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Large-Scale Integer Linear Programming for Orientation Preserving 3D Shape Matching

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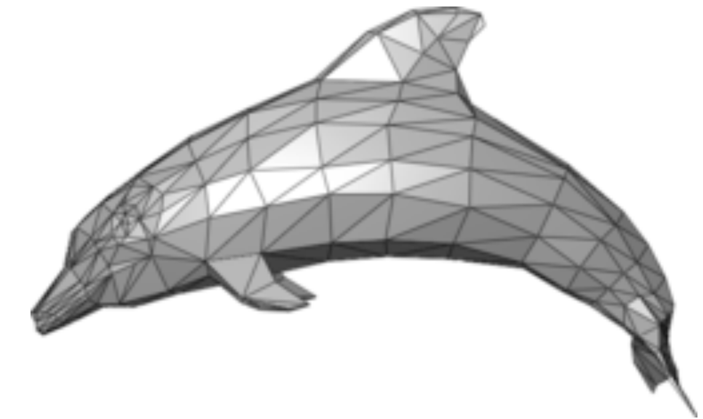
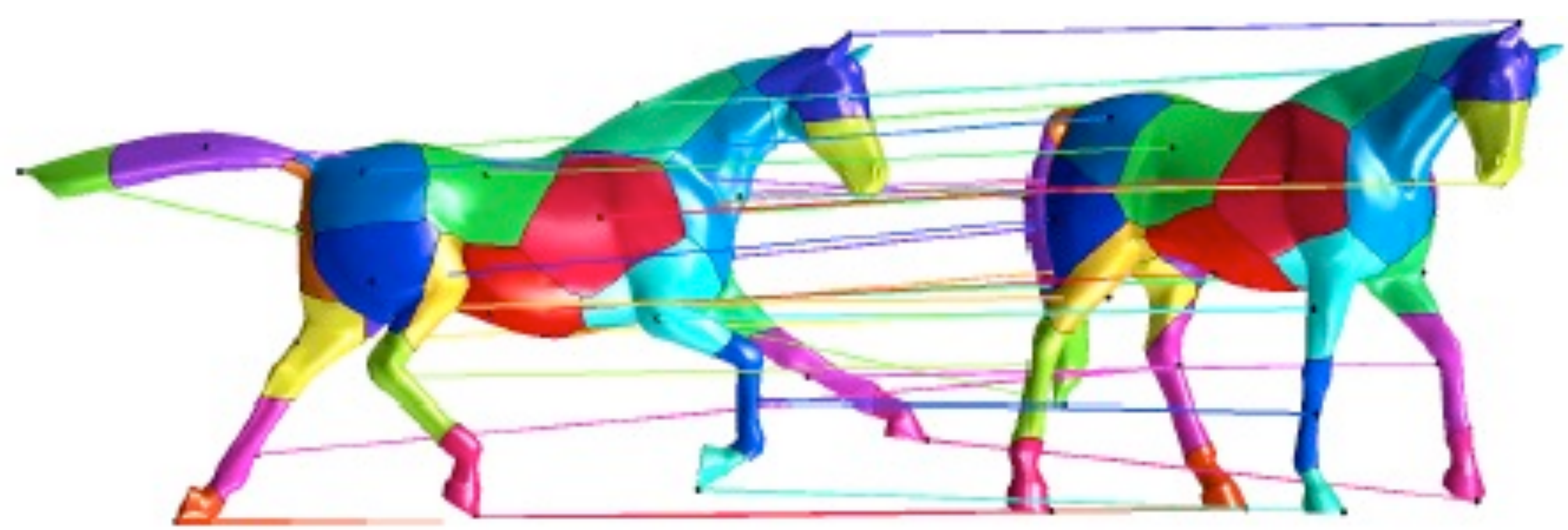
Problem



- Unsupervised non-rigid 3D shape matching
- Meaningful correspondences between:
  - 2 poses of same object
  - 2 objects



Usage

- Relate shapes and their parts
- Fuse different partial scans of single object
- Transfer semantics from one shape to another
- Quantify similarity of shapes
- Interpolate two shapes

Solution

- Match small surface patches, not points
  - Geometric consistency
- Global Optimization
  - Integer Linear Program
- Elastic, non-linear thin-shell energy model
  - Elasticity and bending

