Problem #79

Originator: Mizuhito Ogawa
Date: April 1995

Summary: Does a system that is nonoverlapping under unification with infinite terms have unique normal forms?

Does a system that is nonoverlapping under unification with infinite terms (unification without "occur-check" [MR84]) have unique normal forms? This conjecture was originally proposed in [OO89] with an incomplete proof, as an extension of the result on strongly nonoverlapping systems [Klo80][Che81]. Related results appear in [OO93][TO94][MO94], but the original conjecture is still open. This is related to Problem 58. This problem is also related with modularity of confluence of systems sharing constructors, see [Ohl94].

Remark

The answer is yes if the system is also nonduplicating [Ver96]. A direct technique is given in [Ver96]. The nonduplicating condition can be relaxed under a certain technical condition [Ver96]. Some extensions to handle root overlaps are given in [Ver97] and a restricted version of the result in [Che81] is also proved using the direct technique in [Ver97].

Bibliography

[Che81] Paul Chew. Unique normal forms in term rewriting systems with repeated variables. In *Proceedings of the Thirteenth Annual Symposium on Theory of Computing*, pages 7–18. ACM, 1981.

[Klo80] Jan Willem Klop. *Combinatory Reduction Systems*, volume 127 of *Mathematical Centre Tracts*. Mathematisch Centrum, Amsterdam, 1980.

[MO94] K. Mano and M. Ogawa. A new proof of Chew's theorem. Technical report, IPSJ PRG94-19-7, 1994.

[MR84] A. Martelli and G. Rossi. Efficient unification with infinite terms in logic programming. In *International conference on fifth generation computer systems*, pages 202–209, 1984.

[Ohl94] Enno Ohlebusch. On the modularity of confluence of constructor-sharing term rewriting systems. In Sophie Tison, editor, *Colloquium on Trees in Algebra and Programming*, volume 787 of *Lecture Notes in Computer Science*, Edinburgh, Scotland, 1994. Springer-Verlag.

[OO89] M. Ogawa and S. Ono. On the uniquely converging property of nonlinear term rewriting systems. Technical report, IEICE COMP89-7, 1989.

[OO93] Michio Oyamaguchi and Yoshikatsu Ohta. On the confluent property of right-ground term rewriting systems. *Trans. IEICE*, J76-D-I:39–45, 1993.

[TO94] Yoshihito Toyama and Michio Oyamaguchi. Church-Rosser property and unique normal form property of non-duplicating term rewriting systems. In Nachum Dershowitz and N. Lindenstrauss, editors, *Proceedings of the Fourth International Workshop on Conditional Rewriting Systems*, Jerusalem, Israel, July 1994. Springer-Verlag.

[Ver96] Rakesh M. Verma. Unicity and modularity of confluence for term rewriting systems. Technical report, University of Houston, 1996.

[Ver97] Rakesh M. Verma. Unique normal forms for nonlinear term rewriting systems: Root overlaps. In *Symp. on Fundamentals of Computation Theory*, volume 1279 of *Lecture Notes in Computer Science*, pages 452–462. Springer-Verlag, September 1997.