Problem #88

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Summary: Is there a calculus of explicit substitution that is confluent on open terms, simulates one-step beta-reduction and preserves beta-strong normalization?

There are confluent calculi of explicit substitutions but these do not preserve termination (strong normalization) [CHL92, Mel95], and there are calculi that are not confluent on open terms but which do preserve termination [LRD94]. César Muñoz presented in [Mu n96] a calculus enjoying both properties (answering Problem #78), however, the calculus is not able to simulate one-step of beta-reduction: if a beta-reduces to b in the lambda-calculus then a does not necessarily reduce to b in the calculus of Muñoz. Is there a calculus of explicit substitution that is confluent on open terms, simulates one-step beta-reduction and preserves beta-strong normalization?

Comment sent by Jean Goubault-Larrecq

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This problem was solved positively in [GL99]. The calculus SKInT, introduced in [GGL00], is confluent on open terms and simulates one-step beta-reduction (although in a slightly contorted way, see [GGL00]; the obvious translation only simulates a bit more that one-step call-by-value beta-reduction). The paper [GL99] characterizes strongly normalizing, weakly normalizing and solvable terms through intersection types, and preservation of strong normalization follows. SKInT is also standardizing, has a terminating subcalculus of substitutions ΣT, but is based on an infinite signature and finitely many rule schemes parameterized by integers. Can we lift the latter restriction?

Bibliography

[CHL92] Pierre-Louis Curien, Thérèse Hardin, and Jean-Jacques Lévy. Confluence properties of weak and strong calculi of explicit substitutions. RR 1617, Institut National de Rechereche en Informatique et en Automatique, Rocquencourt, France, February 1992.

[GGL00] Healfdene Goguen and Jean Goubault-Larrecq. Sequent combinators: A Hilbert system for the lambda calculus. *Mathematical Structures in Computer Science*, 10:1–79, 2000.

[GL99] Jean Goubault-Larrecq. Conjunctive types and SKInT. In *Proceedings of the TYPES'98 Workshop*, Lecture Notes in Computer Science. Springer-Verlag, Bad Irsee, Germany, 1999.

[LRD94] Pierre Lescanne and J. Rouyer-Degli. The calculus of explicit substitutions $\lambda\upsilon$. Technical Report RR-2222, INRIA-Lorraine, January 1994.

[Mel95] Paul-André Melliès. Typed λ-calculi with explicit substitutions may not terminate. In *Typed Lambda Calculi and Applications*, 1995.

[Mu n96] César Mu noz. Confluence and preservation of strong normalisation in an explicit substitutions calculus (extended abstract). In Edmund C. Clarke, editor, *Eleventh Symposium on Logic in Computer Science*, pages 440–447, New Brunswick, NJ, July 1996. IEEE.

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