Linear Programming

- A **linear function to be maximized**

  e.g. $\max_{x1,x2} f(x_1, x_2) = c_1x_1 + c_2x_2$

- **Problem constraints** of the following form

  e.g.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &\leq b_1 \\ a_{21}x_1 + a_{22}x_2 &\leq b_2 \\ a_{31}x_1 + a_{32}x_2 &\leq b_3 \end{aligned}$$

- **Non-negative variables**

  e.g.

$$\begin{aligned} x_1 \geq 0 \\ x_2 \geq 0 \end{aligned}$$

- **Non-negative right hand side constants**

$$b_i \geq 0, \ i = 1, 2, 3$$

The problem is usually expressed in *matrix form*, and then becomes:

$$\max\{c^{\mathrm{T}}x \mid 0 \leq Ax \leq b \wedge x \geq 0\}$$

- Widely used
- Polynomial time

Integer LP

- Unknown variables are *integers*
- NP-hard (in general)
- Binary LP and mixed LP are also NP-hard

Problem statement

$$\min_{\Gamma \in \{0,1\}^{|F|}} \quad E^t \cdot \Gamma$$

$$\text{subject to} \quad \begin{pmatrix} \partial \\ \pi_X \\ \pi_Y \end{pmatrix} \cdot \Gamma = \begin{pmatrix} \mathbf{0}_{|E|} \\ \mathbf{1}_{|F_X|} \\ \mathbf{1}_{|F_Y|} \end{pmatrix} .$$

LP relaxation

- Binary LP is NP-hard
- Relaxed LP is polynomial
  - Relax $\Gamma \in \{0,1\}^{|F|}$ to $\Gamma \in [0,1]^{|F|}$
- But this is not enough

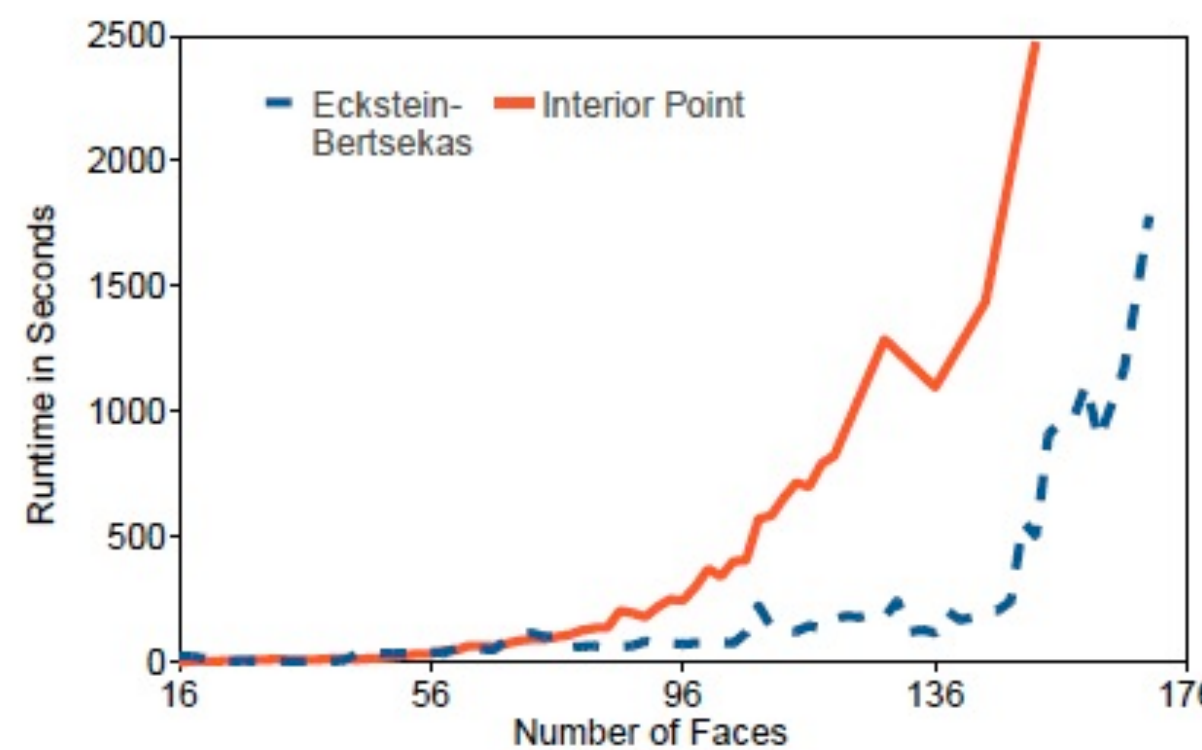
Iterative scheme

- Solve relaxed problem
- Fix the variables with values above 0.5 to 1 (if none is above 0.5, then fix highest one)

