Problem #90

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Summary: *Are* context unification *and* linear second order unification *decidable?*

Context unification and *linear second order unification* are closely related, they both generalize string unification (which is known to be decidable, [Mak77]) and are special cases of second order unification (which is know to be undecidable, [Gol81]).

Context unification ([Com91], [SS94]) is unification of first-order terms with context variables that range over terms with one hole. Linear Second Order Unification is second-order unification where the domain of functions is restricted to λ-terms with exactly one occurrence of any bound variable (there can be several bound variables in contrast to context unification allowing for just one hole) Applications are

- solving membership constraints in completion of contraint rewriting ([Com98a])
- solving constraints occurring in Distributive Unification (Problem #38, [SS97])
- Extended Critical Pairs in Bi-Rewriting Systems ([LA96])
- Semantics of ellipses in natural language ([NPR97])
- One-Step Rewriting constraints ([NPR97])

Some special cases have been solved:

- Hubert Comon [Com98b] solved a special case where any occurrence of the same context variable is always applied to the same term,
- Manfred Schmidt-Schauß [SS94] (see also [SS97]) solved the case of so-called *stratified* context unification, where for any occurrence of the same second-order variable the string of second-order variables from this occurrence to the root of the containing term is the same,
- Jordi Levy [Lev96] (see also [NPR97]) showed that linear-second order unification is decidable when any variable has at most two occurrences.

- Manfred Schmidt-Schauß and Klaus Schulz [SSS99a] showed that solvability is decidable for systems of context equations containing only two context variables (having an arbitrary number of occurrences in the system) and an arbitray number of first-order variables.

Progress towards a decidability proof along the lines of Makanin's proof for string-unification has been reported in [SSS98]. Levy and Villaret [LV00] show how to reduce linear second-order unification to context unification plus membership predicates in regular tree languages, and discuss a possible way of showing decidability of the latter. [LV02] shows that it is sufficient, both for linear 2nd-order and for context unification, to consider signatures consisting of an arbitrary number of constants and one binary function symbol.

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