

Problem #24

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Summary: Is satisfiability of lpo or rpo ordering constraints decidable in case of non-total precedences?

The existential fragment of the first-order theory of the “recursive path ordering” (with multiset and lexicographic “status”) is decidable when the precedence on function symbols is total [Com90, JO91], but is undecidable for arbitrary formulas. Is the existential fragment decidable for partial precedences?

Remark

The Σ_4 ($\exists^*\forall^*\exists^*\forall^*$) fragment is undecidable, in general [Tre92]. The positive existential fragment for the empty precedence (that is, for homeomorphic tree embedding) is decidable [BC93]. One might also ask whether the first-order theory of *total* recursive path orderings is decidable. Related results include the following: The existential fragment of the subterm ordering is decidable, but its Σ_2 ($\exists^*\forall^*$) fragment is not [Ven87]. The first-order theory of encompassment (the instance-of-subterm relation) is decidable [CCD93]. The satisfiability problem for the existential fragment in the total case is NP-complete [Nie93].

Though the first-order theory of encompassment is decidable [CCD93], the first-order (Σ_2) theory of the recursive (lexicographic status) path ordering, assuming certain simple conditions on the precedence, is not [CT97].

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