- Usually converges to binary solution in <10 iterations (never >20 iterations)

GPU acceleration

- Our GPU-based implementation of parallelizable primal-dual alg. by Eckstein-Bertsekas
- compared to Interior Point (from CPLEX)
- up to x100 faster
- linear memory consumption



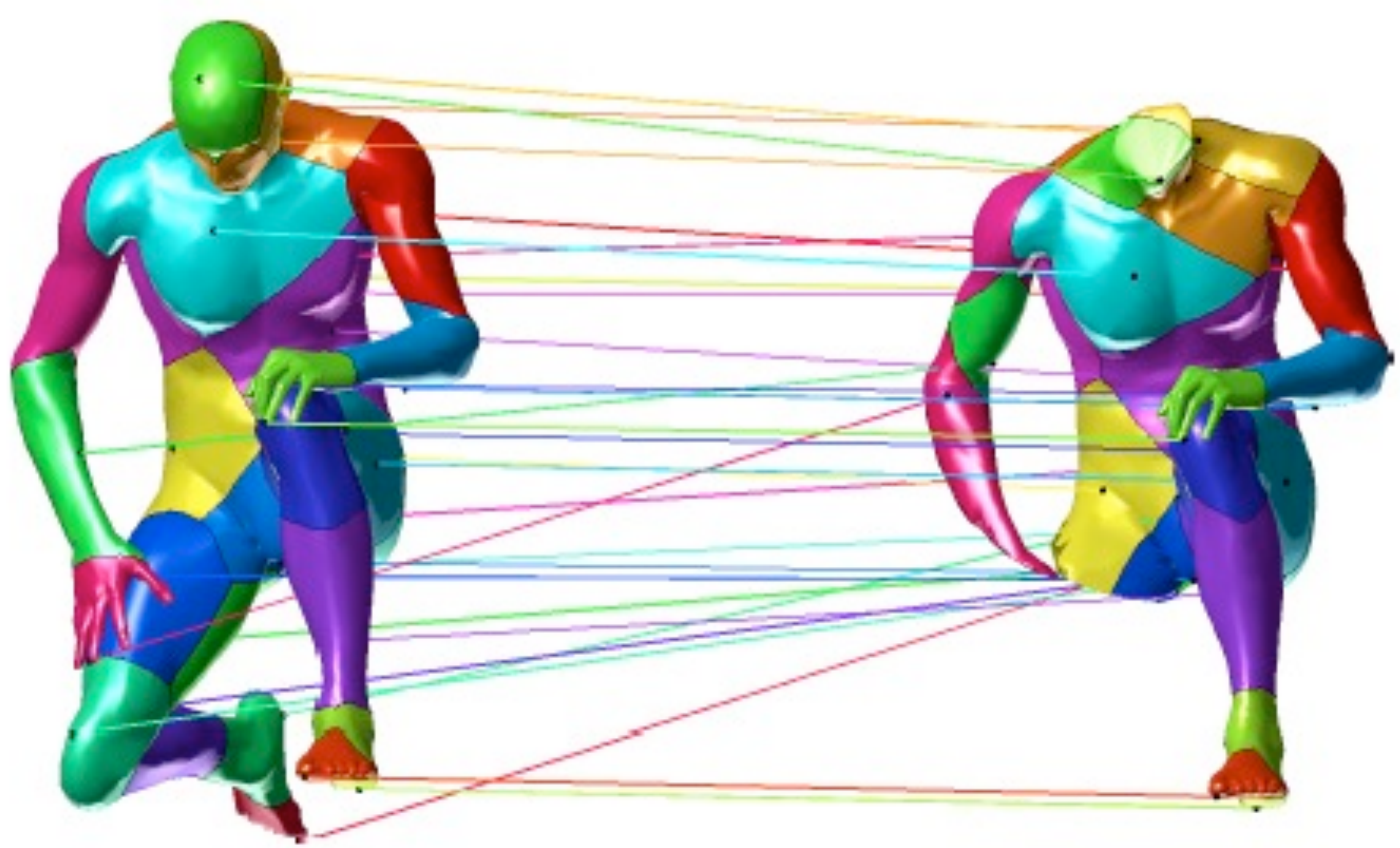
Feature descriptors

- Add Wave Kernel Signatures to the energy function
- Half of the times relaxed solution is binary
- x4 faster on average (less iterations)

