i + \beta_1 x_j) + \frac{x_j}{x_i + x_j} (L + \alpha_2 x_i + \beta_2 x_j)$$

maximize with respect to  $x_i$  :

- Best Response Function:

$$x_i^{BRF} = -x_j + \sqrt{\frac{\{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)\}x_j^2 - (W - L)x_j}{\alpha_1}}$$

enforce symmetric equilibrium ( $x_i^* = x_j^*$ ) + some restrictions on parameters:

- Equilibrium efforts:

$$x_i^* = x_j^* = \frac{W - L}{-(3\alpha_1 + \alpha_2) - (\beta_1 - \beta_2)}$$

# Main theme of the paper

- **Different** contests may be **equivalent** in some aspects:

- Strategically equivalent

same family of BRF

$$x_i^{BRF}$$

- Revenue equivalent

same revenue (for the contest designer)

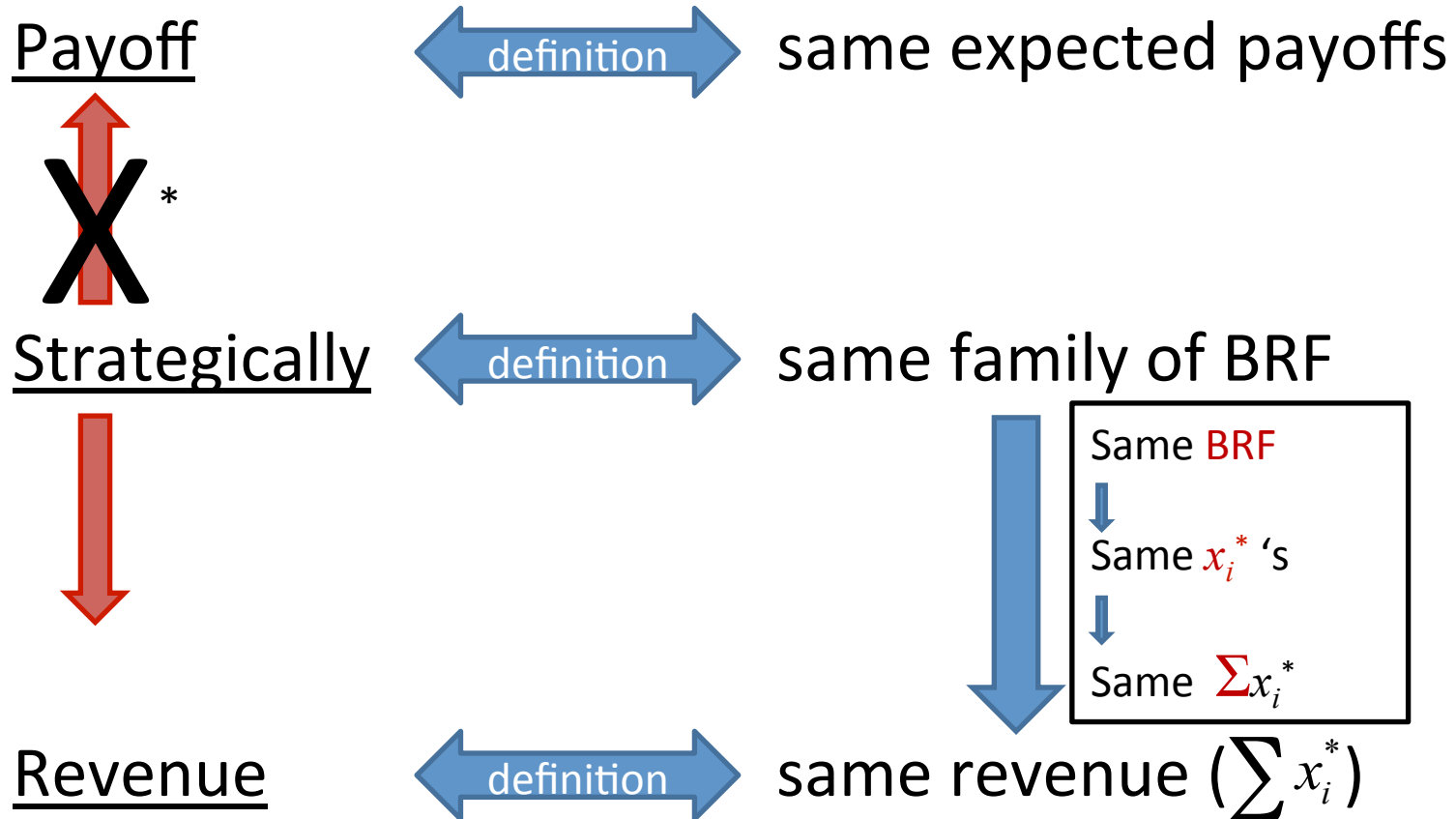
$$\sum x_i^*$$

- Payoff equivalent

same expected payoffs (for players)

$$E(\pi_i(x_i, x_j))$$

# Relation between equivalences



\* If 2 contests are strategically eq. but not payoff eq., contest designer may prefer one contest over the other. This is known as Pareto improvement:  
 A change in the allocation of a resource to a set of individuals that is an improvement for at least one and no worse for any other

# Algorithm for Finding equivalences

Given a contest, to find strategically (→ revenue) equivalent contests:

Step1 Express BRF for the contest  $x_i^{BRF} = \dots$

Step2 Derive **restrictions** on parameters, which generate the **same BRF**

$$x_i^{BRF} = -x_j + \sqrt{\frac{\{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)\}x_j^2 - (W - L)x_j}{\alpha_1}}$$



# Tullock-type contest revisited

- $\{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\} =$   
 $\{W, 0, -1, -1, 0, 0\}$

- Payoff function

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with prob. } P_i(x_i, x_j) \\ -x_i & \text{with prob. } 1 - P_i(x_i, x_j) \end{cases}$$

- BRF:

$$x_i^{BRF} = -x_j + \sqrt{Wx_j}$$

- When is a contest strategically equivalent to  $\{W, 0, -1, -1, 0, 0\}$ ?

