Problem #20 (Solved !)

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Summary: What is the best bound on the length of a derivation for a one-rule length-preserving string-rewriting system?

What is the best bound on the length of a derivation for a one-rule length-preserving string-rewriting (semi-Thue) system? Is it $O(n^2)$ (n is the size of the initial term) as conjectured in [Mét85], or $O(n^k)$ (k is the size of the rule) as proved there.

Remark

The upper bound is $n^2/4$ where n denotes the length of the initiating string [Ber94]. The bound is reached by the derivation from $b^{n/2}a^{n/2}$ for the string rewriting system $\{ba \to ab\}$.

More about the history of this problem in the context of the question of one-rule termination can be found in [Der05].

Bibliography

[Ber94]   A. Bertrand. Sur une conjecture d'Yves Métivier. *Theoretical Computer Science*, 123(1):21–30, 1994.

[Der05]   Nachum Dershowitz. Open. Closed. Open. In Jürgen Giesl, editor, *16th International Conference on Rewriting Techniques*, volume 3467 of *Lecture Notes in Computer Science*, Nara, Japan, April 2005. Springer-Verlag.

[Mét85]   Yves Métivier. Calcul de longueurs de chaînes de réécriture dans le monoïde libre. *Theoretical Computer Science*, 35(1):71–87, January 1985.

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