with feature descriptor without feature descriptor

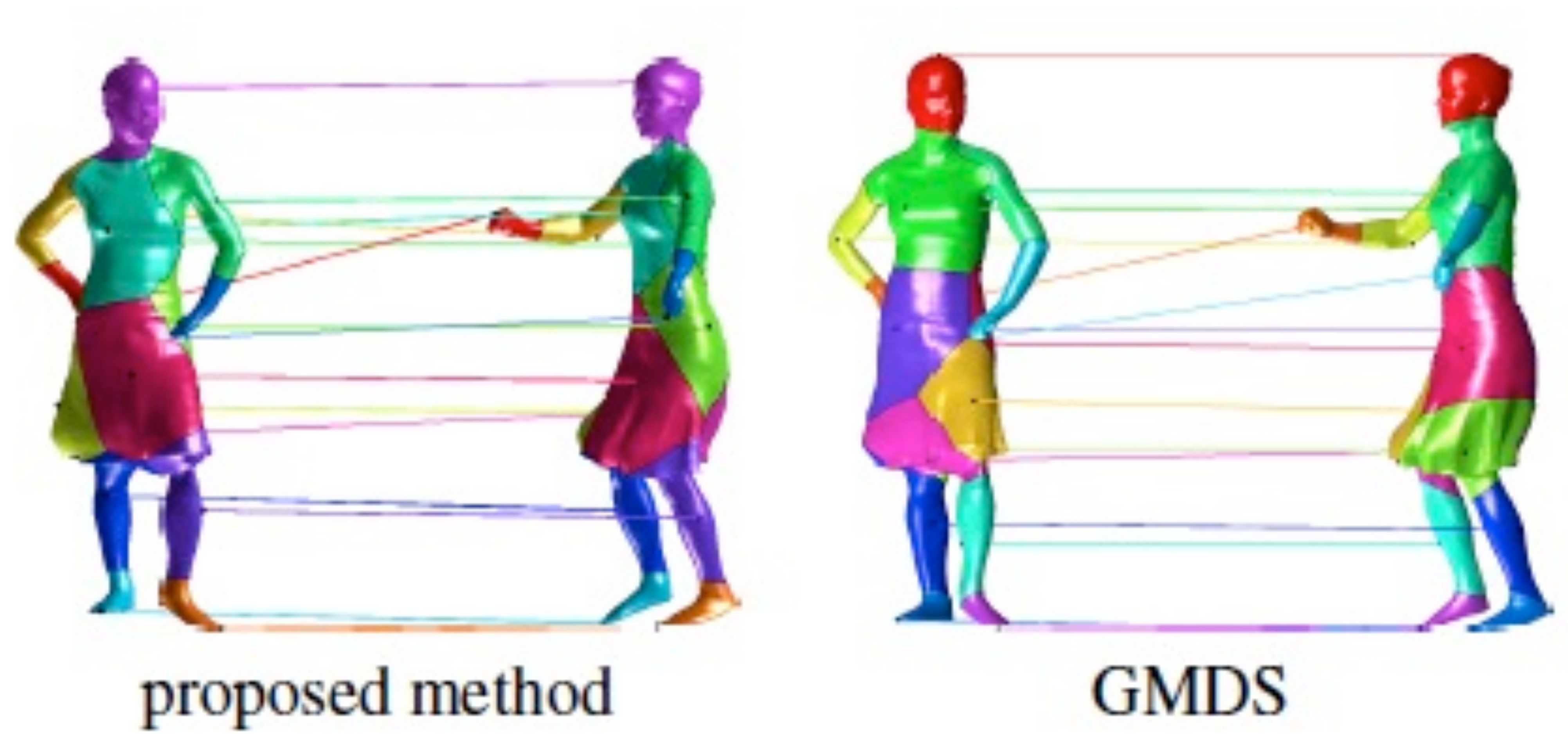
Handling missing parts

- Due to elasticity of the energy function
- Missing parts are shrinked



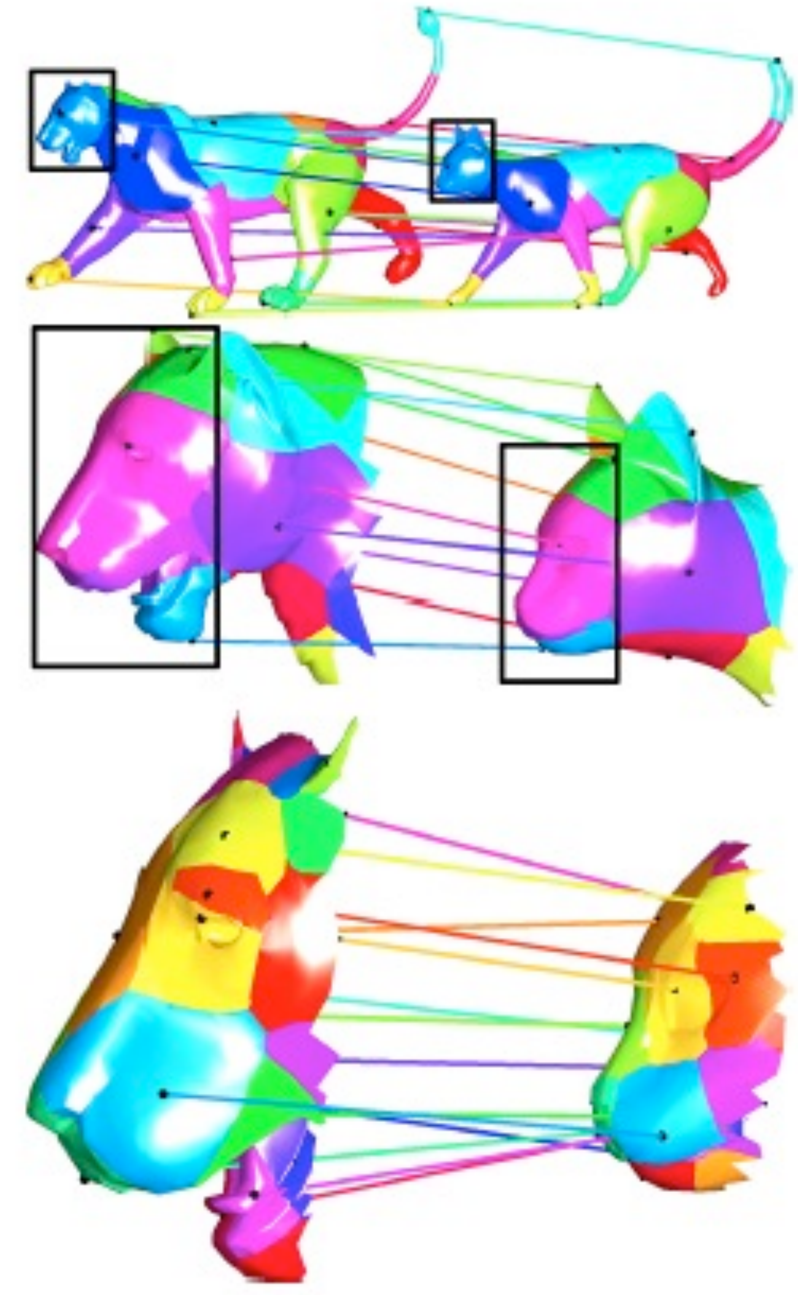
Preserve orientation

- Method guarantees to preserve the orientation



Multiresolution

- The number of variables is quadratic in the number of triangles
- Minimization of LP for more than 250 triangles is infeasible
- Multiresolution allows to handle more than 2000 tris



Summary

- Novel high-resolution 3D shape matching framework
- Globally optimal
- Preserves orientation
- Handles missing parts
- High performance

The end