# Equivalence to Tullock-type contest

When is a contest strategically equivalent to  $\{W, 0, -1, -1, 0, 0\}$ ?

$$\{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\} \stackrel{?}{=} \{W, 0, -1, -1, 0, 0\}$$

$$x_i^{BRF} = x_i^{BRF}$$

$$x_i^{BRF} = -x_j + \sqrt{\frac{\{(\alpha_1 - \alpha_2) - (\beta_1 - \beta_2)\}x_j^2 - (W - L)x_j}{1}} = -x_j + \sqrt{Wx_j}$$

0 α<sub>1</sub> W

$$\left. \begin{aligned} W - L &= W \\ \alpha_1 &= -1 \\ \beta_2 - \beta_1 - \alpha_2 &= 1 \end{aligned} \right\}$$

restrictions on parameters,  
which generate the same BRF

# Example 1

Some interesting cases of Tullock  $\{W, 0, -1, -1, 0, 0\}$  equivalence:

- $\{W + \Delta, \Delta, -1, -1, 0, 0\}$

$$\pi_i(x_i, x_j) = \begin{cases} W + \Delta - x_i & \text{with prob. } P_i(x_i, x_j) \\ \Delta - x_i & \text{with prob. } 1 - P_i(x_i, x_j) \end{cases}$$

Conclusion: the same  $\Delta$  in the winning and losing prize does **not** affect **equilibrium efforts**  $x_i^*$ .

Use: the contest designer may wish to **change the prizes values** while ensuring no change in behavior of players.

# Example 2

Some interesting cases of Tullock  $\{W, 0, -1, -1, 0, 0\}$  equivalence:

- $\{W, 0, -1, 0, -1, 0\}$  that is winner reimburses loser:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i - x_j & \text{with prob. } P_i(x_i, x_j) \\ 0 & \text{with prob. } 1 - P_i(x_i, x_j) \end{cases}$$

(Also payoff equivalent)

Conclusion: if winner reimburses loser, this affects neither **equilibrium efforts**, nor **payoffs**.

Use: a contest where **only the winner expends** resources may be simpler to implement.

# Example 3

Some interesting cases of Tullock  $\{W, 0, -1, -1, 0, 0\}$  equivalence:

- “Input spillover”:  $\{W, 0, -1, -1, \beta, \beta\}$   $\beta \in (-1, 1)$

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i + \beta x_j & \text{with prob. } P_i(x_i, x_j) \\ -x_i + \beta x_j & \text{with prob. } 1 - P_i(x_i, x_j) \end{cases}$$

- Not payoff equivalent:  $\beta > 0 \rightarrow$  payoff  $>$  Tullock

$\beta < 0 \rightarrow$  payoff  $<$  Tullock

Conclusion: symmetric spillovers do **not** affect **equilibrium efforts**

but **do** affect the **payoffs** of players

Use: the contest designer may wish to enhance positive spillovers.

# Contests with complementarities

$E_i$  is the total amount of resources of player  $i$

$x_i$  is the total amount of effort of player  $i$

$\Phi$  is the fraction of resources destroyed during the conflict

## War

$$\{\phi(E_i - x_i), 0, -\phi, -\phi, 0, 0\}$$

$$\pi_i = \begin{cases} \Phi(E_i - x_i) + \Phi(E_j - x_j) & \text{winner} \\ 0 & \text{loser} \end{cases}$$

$$x_i^{BRF} = -x_j + \sqrt{(E_i + E_j)x_j}$$



## Harmful residual

$$\{\phi E_i, -\phi E_j, 0, -\phi, 0, 0, \phi\}$$

$$\pi_i = \begin{cases} \Phi(E_i - x_i) & \text{winner} \\ -\Phi(E_j - x_j) & \text{loser} \end{cases}$$

$\Leftarrow$  *the same*

Skaperdas (1992), Garfinkel and Skaperdas (2000)

Also payoff



# Conditions for equivalence

The **contests shown** assume these **conditions**:

1. **Two**-player contest

2. **Lottery** success function:  $P(x_i, x_j) = \frac{x_i}{x_i + x_j}$

3. **Linear** cost and spillover  $\dots + \alpha x_i + \beta x_j$

4. **Risk** neutrality

When **relaxing** these conditions

(e.g. risk aversion,  $P(x_i, x_j) = \frac{x_i^r}{x_i^r + x_j^r}$ ,  $r \neq 1$ ,  $\dots + \alpha x_i^2 + \beta \sqrt{x_j}$ ),

equivalences shown **may not hold**.

# Conclusions

- Intuitively and structurally different contests can be strategically and revenue equivalent
- Contest designer may replace contest parameter space, keeping the same revenue, while:
  - reducing operational cost.
  - Improving payoffs for contestants
  - Reducing risk for contestants
  - Avoiding regulatory restrictions
- The type of contests shown in the paper cover the majority of contests in the literature.



Thank you !