Problem #99 (Solved !)

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> *Summary: Is the first-order theory of any* Knuth-Bendix order-
> ing *decidable?*

Is there an algorithm which, given a term signature Σ, a weight function w and a precedence $>>$, decides whether a first-order formula is valid in the term-algebra with the Knuth-Bendix ordering defined by $(w, >>)$?

Positive partial results have been given for

- the existential fragment [KV00, KV01];

- signatures consisting only of constants and unary function symbols [KV02a].

Remark

This has been answered in the affirmative [ZSM05].

Bibliography

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[KV02a]   Konstantin Korovin and Andrei Voronkov. The decidability of the first-order theory of the Knuth-Bendix orders in the case of unary signatures. In *Foundations of Software Technology and Theoretical Computer Science*, Lecture Notes in Computer Science, Kanpur, India, December 2002. Springer-Verlag. A preliminary report of the result appeared in [KV02b].

[KV02b]   Konstantin Korovin and Andrei Voronkov. The decidability of the first-order theory of the Knuth-Bendix orders in the case of unary signatures. In Christophe Ringeissen, Cesare Tinelli, Ralf Treinen, and Rakesh M. Verma, editors, *Proceedings of the 16th International Workshop on Unification (UNIF 2002)*, Technical Report 02-05, Department of Computer Science, University of Iowa, pages 45–46, Copenhagen, Denmark, July 2002.

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