

## Problem #59

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*Summary: What are sufficient condition for the modularity of confluence?*

One of the earliest results established on modularity of combinations of term-rewriting systems is the confluence of the union of two confluent systems which share no symbols [Toy87]; if symbols are shared modularity is not preserved by union [KO92]. Some sufficient conditions for modularity of confluence of constructor-sharing systems that are terminating have been found [KO92][MT91]. Are there interesting sufficient conditions that are independent of termination?

### Remark

Left-linearity is a sufficient condition, as shown long ago in [RV80]. In [Ohl94], it is established that confluence is modular in the presence of the weak normalization property. (This result has been extended in [Rao95, Rao98] for hierarchical combinations.) In [Der97], some results are given when only one of the systems is terminating.

There are other sufficient conditions for modularity of confluence that do not require termination of the combined system even when function symbols are shared. One set of conditions, viz., “persistence”, “relative termination”, and *lr*-disjointness, is given in [Ver95, Ver96]. An abstract confluence theorem without termination is given in [Ges90].

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