Problem #21

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Summary: Is termination of one linear rule decidable?

Is termination of one linear (left and right) rule decidable? Left linearity alone is not enough for decidability [Dau89].

Remark

A less ambitious, long-standing open problem (mentioned in [DJ90]) is decidability for *one* (length-increasing) monadic (string, semi-Thue) rule. Termination is undecidable for non-length-increasing monadic systems of rules [Car91]. For one monadic rule, confluence is decidable [Kur90][Wra90]. What about confluence of one non-monadic rule?

Partial results for string rewrite rules have been obtained in [Ges03].

The history of the problem and the attempts to solve it are told in [Der05].

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