

Problem #13

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Summary: Give decidable criteria for left-linear rewriting systems to be Church-Rosser.

By a lemma of Gérard Huet [Hue80], left-linear term-rewriting systems are confluent if, for every critical pair $t \approx s$ (where $t = u[r\sigma] \leftarrow u[l\sigma] = g\tau \rightarrow d\tau = s$, for some rules $l \rightarrow r$ and $g \rightarrow d$), we have $t \rightarrow^{\parallel} s$ (t reduces in one parallel step to s). (The condition $t \rightarrow^{\parallel} s$ can be relaxed to $t \rightarrow^{\parallel} r \leftarrow^{\parallel} s$ for some r when the critical pair is generated from two rules overlapping at the roots; see [Toy88].) What if $s \rightarrow^{\parallel} t$ for every critical pair $t \approx s$? What if for every $t \approx s$ we have $s \rightarrow^= t$? (Here $\rightarrow^=$ is the reflexive closure of \rightarrow .) What if for every critical pair $t \approx s$, either $s \rightarrow^= t$ or $t \rightarrow^= s$? In the last case, especially, a confluence proof would be interesting; one would then have confluence after critical-pair completion without regard for termination. If these conditions are insufficient, the counterexamples will have to be (besides left-linear) non-right-linear, non-terminating, and non-orthogonal (have critical pairs). See [Klo92].

Remark

Significant progress is reported in [OO97].

A new criterion based on so-called *simultaneous critical pairs* has been presented in [Oku98].

The history of the problem and the attempts to solve it are told in [Der05].

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