

Problem #16

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Summary: Under what conditions does confluence of a normal semi-equational conditional term rewriting system imply confluence of the associated oriented system?

For a *normal* conditional term-rewriting system $R = \{s \rightarrow^! t \Rightarrow l \rightarrow r\}$, where t must be a ground normal form of s , we can consider the corresponding semi-equational conditional rewrite system $R_{se} = \{s \leftrightarrow^* t \Rightarrow l \rightarrow r\}$. Under what conditions does confluence of R_{se} imply confluence of R ? In general, this is not the case, as can be seen from the following non-confluent system R (due to Aart Middeldorp):

$$\begin{array}{l} a \rightarrow b \\ a \rightarrow c \\ b \rightarrow^! c \Rightarrow b \rightarrow c \end{array}$$

Remark

Solutions have been provided by [YASM00]. They show that confluence of R follows from confluence of R_{se} if any of the two following conditions is satisfied:

- R_{se} is semi-decreasing
- R_{se} is level-confluent

See [YASM00] for definitions of these properties.

Bibliography

- [YASM00] Toshiyuki Yamada, Jürgen Avenhaus, Carlos Loría Sáenz, and Aart Middeldorp. Logicality of conditional rewrite systems. *Theoretical Computer Science*, 236(1–2):209–232, April 2000.